


Examination seating optimization using graph coloring and combinatorial design

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ABSTRACT

This paper presents an optimization approach for exam seating in universities with limited infrastructure, based on a mixed-course allocation model. Students in different courses share rooms while maintaining spatial separation to improve academic integrity. The model incorporates a theoretical probability of interaction, which decreases as the number of mixed courses in a room increases.

Using real data with 5175 students, the proposed model significantly improves upon the traditional method. Although the traditional approach required 35 rooms with a total capacity of 7269, the proposed model utilized only 12 large rooms, leaving 23 rooms unused and saving about 2475 seats. The unused space within the occupied rooms was minimal (approximately 29 seats), indicating near-optimal utilization.

The invigilation requirement was reduced from at least 70 to 36, achieving nearly 50% savings. Small-enrollment and carryover courses are efficiently integrated and sorted. The model is formulated using graph coloring and combinatorial optimization, supported by a simple allocation algorithm.

1. Introduction

The organization of examination seating arrangements plays an important role in maintaining academic integrity in educational institutions [1,2]. When students taking the same course are seated close to one another during examinations, the likelihood of academic dishonesty, such as copying answers or exchanging information, increases [3]. Consequently, universities often seek systematic methods to allocate seats in a manner that minimizes such risks [4].

From a mathematical perspective, the exam seating problem can be viewed as a constrained allocation problem in which students of different courses must be distributed among available seats while avoiding undesirable adjacencies [5]. This naturally leads to models based on concepts from graph theory and combinatorial design. In particular, graph coloring techniques provide a framework in which seats are represented as vertices of a graph and courses are represented as coloring assigned to these vertices under adjacency constraints [6].

In addition, combinatorial design structures offer systematic ways of constructing seating patterns that evenly distribute course assignments between rows and columns of an examination room [7]. Such structured arrangements can ensure that not only neighboring students are assigned different examinations while maintaining a balanced and efficient use of available seating capacity but also ease invigilation.

Motivated by these ideas, this paper investigates exam seating optimization using graph coloring and combinatorial design principles. The objective is to develop structured seating patterns that minimize adjacent placements of students taking the same examination while making effective use of the existing classroom infrastructure.

2. Background

In the last three decades, higher education in Sub-Saharan Africa has expanded considerably, of which East Africa is not an exception [8]. Tanzania provides a clear example of this growth, where the number of universities increased from fewer than 10 before 2000 to at least 49 universities accredited by TCU in 2025 [9]. This expansion reflects national and regional efforts to expand access to tertiary education and respond to the growing demand for university training [10,11]. Although universities have generally managed to accommodate increased enrollment during regular teaching sessions, the institutional infrastructure has not always expanded at the same pace, particularly with respect to the facilities required for student assessment [12]. Otherwise, universities may produce poor quality education graduates, which may not be able to produce quality research activities [13].

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Fig. 1. Exam hall seating arrangement in alternating chair colors representing different courses.

This challenge has been intensified by larger educational reforms that expanded access to secondary education, thus increasing the pool of students eligible for university admission [14]. As secondary education becomes more widely accessible, the transition rate to tertiary education continues to increase, placing additional pressure on university facilities and examination management systems [15]. Consequently, higher education institutions face increasing logistical challenges in administering examinations for large student populations [16].

Although demand for classroom facilities may be bearable during normal classroom learning, many universities in sub-Saharan Africa struggle to serve students during assessments such as quizzes, tests, and exams [17,18]. Thus, during exams, students registered for the same course are often seated close to each other due to space limitations [19]. Such arrangements may increase the likelihood of academic dishonesty, including copying answers or exchanging information during exams.

One of the possible responses to address these challenges would be to expand the examination infrastructure by building additional classrooms. However, infrastructure expansion requires substantial financial investment and may not be imminently feasible for many public universities operating with limited budgets [20]. Thus, it is cheaper to optimize the use of existing classroom space through structured seating arrangements. By mixing students from different courses in one examination room and applying systematic seating patterns (Fig. 1), institutions can reduce opportunities for academic malpractice while maintaining efficient use of available resources.

Such approaches motivate the development of algorithmic or optimization-based methods to support effective exam seating allocation.

2.1. Literature review

Efficient scheduling in higher education is crucial for optimizing resources and maintaining academic integrity. Various optimization techniques, including mathematical models and algorithms, are employed to address the complexities of exam scheduling and resource allocation. These methods aim to balance multiple criteria such as faculty availability, room allocation, and student preferences, ultimately enhancing the educational process. The following sections delve into specific aspects of these techniques and their applications.

2.1.1. Schedules of student assessment in higher education

Scheduling student assessments in higher education is a complex optimization problem involving multiple constraints such as room ca-

capacity, timetable coordination, invigilation allocation, and fairness in student distribution [21]. As a result, various mathematical and computational techniques have been developed to automate exam scheduling systems, reducing manual errors and improving institutional efficiency [22,23]. Mathematical models are particularly important in balancing constraints such as teacher availability, student satisfaction, and the optimal use of resources.

