

Spatial Inertia-Aware Generation Scheduling Optimization Framework with Maximized Inertia-Less VRES Generation Dispatch for Grid Frequency Stability

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DECLARATION

This thesis is my own work and has not been presented elsewhere for a degree award in any university.

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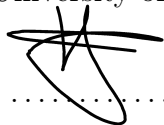
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ABBREVIATIONS AND ACRONYMS

Abbreviation	Description
ACE	Area Control Error
AGC	Automatic Generation Control
ASF	Average System Frequency
COI	Center of Inertia
COP	Center of Power
DMD	Dynamic Mode Decomposition
FCUC	Frequency-Constrained Unit Commitment
GA	Genetic Algorithm
GHG	Greenhouse Gas
HoDMD	Higher-Order Dynamic Mode Decomposition
IDI	Inertia Distribution Index
IEA	International Energy Agency
IEEE	Institute of Electrical and Electronics Engineers
KKT	Karush–Kuhn–Tucker
LRC-SCUC	Locational RoCoF-Constrained Security-Constrained Unit Commitment
LRR	Loss Reduction Ratio
MEFSS	Multi-Energy Frequency-Secure Scheduling

Abbreviation	Description
MILP	Mixed-Integer Linear Programming
MI	Minimum Inertia
MVA	Megavolt-Ampere
MW	Megawatt
NSGA-II	Non-Dominated Sorting Genetic Algorithm II
NZE	Net Zero Emissions
OEDI	Open Energy Data Initiative
OPF	Optimal Power Flow
PFC	Primary Frequency Control
PMU	Phasor Measurement Unit
PSO	Particle Swarm Optimization
PV	Photovoltaic
RoCoF	Rate of Change of Frequency
SBX	Simulated Binary Crossover
SCUC	Security-Constrained Unit Commitment
SG	Synchronous Generator
STEPS	Stated Policies Scenario
TDOA	Time-Delay-of-Arrival
TSO	Transmission System Operator
UC	Unit Commitment
UFLS	Under-Frequency Load Shedding
VRES	Variable Renewable Energy Sources
VSG	Virtual Synchronous Generator

ABSTRACT

The global transition toward low-carbon power systems has accelerated the integration of variable renewable energy sources (VRES). While this transition contributes to carbon-emission reduction, it also displaces synchronous generators (SGs) that inherently provide rotational inertia. The resulting decline in system inertia, together with its non-uniform spatial redistribution across the network, can alter frequency dynamics and increase the risk of local frequency instability following disturbances. Conventional operational and planning approaches commonly rely on aggregated system-level inertia metrics, which may obscure locational vulnerabilities and fail to reveal bus-level violations of frequency stability limits even when global indices appear secure.

This thesis proposes and validates a spatially resolved framework for inertia estimation and inertia-aware generation scheduling in power systems with high penetration of inertia-less VRES. First, an offline method for estimating bus-level inertia distribution is developed. The method uses steady-state network parameters to quantify the effective inertia reflected at individual buses within the power network. This formulation provides a transparent mapping of spatial inertia distribution without requiring online measurements, time-domain simulations, or disturbance data during the estimation stage, making it suitable for planning and operational studies. Building on this foundation, a multi-objective optimization framework is formulated to maximize inertia-less VRES penetration while maintaining frequency stability. The framework enforces minimum inertia thresholds at each bus, rather than relying solely on aggregated system inertia constraints. Transmission losses and synchronous generator commitment are simultaneously considered using the Non-Dominated Sorting Genetic Algorithm II. The resulting Pareto front reveals trade-offs between VRES penetration, network efficiency, and inertia adequacy,

and identifies operating points that balance these competing objectives.

The proposed methods are evaluated on modified IEEE 9- and 39-bus benchmark systems. Results show that, with a bus RoCoF limit of 1 Hz/s, the selected Pareto knee operating point achieves 75% penetration of inertia-less VRES while reducing transmission losses by approximately 56%. Sensitivity analysis performed for RoCoF limits of 1 Hz/s, 2 Hz/s, and 3 Hz/s shows that stricter frequency stability requirements require higher synchronous inertia and consequently limit renewable penetration. Under the 1 Hz/s limit, the maximum achievable inertia-less VRES penetration is approximately 82% with the lowest transmission losses due to reduced line loading. Relaxing the limit to 2 Hz/s increases the achievable penetration to about 86%, while the most relaxed limit of 3 Hz/s permits up to 95% VRES penetration but results in higher transmission losses due to increased line loading. The analytical inertia formulation is benchmarked against kinetic-energy-based frequency stability indicators and time-domain dynamic simulations, showing strong agreement in RoCoF and frequency nadir estimates. In addition, stochastic simulations under time-varying load and VRES conditions indicate that the proposed inertia-aware scheduling framework improves both the average performance and variability of key frequency stability indices relative to the benchmark method.

Overall, this thesis extends existing inertia estimation and inertia-constrained scheduling approaches by linking bus-level spatial inertia assessment with multi-objective VRES-aware generation scheduling. The proposed framework provides a planning-oriented approach for identifying frequency-security limits, inertia bottlenecks, and feasible renewable dispatch levels in low-inertia power systems. The findings support the use of spatially informed inertia management as a complementary tool for maintaining frequency stability in future power systems with high levels of converter-interfaced renewable generation.

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CHAPTER 1

INTRODUCTION

1.1 Background

The continuous emission of greenhouse gases (GHGs) remains the principal cause of global warming [1–3]. This leads to environmental and socio-economic consequences such as sea-level rise, extreme weather events, and food insecurity, which pose risks to sustainable development [4, 5]. The electricity sector is among the major contributors to GHG emissions due to its continued dependence on fossil-fuel-based generation [2].

To mitigate the increasing trend of GHG emissions and their impacts, global frameworks such as the *Paris Agreement* [6] call for deep decarbonization of the power sector so that the global temperature increase is kept well below 2°C above pre-industrial levels. This requires a sharp reduction in energy-related CO₂ emissions by 2050 and a corresponding shift in the generation mix. According to the *International Energy Agency (IEA)*, power-sector emissions need to reduce from approximately 13 Gt CO₂ in 2020 to below 1 Gt CO₂ by 2050 under the *Net Zero Emissions (NZE)* scenario [2, 7].

Recent surveys indicate that renewable generation continues to increase globally. As reported in the *Ember Global Electricity Review 2024*, renewable sources such as wind, solar, and hydropower contributed approximately 30% of total global electricity generation in 2023, while wind and photovoltaic sources alone contributed close to 13.5% [8]. Nonetheless, despite this progress, fossil fuels still contributed more than 60% of the world’s power supply, indicating the need for further structural

transformation of the electricity sector [9, 10].

Historical and projected energy-transition trends show that increasing renewable electricity generation is closely associated with decreasing power-sector CO₂ emissions. Historical data indicate that the share of renewables increased from about 19% in 2000 to about 29% in 2020, while power-sector emissions remained within approximately 9.5–13 Gt CO₂ over the same period [3]. Under the IEA *Stated Policies Scenario (STEPS)*, renewable electricity generation is projected to reach about 60% by 2050, with power-sector emissions reduced by nearly 6 Gt CO₂. Under the *Net Zero Emissions (NZE)* scenario, renewables are projected to exceed 85% of total electricity generation by 2050, while power-sector emissions approach near-zero levels [2, 3, 11]. These trends demonstrate that renewable energy expansion is central to decarbonization of the electricity sector.

However, achieving such a transition requires large-scale replacement of synchronous generators by variable renewable energy sources such as solar photovoltaic (PV) power plants. This transition supports global decarbonization objectives, but it also creates new operational challenges, particularly in relation to system frequency stability. The major technical issue is the reduction in power-system inertia, which represents the ability of the system to resist rapid frequency deviations following a disturbance. Inertia reduces the rate of change of frequency (RoCoF), giving primary frequency-control mechanisms sufficient time to restore power balance [12, 13].

Unlike traditional synchronous machines, inertia-less VRES do not provide natural physical inertia because their electromechanical dynamics are decoupled from the network through power electronic converters. This characteristic has already contributed to frequency-stability concerns in low-inertia grids, such as the 9 August 2019 event in Great Britain, where a lightning strike and subsequent generator tripping caused a rapid frequency decline that could not be adequately arrested by primary controls [14, 15].

As power systems move toward renewable generation shares exceeding 60–90%

by 2050, maintaining adequate and well-distributed inertia becomes a crucial requirement for secure and reliable grid operation. In this context, this study develops a spatial inertia-aware generation scheduling framework for maximizing the dispatch of inertia-less renewable generation while preserving frequency stability in low-inertia power systems.

1.2 Problem Statement

The increasing penetration of inertia-less variable renewable energy sources (VRES) is a key pathway toward global decarbonization of power systems. However, converter-interfaced renewable technologies do not provide the inherent rotational inertia naturally supplied by synchronous generators (SGs). As conventional SGs are progressively displaced, both the magnitude and the spatial distribution of system inertia are altered. Reduced inertia weakens the system's ability to resist rapid frequency deviations following disturbances, leading to increased rate of change of frequency (RoCoF), deeper frequency nadirs, and elevated risk of frequency instability.

Traditional frequency stability assessments commonly represent system inertia using a single aggregated metric. While suitable for highly synchronous systems, this representation neglects how inertial support is distributed across network buses. Consequently, localized regions with insufficient inertial support may remain undetected despite adequate aggregate inertia, creating spatial vulnerabilities in low-inertia power systems.

Existing approaches to inertia estimation primarily focus either on system-wide inertia assessment or on post-disturbance measurement-based techniques. These methods do not provide an analytical framework capable of quantifying how individual synchronous generators contribute inertial support to specific network buses under steady-state operating conditions. Furthermore, current generation scheduling frameworks typically enforce frequency stability constraints using aggregate

inertia measures, without explicitly considering spatial inertia adequacy.

This research addresses these limitations by investigating how inertia can be represented as a spatially distributed network property and incorporated directly into operational planning. To this end, an offline analytical method is developed to estimate an *equivalent bus-level inertia*, representing the effective inertial response observed at each network bus due to electromechanical coupling with synchronous generators. Building upon this spatial characterization, the study further investigates how minimum inertia requirements can be derived locally from RoCoF security criteria and integrated into generation scheduling.

Accordingly, this thesis develops an inertia-aware optimization framework that seeks to maximize inertia-less VRES penetration while ensuring compliance with spatially resolved frequency stability constraints through a multi-objective scheduling formulation solved using the Non-dominated Sorting Genetic Algorithm II (NSGA-II).

Based on the above discussion, the research gap addressed in this thesis can be concisely summarized as follows:

- Existing inertia-constrained scheduling frameworks commonly rely on aggregated or area-level inertia measures, which may not reveal bus-level frequency-security weaknesses in systems with non-uniform VRES penetration.
- Existing spatial inertia estimation methods are mainly used for monitoring, post-disturbance assessment, or inertia ranking, and are not directly embedded into generation scheduling decisions.
- Existing approaches do not sufficiently quantify how each committed synchronous generator contributes effective inertial support to each network bus under a given operating condition.
- Existing RoCoF-based inertia requirements are commonly formulated at

system or area level, with limited consideration of bus-level minimum inertia thresholds.

- Existing scheduling frameworks do not fully integrate inertia-less VRES dispatch, transmission-loss minimization, synchronous-generator commitment, and bus-level spatial inertia adequacy within one multi-objective optimization framework.

These gaps are examined in detail in Section 2.10, where the literature limitations are critically reviewed and linked to the methodological contributions developed in this thesis.

1.3 Objectives of the Study

1.3.1 Main Objective

To develop and validate a spatial inertia-aware generation scheduling optimization framework that maximizes dispatch of inertia-less VRES generation while maintaining grid frequency stability through analytically derived bus-level inertia requirements.

1.3.2 Specific Objectives

- (i) To develop an offline method for estimating bus-level inertia distribution that quantifies the contribution of each synchronous generator to each bus of the power network.
- (ii) To analytically determine minimum bus-level inertia thresholds required to satisfy predefined RoCoF-based frequency stability limits under credible disturbances.

- (iii) To formulate an NSGA-II based optimization framework that maximizes inertia-less VRES penetration while enforcing bus-level minimum inertia thresholds as frequency stability constraints.
- (iv) To validate the proposed optimization framework through comparative case studies and performance evaluation under varying operating conditions.

1.4 Justification of the Study

The transition toward renewable-dominated power systems introduces fundamental challenges for frequency stability due to the displacement of synchronous generation and the associated reduction in rotational inertia. While maintaining adequate inertia is widely recognized as essential for frequency stability, existing operational practices largely rely on aggregate inertia indicators that do not capture spatial variations in inertial support across transmission networks.

As power systems become increasingly converter dominated, uneven distribution of inertia may lead to localized frequency vulnerabilities even when total system inertia appears sufficient. Addressing this challenge requires analytical tools capable of quantifying spatial inertia distribution and translating frequency stability requirements into operational decision-making processes.

The motivation of this research is therefore not only to estimate inertia distribution but to integrate spatial inertia considerations directly into generation scheduling. By linking analytical inertia estimation with optimization-based dispatch, the proposed framework enables operational decisions that simultaneously support renewable integration and frequency stability.

The outcome of this research is a methodological framework applicable to planning and operational studies in low-inertia power systems, providing a systematic means of determining how much inertia is required, where it is required, and how generation scheduling decisions influence spatial frequency stability.