Optimization techniques, especially heuristic and graph-theoretic approaches, are widely used in scheduling and seating allocation [23]. Heuristic algorithms generate near-optimal solutions through iterative improvement under multiple constraints, particularly when exact solutions are computationally expensive [24]. In examination seating, such methods aim to reduce interaction among students by separating candidates from the same or related courses while still improving room utilization and invigilation efficiency [25]. Graph theory is applied through conflict matrices to identify incompatible courses and to group those that can be scheduled simultaneously, thereby minimizing clashes and optimizing room usage [26]. However, these approaches remain largely iterative in nature and do not impose a strict deterministic structure on student separation [23].

Operations research methods are also applied to classroom and seating allocation, focusing on the efficient assignment of rooms and schedules under constraints such as capacity and time availability [27]. These approaches improve resource utilization but often prioritize optimization efficiency over strict control of student interaction patterns [28]. Although such systems are effective in administrative planning, they may still allow for residual probabilities of interaction among students from the same course, depending on the final optimized configuration [29].

In contrast, the approach in this study adopts a structured cyclic allocation strategy, where courses are deliberately mixed within examination rooms to directly control and minimize the interaction probability among candidates of the same course. Unlike heuristic and data-driven optimization methods, which rely on iterative search and computational refinement [30], this method is based on a deterministic combinatorial structure. This clearly differentiates it from existing work such as Patil et al. [23], where the focus is primarily on algorithmic optimization and data-driven allocation, rather than explicit structural control of student interaction.

2.1.2. Literature on examination seating plans

Although extensive research has been conducted on examination timetabling (allocation of exams to rooms and time slots), relatively

little attention has been given to the design of seating plans within examination rooms [31,32]. This within-room seating structure is crucial, as it directly influences the potential for academic dishonesty.

Pomales-Garcia et al. [33] incorporated spatial separation by adjusting inter-student distances to reduce cheating opportunities within a room. Similarly, Kashyap et al. [28] applied a genetic algorithm supported by heuristics to construct seating plans aimed at minimizing malpractice.

2.1.3. Graph coloring and combinatorial optimization approaches

Optimizing examination scheduling and seating has been widely studied using more graph coloring than combinatorial design techniques [34]. These approaches apply mathematical and heuristic strategies to allocate exams to time slots and rooms while minimizing possible candidate interactions and resource usage. In this approach, examination courses are modeled as vertices, and possible candidate interactions between exams as edges, with the objective of assigning different colors to adjacent vertices to prevent scheduling conflicts (possible candidate interactions) [35].

Graph coloring has been extensively applied to examination timetabling to ensure that exams with common students are not scheduled simultaneously. This approach has been shown to effectively reduce scheduling conflicts and improve resource utilization [36]. For example, the application of graph coloring heuristics at the Universiti Utara Malaysia demonstrated improved performance compared to manual scheduling methods while accommodating constraints such as lecturer preferences and room capacities [37]. Furthermore, hybrid heuristic approaches that combine graph coloring with optimization techniques such as simulated annealing have been reported to reduce the total number of time slots required while improving the quality of the solution [38].

Complementary to graph coloring, combinatorial design, and heuristic techniques have been explored to further enhance scheduling efficiency. Strategies such as adaptive decomposition and exam ordering enable the categorization of examinations into difficult and easy sets, allowing harder-to-place exams to be scheduled earlier in the process [37]. Additional heuristic methods, including domain transformation and slot permutation, have also been used to refine initial feasible schedules generated through graph coloring [39].

On the other hand, this paper introduces a deterministic graph-theoretical and combinatorial approach for designing examination seating arrangements within rooms that goes beyond examination scheduling and allocation techniques alone and even extends beyond the use of heuristic algorithms. The technique uses structural conditions in discrete mathematics to minimize undesired proximity between students while also considering factors like room arrangement, distribution of candidates, and supervisory requirements in ordinary classrooms [31, 32]. As most of the literature has primarily concentrated on examination scheduling and allocation, very little research has been done concerning the design of optimal examination seating arrangements. Consequently, the key contribution of this paper lies in the creation of a deterministic model for designing examination seating plans based on graph theory and combinatorics.

3. Methodology

This study approaches the examination seating allocation problem through a structured mathematical framework that combines graph coloring and combinatorial design principles. The methodology models the examination hall as a seat grid and formulates seat assignment as an optimization problem subject to operational constraints. The objective is to construct seating patterns that minimize the likelihood of academic dishonesty while making efficient use of existing examination infrastructure.

3.1. Core objective

The central objective is to arrange students so that no two adjacent students are writing the same examination. This minimizes opportunities for sharing answers while still allowing the university to use the same classroom capacity used during regular teaching. In so doing, the seating space is optimized, including minimizing the number of examination sessions.

3.1.1. Optimization objectives

The proposed seating allocation system aims to optimize examination management according to several operational objectives.

a. Minimizing interaction among students writing the same course

Students registered for the same examination should be spatially separated so that adjacent seats contain students writing different examinations. This reduces opportunities for communication or copying during the examination.

To formalize this idea, the examination room is represented as a rectangular seating grid.

A basic implementation of the proposed system uses alternating seating patterns. Suppose that two courses *A* and *B* share the same examination room. As illustrated in Fig. 2, students from the two courses are arranged in an alternating pattern to minimize adjacency of students writing the same examination.

i. Alternating Seating Arrangement

ii. Multi-Course Seating Extension

In large examination halls, more than two courses may be seated simultaneously. Let *A*, *B*, *C*, and *D* represent four different courses. The cyclic structure shown in Fig. 3 distributes students from each course evenly across the examination hall.

b. Reducing examination time through efficient space utilization

By mixing students from multiple courses within the same examination room, available seating capacity can be fully utilized. This prevents situations where partially filled rooms are assigned to single courses and therefore reduces the number of examination sessions.

c. Minimizing the number of invigilators required

When multiple courses share a room, invigilation responsibilities can be organized through supervision zones. Because the seating arrangement itself discourages collaboration between students writing the same examination, fewer invigilators can effectively monitor a larger group of students. As illustrated in Fig. 4, the examination hall can be divided into supervision zones assigned to different invigilators.

Each zone is supervised by the invigilator responsible for a particular course. Although students from multiple courses appear in each zone, the responsible invigilator focuses on attendance verification and exam-related support for their course.

d. Accommodating carryover or retake students

Carryover students are those who have progressed to a higher academic level but must repeat one or more courses from a previous level after failing them in an earlier examination session. Consequently, these students may be registered simultaneously for examinations belonging to two academic levels: their current level and a previous level.

In many universities, examination timetables are prepared independently for each academic level under the assumption that students belong to only one level. As a result, examinations from different levels may be scheduled at the same time. While this normally creates no conflict, problems arise for carryover students because they may be required to sit two examinations that occur simultaneously.

Table 1
Proposed rotation of academic levels across examination sessions.

Examination session	Academic level 1	Academic level 2
Session 1	First Year	Third Year
Session 2	Second Year	Fourth Year

B A B A B A B A B A
 A B A B A B A B A B
 B A B A B A B A B A
 A B A B A B A B A B

Fig. 2. Alternating seating arrangement for two courses in an examination hall.

A B C D A B C D
 B C D A B C D A
 C D A B C D A B
 D A B C D A B C

Fig. 3. Cyclic seating pattern for four courses sharing an examination hall.

Zone 1 (Inv. A)	Zone 2 (Inv. B)
B A B	A B A
A B A	B A B
B A B	A B A
A B A	B A B

Fig. 4. Division of an examination hall into invigilator supervision zones.

To reduce this problem, the examination timetable can be structured so that **consecutive academic levels do not write examinations at the same time**. Since carryover students most frequently repeat courses from an immediate preceded level, separating consecutive levels greatly reduces the likelihood of timetable conflicts for carry over students.

A practical scheduling rule is therefore to pair examination sessions between **non-consecutive levels**. For example:

- First-year students should write exams at the same time as third-year students.
- Second-year students should write exams at the same time as fourth-year students (Table 1), and so on.

Notable: Writing examinations may be grouped into three different levels if spaces are available as long as there is gap between levels to just ensure that carry over students issues are addressed. For example: First-year students may write examinations at the same time as third-year and fifth year students.

This combined strategy of separating consecutive levels in the timetable and using mixed-course seating within examination halls provides an efficient solution for accommodating carryover students while preserving examination integrity and minimizing administrative complexity.

3.2. Graph theory and seating representation

From a mathematical point of view, the seating allocation problem resembles a *graph coloring problem*. In this interpretation:

- The Seats represent nodes in a graph.
- The adjacent seats represent edges.
- The courses represent colors.

The objective is to assign colors such that adjacent nodes do not share the same color.

This representation allows the seating arrangement problem to be analyzed using well-established techniques from graph theory, particularly graph-coloring algorithms designed to avoid conflicts between neighboring nodes.

3.3. Combinatorial design structure

The seating arrangement also exhibits structures commonly studied in combinatorial design theory. In particular, the cyclic arrangement of the course labels resembles a Latin-square-type design in which symbols are distributed systematically across rows and columns.

Such structured designs ensure balanced placement of courses across the examination hall while preventing undesirable adjacency of identical examinations. Using cyclic or repeating seating patterns, the system can efficiently distribute students from different courses throughout the room while maintaining the required separation between identical examinations. This can simplify invigilation.