1.5 Scope of the Study

This research focuses on the assessment and management of frequency stability in power systems with high penetration of inertia-less variable renewable energy sources, with particular emphasis on the spatial distribution of synchronous inertia and its operational implications. The scope of the study is defined as follows.

1.5.1 System and Modeling Scope

The proposed methods are implemented and validated using benchmark power system models, primarily focusing on the modified IEEE 39-bus (New England) test system. Additional simulations on a smaller network (IEEE 9-bus system) are conducted for conceptual validation and interpretation of spatial inertia behavior. Synchronous generators are modeled using classical inertia representations, while VRES are modeled as converter-interfaced resources without inherent rotational inertia. To represent the inertia-less characteristics of VRES, the study focuses specifically on solar photovoltaic (SPVs), which offer no natural inertial response. SPVs thus, serve as the benchmark for the worst-case inertia-less scenario. Wind power plants are excluded from this analysis because their rotating turbine components can store kinetic energy and provide partial or “hidden” inertia, which would obscure the evaluation of purely converter-dominated systems.

1.5.2 Operational Scope

The proposed framework is intended for offline planning and operational scheduling applications, such as day-ahead or intra-day generation scheduling studies. Real-time control, protection design, and sub-second corrective actions are not explicitly addressed. However, the results are intended to inform operational decisions that improve local and system-wide frequency stability in low-inertia power systems.

1.5.3 Practical Relevance

While the methodology is validated using standard IEEE 9 and 39 bus test systems, the research is motivated by the operational challenges faced by emerging and transitioning power systems with rapidly increasing renewable energy technologies penetration. The proposed framework is designed to be transferable to real-world systems, subject to availability of network and generator data.

1.5.4 Disturbance Modeling for Minimum Inertia Derivation

From the perspective of deriving a robust and consistent minimum inertia constraint, the disturbance used in this study is represented as a load change event rather than a synchronous generator outage. The primary objective of the work is to maximize the penetration of inertia-less VRES, which progressively displaces the loading of synchronous generators. If a generator outage were used to define the disturbance, the magnitude of the resulting power imbalance would depend on the operating level of the generator at the time of the event. As VRES penetration increases and synchronous generator loading decreases, the associated outage would produce a smaller power deficit and therefore lead to a lower minimum inertia requirement. This would cause the inertia constraint to relax as VRES penetration increases, undermining its role as a consistent frequency stability requirement. In contrast, a load change disturbance introduces an active-power imbalance without directly altering the system inertia, allowing the minimum inertia thresholds to be derived independently of generator dispatch and ensuring that the resulting constraint remains consistent across different operating conditions.

1.6 Thesis Organization

This thesis is organized into five chapters, each addressing a distinct aspect of the research problem.

Chapter 1: introduces the research background and motivation, outlines the problem statement, and defines the study objectives. It also presents the scope, justification and significance of the research, emphasizing the need for optimized inertia distribution in low-inertia power systems with high renewable energy penetration.

Chapter 2: provides a comprehensive review of the existing literature on power system inertia, estimation techniques, spatial inertia distribution, and optimization-based approaches for renewable energy integration. The chapter identifies the key knowledge gaps that form the basis for the proposed research methodology.

Chapter 3 presents the research methodology adopted in this study. The chapter details the overall simulation and implementation framework, including the analytical formulation of the offline bus-level inertia estimation method based on network reduction and electrical-distance modeling. It further describes the development of the inertia-aware multi-objective optimization framework, the formulation of bus-level minimum inertia constraints, and the integration of these constraints into the generation scheduling problem. The chapter also outlines the validation methodology, simulation setup, and computational procedures employed for subsequent analysis.

Chapter 4 presents and discusses the results obtained from the application of the proposed framework. It first analyzes the spatial distribution of inertia under different operating conditions and validates the offline inertia estimation method using frequency-domain and time-domain indicators. The chapter then evaluates the performance of the inertia-aware optimization framework through detailed case studies on the modified IEEE 39-bus test system, examining the trade-offs between inertia-less VRES penetration, transmission power losses, and synchronous generator commitment. Comprehensive validation results are provided using rate of change of frequency patterns, bus-level frequency dynamics, and sensitivity analyses, demonstrating the effectiveness of enforcing bus-level inertia constraints for local frequency stability.

Chapter 5 summarizes the key findings and contributions of the research. It

consolidates the insights gained from the spatial inertia assessment and optimization results, highlights the practical implications for frequency-stable operation of low-inertia power systems, and presents recommendations for future research. Potential extensions include the incorporation of synthetic inertia and grid-forming converter technologies, nonlinear time-domain validation under large disturbances, and application of the proposed framework to large-scale power systems.

CHAPTER 2

LITERATURE REVIEW

2.1 Overview

Increasing penetration of inertia-less VRES, has become a core strategy for reducing greenhouse gas emissions and advancing global de-carbonization efforts. However, the displacement of SGs by inertia-less VRES reduces the total system inertia and alters its spatial distribution as synchronous units are displaced non-uniformly across the network. This shift can increase RoCoF, accelerate the time-to-nadir, and create location-dependent frequency vulnerabilities following disturbances.

This chapter reviews the literature with a focus on three themes that directly motivate the thesis contributions: (i) how inertia and control mechanisms shape frequency dynamics in low-inertia systems, (ii) how inertia is estimated in practice at both aggregated and spatially resolved levels, and (iii) how inertia-related constraints are embedded in operational planning and scheduling. Throughout the review, methods are compared using four recurring criteria: required data (event-based, ambient, or model-based), disturbance dependence, spatial resolution (system, zonal, or bus-level), and suitability for iterative optimization studies under high VRES penetration.

2.2 Variable Renewable Energy Sources and Frequency Stability Challenges

Variable renewable energy sources, particularly solar photovoltaic (PV) and wind energy conversion systems (WECS), are central to the transition toward low-carbon power systems. Their increasing deployment reduces dependence on fossil-fuel-based generation and supports global decarbonization objectives. However, unlike conventional synchronous generators, most VRES technologies are connected to the grid through power electronic converters. This changes the physical relationship between generation, grid frequency, and stored kinetic energy, thereby introducing new challenges for frequency stability in modern power systems [16–18].

2.2.1 Characteristics of Solar PV and Wind Energy Conversion Systems

Solar PV and wind energy systems differ significantly from synchronous generators in their energy conversion mechanisms and their interaction with grid frequency. A synchronous generator is electromechanically coupled to the grid through its rotating magnetic field. As a result, its stored kinetic energy is naturally released or absorbed during active-power disturbances, providing an immediate inertial response. In contrast, solar PV generation has no rotating mass and therefore does not provide inherent mechanical inertia. Its active power output is governed mainly by solar irradiance, module temperature, maximum power point tracking, inverter control, and grid-code requirements [19].

Wind energy systems contain rotating components and therefore store kinetic energy in the turbine-generator shaft. However, modern variable-speed wind turbines are commonly interfaced through power electronic converters, which partially or fully decouple the mechanical rotor speed from the grid frequency. Consequently, the kinetic energy stored in wind turbines is not automatically available to the

power system in the same way as synchronous-generator inertia. Wind turbines can contribute inertial or fast frequency response only when appropriate control strategies, such as synthetic inertia, virtual inertia, or deloaded operation, are implemented [18, 20].

This distinction is important for the present study. Solar PV represents a purely inertia-less VRES technology from the perspective of natural rotational inertia, making it suitable for assessing the worst-case effect of inertia-less VRES on system inertia. Wind generation, on the other hand, may provide frequency support depending on turbine type and control strategy. For this reason, wind power plants require additional modeling assumptions related to rotor kinetic energy extraction, converter control, and recovery dynamics. These characteristics justify the use of solar PV as the benchmark inertia-less VRES technology in this study, while wind-based inertial support is treated as an important extension for future work.

2.2.2 Variability, Uncertainty, and Dispatchability of VRES

A major operational challenge associated with VRES is their dependence on weather-driven primary energy sources. Solar PV output varies with solar irradiance, cloud movement, temperature, time of day, and seasonal conditions. Wind generation varies with wind speed, air density, turbine characteristics, and site-specific meteorological conditions. These variations introduce uncertainty into generation scheduling because the available renewable power may differ from forecasted values over operational time scales [21–23].

In conventional generation scheduling, dispatchable synchronous units are committed and dispatched to meet demand while satisfying operational constraints such as generation limits, ramping limits, reserve requirements, and network constraints. With high VRES penetration, the scheduling problem becomes more complex because renewable generation is variable, partially uncertain, and often prioritized due to its low marginal cost. Increased VRES dispatch may reduce fuel cost and emissions, but it may also displace synchronous generators that provide inertia,

damping, and primary frequency response. Therefore, maximizing VRES penetration without considering frequency-security constraints may lead to operating points that are economically attractive but dynamically weak [24–26].

This creates a direct link between VRES variability and inertia-aware generation scheduling. When renewable output is high, fewer synchronous units may be required for active-power balance, resulting in lower online inertia. Conversely, when renewable output is low, synchronous generation may increase, improving inertial support but reducing renewable utilization. A frequency-secure scheduling framework must therefore coordinate renewable dispatch, synchronous generator commitment, network losses, and inertia adequacy within the same decision-making process.

2.2.3 Effect of inertia-less VRES on Frequency Dynamics

The displacement of synchronous generators by inertia-less VRES affects frequency dynamics in two related ways. First, it reduces the aggregate amount of synchronous inertia available in the system. Since the initial RoCoF following a disturbance is inversely related to the available inertia, low-inertia operating conditions produce faster frequency deviations for the same active-power imbalance [19, 27]. Therefore, low inertia reduces the time available for primary frequency response, fast frequency response, and protection systems to act before frequency reaches critical limits [28, 29].

Second, VRES integration changes the spatial distribution of inertia. Renewable generation is not necessarily installed at the same buses or areas where synchronous generators are displaced. As a result, inertia may become concentrated around the remaining synchronous machines, while renewable-rich or electrically remote areas may experience weaker inertial coupling. Under such conditions, frequency response cannot always be represented adequately by a single center-of-inertia frequency. Local frequency deviations and local RoCoF may differ across the network, particularly following disturbances in weakly coupled or low-inertia

areas [30–33].

This spatial effect is especially important for operational planning. A system may satisfy a minimum aggregate inertia requirement while still exposing specific buses to unacceptable local RoCoF. Similarly, a dispatch decision that maximizes VRES penetration may appear feasible from a power-balance perspective but may create local frequency-security risks if it removes synchronous inertia from electrically critical locations. These challenges motivate the need for spatially resolved inertia assessment and for generation scheduling methods that enforce inertia adequacy at the bus level rather than only at the system level.

2.3 Power System Stability in Low-Inertia Grids

Power system stability refers to the ability of an electric power system to remain in a state of operating equilibrium under normal conditions and to regain an acceptable state of equilibrium following a disturbance. Traditionally, stability has been classified into rotor-angle stability, frequency stability, and voltage stability, with each category associated with distinct physical mechanisms and time scales [16, 34]. In conventional power systems dominated by synchronous generators, these stability phenomena are strongly influenced by the electromechanical behavior of rotating machines, network synchronizing power, excitation systems, turbine-governor control, and load-frequency response.

The increasing penetration of inertia-less resources has expanded the classical view of power system stability. Modern low-inertia systems are not governed only by synchronous-machine electromechanical dynamics, but also by the interaction of power electronic converters, phase-locked loops, current controllers, grid-forming controls, protection systems, and fast frequency-response mechanisms [35–37]. As a result, frequency stability in renewable-rich systems must be understood not only as a system-wide active-power balance problem, but also as a spatial and control-dependent phenomenon shaped by the location of inertia, converter behavior, and

the strength of electrical coupling across the network.

2.3.1 Classical Frequency Stability in Synchronous-Generator-Dominated Systems

In synchronous-generator-dominated power systems, frequency stability is primarily governed by the balance between mechanical input power and electrical output power. When a disturbance occurs, such as a generation outage or a sudden load increase, the mismatch between generation and demand is initially supplied by the kinetic energy stored in the rotating masses of synchronous machines. This inertial response is automatic and occurs before governor action becomes effective [38].

The classical interpretation of early frequency dynamics is commonly based on the swing-equation concept. For a given active-power imbalance, higher synchronous inertia reduces the initial RoCoF, delays the frequency nadir, and provides additional time for primary frequency response to act. Conversely, lower inertia causes frequency to change more rapidly, reducing the available time margin for governors, fast frequency-response resources, and protection systems [39].

In classical power system, the main sources of frequency stability are synchronous inertia, governor-based primary frequency response, load damping, and secondary control through automatic generation control. Inertia limits the initial RoCoF, primary control arrests the frequency decline, load damping naturally reduces part of the imbalance, and secondary control restores frequency toward its scheduled value [16, 17]. These mechanisms have historically allowed system operators to assess frequency security using aggregated indicators such as total online inertia, center-of-inertia frequency, spinning reserve, and frequency-response reserve [26].

However, the aggregated view is most reliable when the system remains strongly coherent and when synchronous machines are widely distributed across the network. Under such conditions, local bus frequencies tend to move together, and the center-of-inertia frequency provides a reasonable representation of the system re-

sponse. This assumption becomes weaker in low-inertia systems where synchronous generation is displaced non-uniformly and inertia-less VRES dominate some areas of the grid.