3.4. Modeling the interaction probability

In this study, interaction is modeled through structural adjacency in a graph-based seating arrangement, where vertices represent students and edges represent possible interaction paths between nearby seats.

3.4.1. Probability of interaction

The interaction probability P_k denotes the allowable probability that a candidate may interact dishonestly during an examination, considering only the seating arrangement. The model assumes no candidate movement, no strict invigilation, and no similarity between different examination courses.

It is assumed that the interaction probability is inversely proportional to the average number of separating graph edges between a student and the nearest students taking the same examination. Thus,

$$P_k \propto \frac{1}{n(E)} \tag{1}$$

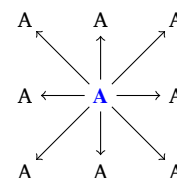
where $n(E)$ denotes the average number of separating graph edges between a student and the nearest same-course students. Introducing a proportionality constant κ gives $P_k = \frac{\kappa}{n(E)}$. For a room containing only one course ($k = 1$), the nearest same-course student is separated by one edge only, so that $n(E) = 1$.

Assuming maximal interaction probability in this case, $P_1 = 1$. Hence, $1 = \frac{\kappa}{1}$, implying $\kappa = 1$. Therefore,

$$P_k = \frac{1}{n(E)} \tag{2}$$

The quantity $n(E)$ is evaluated from the seating graph. For one course ($k = 1$),

A A A
 A A A
 A A A



The central candidate has eight nearest same-course neighbors, each separated by one edge. Thus, $n(E) = \frac{1+1+1+1+1+1+1+1}{8} = 1$, and therefore, $P_1 = 1$. For two courses ($k = 2$), $n(E) = \frac{2+2+2+2+1+1+1+1}{8} = \frac{3}{2}$, giving $P_2 = \frac{2}{3}$. For three courses ($k = 3$), $n(E) = \frac{3+3+3+3+1+1+1+1}{8} = 2$. Thus, $P_3 = \frac{1}{2}$. From the observed pattern, $n(E) = \frac{k+1}{2}$, where $k \geq 1$ denotes the number of courses assigned to a room. Hence,

$$P_k = \frac{1}{(k+1)/2} = \frac{2}{k+1}, \quad k \geq 1. \tag{3}$$

Furthermore,

$$\lim_{k \rightarrow \infty} P_k = 0, \tag{4}$$

Showing that interaction probability decreases as the number of courses sharing the examination room increases.

Theorem 1. *Let P_n denote the probability of interaction between candidates of the same course in an examination room under a cyclic seating arrangement involving n mixed courses. Then*

$$P_n = \frac{2}{n+1}, \quad n \geq 1.$$

Proof. We prove the result by mathematical induction on n .

Base Case: For $n = 1$,

$$P_1 = \frac{2}{1+1} = \frac{2}{2} = 1.$$

Thus, the formula holds for $n = 1$.

Inductive Hypothesis: Assume that for some positive integer $k \geq 1$,

$$P_k = \frac{2}{k+1}.$$

Inductive Step: We show that the formula also holds for $n = k + 1$. Starting from the proposed formula,

$$P_{k+1} = \frac{2}{(k+1)+1}.$$

Simplifying,

$$P_{k+1} = \frac{2}{k+2}.$$

Observe that increasing the number of mixed courses from k to $k + 1$ increases the spacing diversity among candidates, thereby reducing the interaction probability according to the same structural rule established for P_k . Hence, the same proportional relationship remains valid.

Therefore,

$$P_{k+1} = \frac{2}{k+2},$$

which is exactly the required form.

Hence, by the principle of mathematical induction,

$$P_n = \frac{2}{n+1}$$

for all integers $n \geq 1$. \square

3.5. Model formulation for optimization of objectives

The proposed examination seating system aims to simultaneously optimize several operational objectives in the examination management process.

3.5.1. Decision variables and parameters

To formulate the examination seating optimization model, the following decision variables and parameters are defined:

- $r \in R$ = one of a room from a set of rooms, R
- $x_{c,r}$ = number of students of course c assigned to room r
- S_{ij} = course assigned to seat position (i, j)
- k_r = number of courses assigned to room r
- Inv_r = number of invigilators assigned to room r
- Cap_r = seating capacity of room r
- C^r = set of courses assigned to room r
- $|C^r|$ = k_r
- P_r = probability of interaction in room r
- P = sum of interaction probabilities across all examination rooms
- f_1 = total number of examination sessions required
- f_2 = total number of invigilators required
- N = total number of students scheduled for examinations

The probability of interaction between students taking the same examination in room r is defined by

$$P_r = \frac{2}{k_r + 1}, \quad k_r \geq 1 \quad \text{from theorem 1}$$

The overall interaction probability, therefore, is given by

$$P = \sum_{r \in R^*} P_r.$$

3.5.2. Objective function

The proposed examination seating model aims to minimize interaction among students taking the same examination while simultaneously reducing the number of examination sessions and invigilators required. The optimization objectives are aggregated into a weighted objective function given by

$$\min Z(P, f_1, f_2) = w_0 P + w_1 f_1 + w_2 f_2$$

where

$$w_0 + w_1 + w_2 = 1, \quad 0 \leq w_i \leq 1,$$

and

$$w_0, w_1, w_2 = \text{weights assigned to the respective objectives}$$

The objective values are normalized before aggregation to ensure comparability among the different optimization criteria. The weights may be adjusted according to institutional priorities such as examination security, operational efficiency, and cost minimization.