2.3.2 Modern Stability Concepts in Converter-Dominated Power Systems

Converter-dominated power systems introduce stability mechanisms that differ from those of conventional synchronous-machine-based grids. Inertia-less VRES do not inherently synchronize through rotor electromechanics. Instead, their interaction with the grid depends on control algorithms, current limits, phase-locked loops, dc-link dynamics, and grid-code requirements. Consequently, the stability of such systems is strongly influenced by converter control modes and by the electrical strength of the network at the point of interconnection [36, 40, 41].

A useful distinction is commonly made between grid-following and grid-forming converter operation. Grid-following converters rely on an existing voltage waveform and typically synchronize through a phase-locked loop. They inject controlled active and reactive currents but do not naturally establish grid voltage or frequency. In weak grids or during severe disturbances, their dependence on voltage-angle tracking can introduce control interactions and may limit their ability to support frequency and voltage stability. Grid-forming converters, in contrast, are controlled to establish a voltage magnitude and frequency reference. With appropriate control design, they can contribute to frequency support, voltage regulation, damping, and black-start capability [37, 42].

The concept of stability has therefore expanded beyond rotor-angle, voltage, and frequency stability alone. Converter-driven stability, resonance stability, and control-interaction stability have become increasingly important in systems with high penetration of inverter-based resources [35]. For frequency stability studies, this means that the availability of active-power support is not sufficient by itself.

The location, response speed, control mode, energy availability, and dynamic limitations of converter-based resources also influence whether frequency can be contained after a disturbance [43, 44].

This has direct relevance to inertia-aware scheduling. A dispatch solution that replaces synchronous generators with inertia-less VRES may satisfy steady-state power-flow constraints but still weaken the dynamic security of the system. The loss of synchronous inertia may increase RoCoF, while the removal of synchronous voltage sources may reduce system strength and affect converter behavior. Therefore, generation scheduling in low-inertia systems must account for both power balance and frequency-support adequacy.

2.3.3 Sources of Frequency Stability in Low-Inertia Systems

Frequency stability in low-inertia systems is supported by several complementary mechanisms. The first is physical synchronous inertia, which remains the most immediate natural response to an active-power imbalance. It is provided by online synchronous generators, and other rotating machines directly coupled to the grid [13, 16]. Its main advantage is that it responds autonomously without measurement, communication, or control delay. Its limitation is that it depends on the commitment and location of synchronous machines, which may decline as VRES penetration increases.

The second source is primary frequency response from governor-controlled synchronous generators. Primary response adjusts mechanical power input following frequency deviations and helps arrest the frequency decline. However, this response is not instantaneous because it is affected by sensing, actuator, and turbine dynamics. In very low-inertia systems, frequency may decline rapidly before primary response becomes fully effective [13, 23].

The third source is load damping and demand-side frequency response. Some loads naturally reduce their power consumption when frequency falls, while controllable

loads can be designed to provide fast demand-side support. This mechanism can improve frequency containment, but its contribution depends on load composition, control coordination, customer participation, and protection settings [45].

The fourth source is fast frequency response from inertia-less VRES, including battery energy storage systems, curtailed renewable plants, and flexible demand. Fast frequency response can inject or reduce active power within a short time following a disturbance, thereby improving RoCoF and frequency nadir performance. Battery energy storage systems are particularly attractive because they can respond rapidly and can be controlled to provide frequency regulation, synthetic inertia, and reserve services [39, 46].

The fifth source is synthetic or virtual inertia. Synthetic inertia emulates the inertial behavior of synchronous machines by modifying converter active-power output in response to frequency or RoCoF measurements. Wind turbines, battery energy storage systems, and deloaded PV plants can provide such support when suitable control strategies and energy margins are available [18, 20, 47]. However, synthetic inertia is not identical to physical inertia. It depends on measurement accuracy, converter control design, available headroom, energy recovery dynamics, current limits, and grid-code requirements.

These sources show that system stability in modern grids is no longer supplied only by synchronous machines. Stability can be supported by a portfolio of physical inertia, fast converter-based controls, reserve services, demand response, and energy storage. Nevertheless, synchronous inertia remains important because it provides an immediate response and influences the spatial propagation of frequency dynamics before controlled resources fully respond.

2.3.4 Implications for Spatial Inertia-Aware Scheduling

The reviewed stability concepts have important implications for generation scheduling. In conventional scheduling, synchronous generator commitment is mainly

determined by energy balance, operating cost, reserve requirements, and network constraints. In low-inertia systems, the commitment and dispatch of synchronous generators also determine the amount and location of inertial support. Therefore, replacing synchronous generation with inertia-less VRES changes not only the generation mix but also the dynamic capability of the system to withstand disturbances.

A purely aggregated inertia requirement may ensure that the total online inertia is above a minimum value, but it does not indicate whether that inertia is electrically close to vulnerable buses. This is a critical limitation in systems where renewable generation is concentrated in specific areas and synchronous generators are displaced unevenly. Local RoCoF may become excessive even when the aggregate inertia appears sufficient. Therefore, frequency-security constraints should account for where inertia is located and how it is coupled to the rest of the network.

This thesis builds on this observation by treating inertia as a spatially distributed network property rather than only as a system-wide scalar quantity. The scheduling problem is therefore framed not only as a problem of maximizing inertia-less VRES dispatch, but also as a problem of preserving sufficient bus-level inertia support to maintain local frequency stability. This perspective provides the foundation for the subsequent review of inertia representation and inertia estimation techniques.

2.4 Inertia Representation and Spatial Frequency Dynamics

The representation of inertia is central to frequency-stability analysis in power systems. In conventional systems, inertia is commonly treated as an aggregated system property because synchronous generators are strongly coupled and frequency deviations are often assumed to be sufficiently coherent across the network. Under this assumption, the post-disturbance frequency response can be described using an equivalent center-of-inertia (COI) frequency and a single equivalent inertia

value [13]. Aggregated inertia assumption has been widely used for frequency-security assessment, RoCoF evaluation, reserve sizing, and inertia-constrained scheduling [48, 49].

However, the increasing penetration of inertia-less VRES weakens the validity of purely aggregated inertia representation. As synchronous generators are displaced non-uniformly across the network, inertia becomes geographically and electrically uneven. The remaining synchronous machines may be concentrated in specific areas, while renewable-rich areas may become weakly coupled to the main sources of inertial response. Consequently, local bus frequencies may not follow the COI frequency closely during the early post-disturbance period. This creates the need to interpret inertia not only as a system-wide scalar quantity, but also as a spatially distributed network property.

2.4.1 Aggregated Inertia Representation

Aggregated inertia representation is based on the assumption that the collective inertial behavior of all online synchronous machines can be expressed through an equivalent inertia constant. If the system contains N_{SG} online synchronous generators, the COI frequency can be expressed as an inertia-weighted average of generator frequencies:

$$f_{\text{COI}} = \frac{\sum_{k=1}^{N_{SG}} H_k S_k f_k}{\sum_{k=1}^{N_{SG}} H_k S_k}, \quad (2.1)$$

where H_k is the inertia constant of generator k , S_k is its rated apparent power on a common base, and f_k is the electrical frequency associated with the generator rotor. The equivalent stored inertial energy is therefore represented by the total inertia contribution of all synchronized generators:

$$H_{\text{sys}} = \sum_{k=1}^{N_{SG}} H_k S_k. \quad (2.2)$$

For an active-power imbalance ΔP , the initial RoCoF is commonly approximated from the aggregated swing-equation relationship:

$$\left. \frac{df_{\text{COI}}}{dt} \right|_{t_0^+} \approx -\frac{f_0 \Delta P}{2H_{\text{sys}}}, \quad (2.3)$$

where f_0 is the nominal frequency and t_0^+ denotes the instant immediately after the disturbance. This expression shows that, for a fixed disturbance size, higher aggregated inertia reduces the initial RoCoF. It provides a simple and useful basis for deriving minimum inertia requirements in system-level frequency-security studies.

The main strength of aggregated inertia representation is its simplicity. It requires limited system information, can be incorporated into optimization models with relatively low computational burden, and provides a clear link between disturbance size, inertia, and RoCoF. For this reason, many inertia-constrained unit commitment and dispatch formulations use aggregated inertia constraints to maintain frequency security under high renewable penetration [25, 26, 50].

Despite these advantages, aggregated inertia representation assumes that all buses experience frequency deviations that are sufficiently close to the COI response. This assumption becomes less reliable in low-inertia systems, especially where synchronous generation is unevenly distributed, VRES penetration is geographically concentrated, or the network contains weak electrical corridors.

2.4.2 Limitations of Aggregated Inertia

The principal limitation of aggregated inertia is that it masks the spatial distribution of inertial support. Two operating conditions may have the same total online inertia but very different frequency-security characteristics if the inertia is located at different parts of the network. For example, inertia concentrated near one group of synchronous generators may provide strong support to electrically nearby buses, while remote buses with high inertia-less VRES generation may experience higher

local RoCoF.

This limitation becomes important because frequency is not perfectly uniform during the early transient period after a disturbance. Although the steady-state frequency of an interconnected system is common after synchronization effects settle, the immediate post-disturbance response can exhibit local differences. These differences arise from network impedance, generator location, disturbance location, and the electrical distance between buses and synchronous inertia sources [30–33].

A system-level inertia constraint may therefore be satisfied while some buses remain locally weak. This can occur when the aggregate inertia is adequate in total magnitude but poorly distributed relative to the disturbance location or renewable generation areas. Under such conditions, local RoCoF protection, converter controls, and under-frequency load-shedding schemes may respond to local frequency behavior rather than to the COI frequency. Therefore, a scheduling solution based only on aggregated inertia may overestimate the true frequency-security margin at vulnerable buses.

The limitation of aggregated inertia can be summarized through the distinction between system-level and local frequency deviation:

$$\Delta f_j(t) = f_j(t) - f_0, \quad \Delta f_{\text{COI}}(t) = f_{\text{COI}}(t) - f_0, \quad (2.4)$$

where $f_j(t)$ is the local frequency at bus j . In a strongly coherent system,

$$\Delta f_j(t) \approx \Delta f_{\text{COI}}(t), \quad (2.5)$$

and aggregated inertia is generally adequate. However, in a spatially non-uniform low-inertia system,

$$\Delta f_j(t) \neq \Delta f_{\text{COI}}(t), \quad \left| \frac{df_j}{dt} \right| \neq \left| \frac{df_{\text{COI}}}{dt} \right|, \quad (2.6)$$

especially during the initial post-disturbance interval. This motivates the need for local or bus-level inertia indicators that can identify whether each bus has sufficient inertial support relative to its frequency-security requirement.

2.4.3 Spatial Inertia and Local Frequency Dynamics

Spatial inertia refers to the location-dependent availability and effectiveness of inertial support across a power network. It is not determined only by the inertia constants of online synchronous generators, but also by their electrical coupling to different buses. A synchronous generator that is electrically close to a bus provides stronger inertial influence at that bus than a generator that is electrically remote or weakly coupled. Therefore, bus-level inertia can be interpreted as the effective inertial support observed at a network bus due to the combined influence of online synchronous generators.

The idea of spatial inertia is closely related to the spatial behavior of frequency following a disturbance. Immediately after an active-power imbalance, frequency deviations propagate through electromechanical coupling rather than appearing uniformly everywhere at the same magnitude. Buses near the disturbance or far from strong inertial sources may experience steeper frequency gradients, while buses close to large synchronous units may experience slower frequency deviations. This spatial variation has been observed and analyzed using inertia distribution indices, PMU-based frequency measurements, nodal inertia mapping, and locational RoCoF formulations [30, 31, 33, 51, 52].

A general way to express the spatial nature of frequency is to consider the vector of bus-frequency deviations:

$$\Delta \mathbf{f}(t) = \begin{bmatrix} \Delta f_1(t) & \Delta f_2(t) & \cdots & \Delta f_{N_b}(t) \end{bmatrix}^T, \quad (2.7)$$

where N_b is the number of buses. In aggregated frequency studies, this vector is reduced to a single scalar frequency, usually the COI frequency. In spatial frequency

analysis, however, the individual elements of $\Delta \mathbf{f}(t)$ are retained to describe local frequency behavior.

The corresponding local RoCoF vector is given by

$$\frac{d\Delta \mathbf{f}(t)}{dt} = \left[\frac{d\Delta f_1(t)}{dt} \quad \frac{d\Delta f_2(t)}{dt} \quad \dots \quad \frac{d\Delta f_{N_b}(t)}{dt} \right]^T. \quad (2.8)$$

This representation shows that frequency security can be evaluated locally by comparing each bus RoCoF against an admissible limit:

$$\left| \frac{d\Delta f_j(t)}{dt} \right| \leq \text{RoCoF}_j^{\max}, \quad j = 1, 2, \dots, N_b. \quad (2.9)$$

Equation (2.9) provides the conceptual basis for spatial frequency-security assessment. Instead of requiring only that the COI RoCoF remains within a system-wide limit, each bus is assessed according to its local frequency response. This is particularly relevant in low-inertia networks where local RoCoF may exceed acceptable limits even when the COI RoCoF appears secure.

In this context, bus-level inertia is useful because it provides a planning-oriented measure of the local inertial strength available at each bus. If a bus has low effective inertia, it is more likely to experience rapid frequency deviations following disturbances. Conversely, a bus with strong inertial coupling to online synchronous generators is expected to have better local frequency containment. This interpretation links spatial inertia estimation directly to operational scheduling: generator commitment and dispatch decisions change not only total inertia, but also the spatial pattern of effective inertia across the network.