3.5.3. Model constraints

The optimization model is subject to operational and physical constraints associated with examination management and seating allocation.

Invigilator Constraint

The number of invigilators assigned to each examination room must be equal to the number of courses allocated to that room:

$$Inv_r = k_r, \quad \forall r \in R.$$

The total number of invigilators required is therefore given by

$$f_2 = \sum_{r \in R^*} k_r,$$

where

$$k_r \in \{2, 3, 4, \dots\}.$$

Seating Capacity Constraint

The number of students assigned to a room must not exceed its seating capacity:

$$\sum_{c \in C^r} x_{c,r} \leq Cap_r, \quad \forall r \in R.$$

Student Allocation Constraint

All students scheduled for examinations must be assigned to examination rooms:

$$\sum_{r \in R^*} \sum_{c \in C^r} x_{c,r} = N.$$

Adjacency Constraint

Students taking the same examination must not occupy horizontally or vertically adjacent seats:

$$S_{ij} \neq S_{i,j+1},$$

$$S_{ij} \neq S_{i+1,j}.$$

Cyclic Seat Allocation Constraint

Seat assignments must follow the cyclic allocation pattern

$$S_{ij} = C_{((i+j-2) \bmod k_r)+1}^r,$$

where C_m^r denotes the m th course in the ordered set of courses assigned to room r . This arrangement ensures a uniform distribution of courses throughout the room and minimizes the clustering of students taking the same examination.

4. Simulation of examination seating optimization

4.1. Algorithm for automatic seat generation

To implement the proposed seating strategy efficiently, a computational algorithm can be used to automatically generate seating arrangements while minimizing the number of examination sessions and the number of invigilators required.

The algorithm determines:

- How courses should be grouped within examination sessions,
- How students should be distributed across available rooms,
- The seating coordinates of each student,
- The minimum number of invigilators required.

4.1.1. Required input data

The algorithm requires the following input information:

- List of courses to be examined;
- Number of students registered in each course;
- Available examination rooms;
- Seating capacity of each room;
- Dimensions of the seating grid ($L \times W$) for each room

4.1.2. Room allocation strategy

The algorithm determines the smallest number of rooms $R^* \subseteq R$ such that

$$\sum_{r \in R^*} Cap_r \geq N$$

$$\begin{aligned} N &= \text{total number of students in a given examination session} \\ Cap_r &= \text{seating capacity of examination room } r \end{aligned}$$

Using the minimum number of rooms contributes to reducing the total number of invigilators required.

4.1.3. Minimum invigilator requirement

Let k_r denote the number of courses assigned to room r , where $k_r \in \{2, 3, 4, 5\}$.

for simulation purposes. Since each course in a room requires dedicated supervision, the minimum number of invigilators required in room r is equal to the number of courses assigned to that room: $Inv_r = k_r$. Therefore, the total number of invigilators required in all rooms is $Inv_{total} = \sum_{r \in R^*} k_r$

subject to the constraint that each room must have at least two invigilators:

$$k_r \geq 2, \quad \forall r \in R^*.$$

By minimizing both the number of rooms and the number of courses assigned per room (where feasible), the system achieves an overall reduction in the requirements of the invigilation.

4.1.4. Seat matrix generation

Each examination room is represented as a grid of seats:

$$Room = \{(i, j) \mid i = 1, \dots, L_r, j = 1, \dots, W_r\}$$

Let the ordered set of available courses be defined as $C = \{A, B, C, D, E\}$.

Let k denote the number of courses assigned to a given examination room, where

$$k_r \in \{2, 3, 4, 5\}.$$

For a given room, the active set of courses is taken as the first k elements:

$$C^r = \{A, B, \dots, C_k\}.$$

Then a cyclic seating pattern is constructed by repeating the ordered sequence

$$A, B, \dots, C_k$$

across the seating grid. Formally, the course assigned to seat (i, j) is given by

$$S_{ij} = C_{((i+j-2) \bmod k_r)+1}^r$$

This formulation allows each examination room to accommodate 2, 3, 4, or 5 courses while ensuring that adjacent seats contain different course labels and that students are distributed as uniformly as possible throughout the room.