2.4.4 Conditions Under Which Spatial Inertia Becomes Critical

Spatial inertia representation becomes especially important under operating conditions where the assumptions behind aggregated inertia are weak. The first condition is high VRES penetration. When renewable generation displaces synchronous machines, the total amount of online inertia declines, and the remaining inertia may be concentrated in fewer locations. This increases the likelihood of local inertia deficiencies.

The second condition is non-uniform displacement of synchronous generation. If synchronous units are removed from one area while retained in another, the system may preserve an acceptable aggregate inertia value but develop local weak-inertia regions. This situation is common when renewable resources are geographically concentrated or when economic dispatch decommits synchronous units based on cost rather than dynamic location.

The third condition is weak electrical coupling. Long transmission corridors, high reactance paths, constrained tie-lines, or radial network sections reduce the ability of remote synchronous generators to support local frequency behavior. In such systems, electrical proximity becomes as important as the magnitude of generator inertia.

The fourth condition is disturbance location. A disturbance occurring near a low-inertia or weakly coupled area may produce higher local RoCoF than the same disturbance occurring near strong synchronous generation. Therefore, the severity of a disturbance is not determined only by its size, but also by its location relative to the spatial distribution of inertia.

The fifth condition is operation close to protection or grid-code limits. In low-inertia systems, high local RoCoF may trigger RoCoF relays, under-frequency load shedding, or converter protection before system-wide frequency indicators show severe instability. This makes local frequency-security assessment important for

both planning and operation.

Nevertheless, spatial inertia representation is not always necessary. If a system has uniformly distributed synchronous generation, operates with low VRES penetration, and exhibits coherent frequency response across buses, aggregated inertia may remain adequate for first-level frequency-security screening. Spatial inertia becomes most valuable when the system is weakly coupled, renewable-rich, dynamically non-uniform, or when operational decisions significantly change the location of online synchronous machines.

Therefore, aggregated and spatial inertia representations should not be viewed as competing concepts, but as tools with different levels of resolution. Aggregated inertia is useful for system-level screening, while spatial inertia is needed when local frequency-security constraints become operationally relevant. This thesis adopts the spatial view because the research objective is not only to maintain sufficient total inertia, but to ensure that each bus has adequate effective inertia support while maximizing inertia-less VRES dispatch.

2.5 Generation Scheduling under High VRES Penetration

Generation scheduling is a core operational function in power systems. Its purpose is to determine which generating units should be committed and how much power each unit should produce over a specified scheduling horizon while satisfying demand, reserve, technical, and network constraints. In conventional power systems, generation scheduling is mainly driven by economic dispatch, fuel cost minimization, unit commitment constraints, ramp-rate limits, spinning reserve requirements, and transmission security [53,54]. Under this paradigm, synchronous generators provide not only active power but also essential stability services such as inertia, damping, voltage support, and primary frequency response.

The increasing penetration of VRES has changed the nature of the scheduling problem. Solar PV and wind generation are characterized by variability, partial uncertainty, and low marginal operating cost. Consequently, system operators are encouraged to dispatch as much renewable generation as possible to reduce fuel consumption and emissions. However, higher VRES dispatch often displaces synchronous generation, thereby reducing the amount of online synchronous inertia and primary frequency-response capability [49, 55, 56]. This creates a conflict between renewable energy maximization and frequency-security preservation [57, 58]. Therefore, generation scheduling in renewable-rich systems can no longer be treated only as an economic or energy-balance problem; it must also account for the dynamic adequacy of the resulting generation mix.

2.5.1 Conventional Generation Scheduling

Conventional generation scheduling generally involves two related problems: unit commitment and economic dispatch. Unit commitment determines the on/off status of generators over the scheduling horizon, while economic dispatch determines the output power of committed units. The optimization normally considers generator capacity limits, ramping constraints, minimum up/down time constraints, reserve margins, power balance, and transmission limits [53, 54, 59].

In synchronous-generator-dominated systems, frequency security is often treated indirectly through reserve requirements and operating rules. This approach is reasonable when enough synchronous machines remain online because the system naturally retains adequate inertia and governor response. In such systems, the total amount of online synchronous generation usually provides sufficient inertial support, and frequency dynamics can often be approximated using aggregated system indicators.

However, this assumption becomes weaker as VRES penetration increases. A low-cost scheduling solution may decommit synchronous generators and replace their output with inertia-less renewable generation. Although such a solution may

satisfy steady-state power-flow and reserve constraints, it may leave the system with insufficient inertia to withstand credible disturbances. Therefore, conventional scheduling formulations may produce dynamically vulnerable operating points when applied directly to low-inertia systems.

2.5.2 VRES-Aware Generation Scheduling

VRES-aware scheduling extends conventional scheduling by explicitly considering the availability, variability, and uncertainty of renewable generation. In such formulations, renewable power may be represented using forecast profiles, time-series scenarios, probabilistic models, or robust uncertainty sets. The objective is commonly to maximize renewable utilization, minimize curtailment, reduce operating cost, or reduce emissions while maintaining operational feasibility [21,24].

The main strength of VRES-aware scheduling is that it recognizes the time-varying nature of renewable generation. By incorporating solar and wind availability profiles, the scheduler can reflect the actual operating conditions under which synchronous generators are displaced. This is important because the inertia level of a power system is not fixed; it changes with the commitment and dispatch of synchronous machines.

Nevertheless, many VRES-aware scheduling approaches focus primarily on energy adequacy, reserve adequacy, and cost minimization. Frequency stability is often addressed only indirectly, for example by maintaining spinning reserve or requiring a minimum number of synchronous units online. Such measures may improve security, but they do not explicitly quantify whether the scheduled operating point provides enough inertia to limit RoCoF or frequency nadir following a disturbance. As a result, renewable dispatch may be maximized without a clear guarantee of frequency-security adequacy.

Another limitation is that VRES-aware scheduling often treats renewable penetration as a system-level quantity. While total renewable penetration is useful for

assessing decarbonization performance, it does not show how renewable siting and synchronous-generator displacement affect local inertial support. This becomes important when renewable generation is concentrated in particular areas of the network, causing spatially non-uniform inertia distribution.

2.5.3 Frequency-Constrained Generation Scheduling

To address the dynamic limitations of conventional scheduling, several studies have incorporated frequency-security constraints into unit commitment and dispatch formulations. These approaches commonly derive analytical constraints from simplified frequency-response models. Typical constraints include RoCoF limits, frequency nadir limits, steady-state frequency deviation limits, and minimum inertia requirements [21, 25, 50].

Frequency-constrained scheduling provides an important improvement because it links scheduling decisions to post-disturbance frequency performance. For example, the commitment of synchronous generators can be constrained so that the system retains sufficient inertia to keep RoCoF within an acceptable limit. Similarly, primary frequency-response constraints can be included to ensure that the frequency nadir remains above the required security threshold.