4.1.5. Student allocation

Students in each course are assigned sequentially to seats labeled with their respective course code until all students have been seated.

4.2. Algorithm pseudo-code

- Step 1: Input course enrollments $\{N_c\}$ and room capacities $\{Cap_r\}$.
- Step 2: Compute total number of students: $N = \sum_c N_c$
- Step 3: Sort available rooms in decreasing order of capacity.
- Step 4: Select the smallest subset of rooms $R^* \subseteq R$ such that $\sum_{r \in R^*} Cap_r \geq N$
- Step 5: Initialize total invigilators: $Inv_{total} = 0$
- Step 6: For each room $r \in R^*$:

- Determine number of courses to assign: $k_r \in \{2, 3, 4, 5\}$ (based on enrollment balance and room capacity)
- Select course subset: $C^r = \{A, B, \dots\}$
- Generate seating grid: $(i, j), \quad i = 1, \dots, L_r, j = 1, \dots, W_r$
- Generate cyclic seating pattern: $S_{ij} = C_{((i+j-2) \bmod k_r)+1}^r$
- Assign students to seats ensuring approximately equal distribution per course.
- Assign invigilators: $Inv_r = k_r$
- Update total invigilators: $Inv_{total} = Inv_{total} + k_r$

Step 7: Output:

- seating plan for each room
- student-to-seat allocation list
- total invigilators required: $Inv_{total} = \sum_{r \in R^*} k_r$

The algorithm can be implemented using spreadsheet software or programming languages such as Python or MATLAB to automatically generate seating charts for each examination session.

Table 2
Courses scheduled in a single exam session under the traditional method.

Course	Students	Course	Students	Course	Students
AA 8307	80	AS 8321	45	CE 8302	385
AA 8324	62	BM 6122	35	CS 6305	34
AS 6129	45	BM 8341	105	CS 6371	47
AS 6301	62	CE 6302	198	DS 6106	55
AS 6316	12	CE 6324	25	DS 6117	5
EE 6308	124	EE 8305	357	ET 8303	82
GM 6304	32	HS 6106	90	HS 6114	200
HS 6116	392	HS 6141	557	IT 6312	38
IT 6324	35	IT 8301	114	MB 6122	5
ME 6301	112	ME 6319	42	ME 8301	308
MIS 6125	89	MS 6121	617	MT 6303	185
NS 8354	150	NSCH 3118	78	NSLTO 8106	190
NSPH 3115	106	ST 8122	77		
Total					5175

4.2.1. Administrative implementation

Before the students are allowed into the examination hall, the exam papers could be laid directly on the chairs in accordance with the already fixed seating pattern. Another way to organize the seating pattern could be to number or mark chairs or tables in such a manner that students will find it easy to identify the assigned chairs, as shown in (Fig. 1). The fact that seating positions will be arranged based on the courses' labels means that the exam papers can be distributed systematically based on the seating position.

Implementation of the proposed system requires the following information:

- Rooms : list of available examination rooms
 - Cap_r : seating capacity of each room *r*
 - N_c : number of students registered in course *c*
 - S : set of examination sessions
 - k_r : number of courses assigned to room *r*, k_r ∈ {2, 3, 4, 5}
- Using this information, the scheduling office can:

- determine the minimum number of rooms required,
- assign an appropriate number of courses to each room,
- generate cyclic seating patterns for each room,
- distribute students approximately evenly across assigned rooms,
- determine invigilator requirements using: $Inv_r = k_r, Inv_{total} = \sum_{r \in R^+} k_r$
- produce seating charts and student allocation lists for invigilators.

This structured approach ensures efficient space utilization, improved examination integrity, and alignment between administrative procedures and the underlying optimization model.

5. Simulation example from a specific university

To further demonstrate the effectiveness of the proposed seating optimization system, a realistic examination scenario is considered based on actual data from a specific university in Tanzania. This example incorporates courses with varying enrollments, including large class courses distributed across multiple rooms. In this situation, this method highlights efficiency gains compared to the traditional allocation method.

5.1. Tradition method of one exam session from a university in tanzania

According to the traditional method, the following courses were scheduled in one exam session, together with their total number of students (Table 2).

Total number of students: N = 5175

The classrooms used to seat 5175 students in Table 2. Following the fact that the traditional method distances students of the same course in the same room to avoid interactions, the total capacity of the rooms

Table 3
Rooms scheduled in a single exam session under the traditional method.

Room	Capacity	Room	Capacity	Room	Capacity
102-C	130	202-A	50	A-102	88
105-C	50	202-C	80	A-103	120
107-C	90	203-A	50	A-108	95
109-C	80	207-A	90	A-109	93
209-A	160	211-C	70	A115	139
A-204	306	A-210	342	C.BSMT	240
C110	130	CLR 1	120	CLR 2	120
CLR 3	120	D 005	130	D006	130
D110	130	LBH-6	450	LGH 7	450
LP11-BF	450	LP11-BF-R	110	LP11-FF	450
LP11-GF	450	NLH 4	378	NLH 5	378
NYERR.HL	500	SP.HL	500		
Total Capacity					7269

is greater than the actual number of students. Thus, below are the classrooms used and their total capacity calculated Table 3

Total available capacity: 7269

5.2. Proposed mixed-course allocation of same students

Using the proposed model, the courses are distributed as follows (Table 4):

Remark on Enrollment Approximation in Mixed-Course Allocation: In the mixed-course allocation model, the number of students per course within a room is sometimes presented using approximate values (e.g., average or rounded figures) rather than exact enrollments. This is done to simplify the allocation process and highlight general room utilization patterns.

It should be noted that courses with very small enrollments (e.g., fewer than 10 students) are not omitted; rather, they are grouped together with other courses with a relatively larger number of students within the same room. Their combined contribution is incorporated into the estimated average number of students per course reported for that room, for a course with a larger number of students.

For instance, a room assigned multiple courses may display an approximate value such as “≈ 120 students per course”, which reflects the average load based on the total number of students allocated to that room. This approximation remains representative even when some individual courses have significantly smaller enrollments, as their effect on the overall average is minimal.

Therefore, the estimation method preserves clarity and convenience in presentation while still maintaining a reasonable reflection of the actual distribution of students across courses.

Table 4
Mixed-course allocation of the proposed model.

Room	Courses assigned	k_r	# of students
SP.HL	MS 6121, HS 6141, CE 8302	3	≈ 165 each
NYERR.HL	EE 8305, ME 8301, HS 6116	3	≈ 165 each
LBH-6	MS 6121, HS 6114, MT 6303	3	≈ 150 each
LGH 7	HS 6141, EE 8305, ME 8301	3	≈ 150 each
LPII-BF	CE 6302, NSLT0 8106, NS 8354	3	≈ 150 each
LPII-FF	HS 6116, CE 8302, EE 6308, BM 8341	4	≈ 110 each
LPII-GF	IT 8301, ME 6301, MIS 6125	3	≈ 150 each
NLH 4	ET 8303, AA 8307, AA 8324, ST 8122	4	≈ 95 each
NLH 5	NSCH 3118, NSPH 3115, AS 6301	3	≈ 125 each
A-210	AS 6129, AS 8321, ME 6319	3	≈ 114 each
A-204	IT 6312, IT 6324	2	≈ 150 each
C.BSMT	CS 6305, CS 6371	2	≈ 120 each
Total Students			5175

Table 5
Unused space from already occupied classrooms.

Room	Capacity	Approx. unused seats
SP.HL	500	≈ 5
NYERR.HL	500	≈ 5
LBH-6	450	≈ 0
LGH 7	450	≈ 0
LPII-BF	450	≈ 0
LPII-FF	450	≈ 10
LPII-GF	450	≈ 0
NLH 4	378	≈ 0
NLH 5	378	≈ 3
A-210	342	≈ 0
A-204	306	≈ 6
C.BSMT	240	≈ 0
Total Partial Unused		≈ 29

Table 6
22 unused classrooms out of classrooms following proposed model.

Room	Capacity	Room	Capacity	Room	Capacity
102-C	130	202-A	50	A-102	88
105-C	50	202-C	80	A-103	120
107-C	90	203-A	50	A-108	95
109-C	80	207-A	90	A-109	93
209-A	160	211-C	70	A115	139
C110	130	CLR 1	120	CLR 2	120
CLR 3	120	D 005	130	D006	130
D110	130	LPII-BF-R	110		
Total Unused Capacity				2475	

5.3. Unused space following proposed model

Partially Utilized classrooms: Table 5 represent spaces left in classes selected for student occupation by the proposed model.

Classrooms completely unused: Table 6 represents classrooms saved following the proposed model.

Total unused capacity following the proposed model:

Therefore, total unused capacity $29 + 2475 = 2504$

and invigilator requirement for the proposed model:

$$Inv_{total} = 3 + 3 + 3 + 3 + 3 + 4 + 3 + 4 + 3 + 3 + 2 + 2 = 36$$

5.4. Merits of proposed model over traditional allocation method

In the traditional allocation method, each course is assigned to separate examination rooms without mixing with other courses. Consequently, large courses must be split into multiple rooms to accommodate all registered students. This increases the number of rooms required and leads to increased supervision demand. For example, at this university, every exam room is required to have at least two

invigilators; therefore, the Rooms used to run this session reflect the invigilation requirement as follows:

Total rooms required = 35.

$$\text{Minimum invigilators: } Inv_{\text{traditional}} \geq 2 \times 35 = 70$$

This suggests that the proposed model saves about 50% of the invigilation manpower required during the traditional method.

Moreover, since a significant part of the classroom or space remained unused under the proposed model, this suggests that the number of sessions can be reduced, and so can the total time to complete the whole examinations.

6. Results and discussion

This section presents the results of the proposed examination seat optimization model and discusses its implications in comparison to traditional seating allocation methods. The model successfully accommodates all 5175 students within the available 12 large classroom spaces, while 23 small rooms remain unused, resulting in a total seating capacity of 2475 seats unused. The mixed-course allocation strategy ensures a balanced distribution of students and improved spatial separation, thereby reducing potential interactions among candidates of the same course. In addition, large courses are distributed across multiple rooms, enhancing supervision efficiency and examination control.

6.1. Reduction in examination sessions

The simulation results indicate that mixing multiple courses within the same exam room significantly reduces the number of exam sessions required. In the traditional approach, courses are often scheduled in separate rooms or sessions due to adjacency concerns.

By applying structured seating patterns based on cyclic and combinatorial design principles, multiple courses can be examined simultaneously within the same room without violating adjacency constraints. This leads to fewer sessions, reduced examination duration, and improved scheduling efficiency.

6.2. Improved classroom space utilization

The proposed model demonstrates a high level of space utilization. Instead of allocating rooms to single courses, the mixed-course strategy ensures that the available seating capacity is used more efficiently.

In the simulation example, all students were accommodated using fewer rooms, with at least one room remaining unused. In contrast, the traditional method would require additional rooms due to course separation, resulting in underutilized seating capacity. This observation is consistent with the principle of optimization in resource allocation, where maximization of utilization is a key objective [40].

6.3. Invigilator requirements

The number of invigilators in the proposed model is directly related to the number of courses assigned to each room. In traditional methods, it is advised to have at least two invigilators per room to ensure availability of the invigilation personnel at all times during examination [41]. Sometimes, this leads to a larger number of invigilators, particularly if exam rooms are abnormally large. In the simulated example, the invigilator requirements were higher in the traditional method than in the proposed method. Nevertheless, the invigilation requirement may depend on the policies that follow university requirements.

However, in the proposed model, the structured arrangement improves supervision effectiveness, as each invigilator can focus on a specific subset of students associated with a particular course. This targeted supervision can improve quality control and reduce the likelihood of academic misconduct.

6.4. Integration of small and carryover classes

A key advantage of the proposed approach is its flexibility in accommodating small or carryover classes. Such classes are often difficult to schedule efficiently using traditional methods, leading to wasted space or additional sessions.

By incorporating these smaller groups into mixed-course seating arrangements, the model ensures a balanced distribution across rooms without requiring dedicated spaces. This contributes to both logistic efficiency and cost reduction.

6.5. Impact on examination integrity

The primary objective of the seating optimization model is to minimize adjacency between students enrolled in the same course. The use of cyclic seating patterns ensures that no two adjacent seats are assigned the same course, thus reducing opportunities for copying or unauthorized collaboration.

From a theoretical perspective, this corresponds to both a graph coloring problem in which adjacent nodes (seats) are assigned different colors (courses), a well-established approach to graph coloring [42], and combinatorial design for cyclic seating patterns of seating arrangements [43].

Thus, the proposed model enhances the integrity of the examination while maintaining practical feasibility.

7. Conclusion

The proposed seating allocation strategy provides a practical and efficient method for managing university exams without expanding the physical infrastructure. By systematically mixing students from different courses and using structured seating patterns, universities can reduce academic misconduct, improve the utilization of available space, minimize invigilation requirements, reduce examination sessions, and efficiently accommodate carryover courses.

The approach can be implemented manually through seating templates or automatically through scheduling software.

In general, the results demonstrate that the proposed examination seating optimization model provides a practical and mathematically grounded solution to challenges in examination management. By integrating graph coloring and combinatorial design principles, the model achieves improved efficiency, enhanced integrity, and better utilization of institutional resources.

CRedit authorship contribution statement

Isack E. Kibona: Writing – review & editing, Writing – original draft, Visualization, Validation, Software, Project administration, Methodology, Investigation, Formal analysis, Data curation, Conceptualization. **Jeremiah January:** Writing – review & editing, Validation, Methodology. **Hadija Matimbwa:** Writing – review & editing, Writing – original draft, Validation, Methodology, Investigation, Formal analysis, Conceptualization. **Edwin Nchia:** Writing – review & editing, Writing – original draft, Validation, Methodology, Investigation, Conceptualization. **William Matungwa:** Writing – review & editing, Writing – original draft, Visualization, Validation, Methodology, Investigation, Conceptualization. **Said A.H. Vuai:** Writing – review & editing, Writing – original draft, Supervision, Project administration, Methodology, Investigation, Conceptualization.

Declaration of competing interest

The authors declare that there are no known competing financial interests or personal relationships that could have appeared to influence the work reported in this document.

Data availability

Data will be made available on request.

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