The advantage of these formulations is their operational practicality. By converting frequency-security requirements into algebraic constraints, they can be embedded into optimization models without running a full time-domain simulation for every candidate dispatch. This makes them attractive for day-ahead and operational planning studies.

However, most frequency-constrained scheduling approaches rely on aggregated frequency models, such as center-of-inertia or average system frequency representations. This means that frequency security is assessed at the system level rather than at individual buses. While such methods can ensure that the system-wide frequency response satisfies a prescribed criterion, they may not identify local

frequency vulnerabilities caused by uneven inertia distribution or weak electrical coupling. Therefore, a dispatch solution may satisfy an aggregated RoCoF constraint while some buses still experience excessive local RoCoF.

2.5.4 Inertia-Constrained Scheduling

Inertia-constrained scheduling directly treats inertia as an operational security requirement. In these formulations, the commitment of synchronous generators is constrained so that the total online inertia remains above a minimum value derived from RoCoF or frequency-nadir criteria [26, 60, 61]. This approach is conceptually simple and computationally attractive because it converts frequency-security requirements into a schedulable constraint.

System-level inertia constraints are useful for first-level frequency-security screening. They can prevent excessive decommitment of synchronous machines and ensure that renewable dispatch does not reduce system inertia below a minimum secure level. They are also compatible with existing unit commitment and economic dispatch frameworks.

Despite these strengths, system-level inertia constraints have an important limitation: they do not consider where inertia is located. The total online inertia may be sufficient, but if the committed synchronous generators are electrically remote from vulnerable load or renewable-rich areas, local frequency performance may still be poor. This limitation is especially relevant in systems with high VRES penetration, where synchronous generation is displaced non-uniformly and the remaining inertia may be concentrated in a few areas.

Area-level inertia-constrained scheduling improves this representation by dividing the system into zones and enforcing minimum inertia requirements within each zone [61, 62]. Such approaches provide better spatial awareness than purely system-level constraints and can help identify regional inertia deficiencies. However, zonal aggregation still masks bus-level variations. Within a zone, some buses may be

strongly connected to synchronous inertia sources while others may be weakly coupled. Therefore, zonal inertia constraints may still be insufficient for identifying local RoCoF vulnerabilities.

Bus-level frequency-constrained scheduling represents a further advancement because it attempts to enforce frequency-security criteria at individual buses. Locational RoCoF-constrained formulations have shown that local frequency behavior can be included in security-constrained unit commitment [52]. However, these methods often require disturbance-specific frequency-response expressions, modal decomposition, or repeated evaluation of locational RoCoF over selected time windows. This may increase computational complexity and reduce suitability for iterative multi-objective renewable dispatch studies.

2.5.5 Critical Link to Spatial Inertia-Aware Scheduling

The reviewed scheduling literature shows a clear progression from conventional cost-based scheduling to VRES-aware scheduling, frequency-constrained scheduling, and inertia-constrained scheduling. Each development addresses an important limitation of the previous approach. Conventional scheduling ensures economic operation but may ignore low-inertia frequency risks. VRES-aware scheduling captures renewable variability but may not guarantee frequency security. Frequency-constrained scheduling introduces dynamic security requirements but often relies on aggregated frequency models. Inertia-constrained scheduling treats inertia as a schedulable resource but usually represents it at system or zonal level.

The remaining gap is therefore not the absence of inertia constraints in scheduling, but the limited spatial resolution with which inertia is represented and enforced. In renewable-rich systems, the security value of synchronous inertia depends not only on its magnitude but also on its location and electrical coupling to network buses. A scheduling framework that maximizes inertia-less VRES dispatch should therefore ensure that the resulting operating point preserves adequate bus-level inertia support, not only adequate total system inertia.

This observation directly motivates spatial inertia-aware generation scheduling. In such a framework, synchronous generator commitment and dispatch decisions influence both active-power balance and the spatial distribution of effective inertia. The scheduler must therefore determine the maximum feasible VRES penetration while ensuring that each bus satisfies a minimum inertia requirement consistent with RoCoF-based frequency-security limits. This provides a stronger locational security criterion than aggregated inertia constraints and directly supports the objective of maintaining frequency stability under high penetration of inertia-less VRES.

Accordingly, this thesis extends the existing literature by embedding bus-level inertia adequacy into a multi-objective generation scheduling framework. The proposed approach differs from conventional aggregated inertia-constrained scheduling by explicitly treating inertia as a spatially distributed network property and by enforcing local inertia requirements during VRES dispatch optimization.

2.6 Inertia Estimation Techniques

Accurate inertia estimation is essential for assessing frequency stability in power systems with increasing penetration of inertia-less renewable generation. As synchronous generators are displaced by inertia-less or low-inertia resources, the available inertial support becomes time-varying, operating-point dependent, and spatially non-uniform. This has motivated extensive research on methods for estimating inertia at different levels of system resolution, ranging from system-wide aggregated inertia to area-level, nodal, and bus-level inertia representations.

Inertia estimation techniques differ significantly in terms of required data, modeling assumptions, spatial resolution, computational complexity, and suitability for operational planning. Some methods rely on large disturbance records, while others use ambient measurements, phasor measurement unit (PMU) data, dynamic simulation models, or data-driven learning techniques. Although these approaches

have improved the visibility of inertia in modern power systems, their usefulness for generation scheduling depends on whether they can provide quantitative, repeatable, and computationally efficient inertia indicators under different dispatch conditions.

2.6.1 Aggregated Inertia Estimation Methods

Aggregated inertia estimation methods represent the power system using a single equivalent inertia value. This value is usually associated with the center-of-inertia frequency or average system frequency and reflects the combined inertial contribution of all online synchronous machines [16]. Aggregated inertia estimation is attractive because it provides a simple and physically interpretable measure of the system’s ability to resist rapid frequency change after an active-power disturbance.

Early disturbance-based approaches estimate aggregated inertia from recorded frequency transients following events such as generator outages, load changes, or other large disturbances. In these methods, the measured initial rate of change of frequency is combined with the estimated active-power imbalance to infer the equivalent system inertia [63, 64]. The main strength of this approach is its direct physical connection to the swing-equation interpretation of frequency dynamics. It is also relatively simple and can be applied using post-event data.

However, disturbance-based estimation has important limitations. It requires a sufficiently large and clearly identifiable disturbance, accurate knowledge of the disturbance magnitude, and high-quality frequency measurements. The estimated inertia may also be biased if primary frequency control, load damping, or converter-based fast frequency response becomes active within the estimation window. These limitations make disturbance-based aggregated inertia estimation useful for post-event assessment and model validation, but less suitable for continuous scheduling studies where inertia must be evaluated repeatedly for many candidate dispatch solutions.

Other aggregated inertia estimation methods use simulation-based fitting, distur-

bance replay, modal identification, or system identification. In these approaches, inertia is treated as an unknown parameter and is adjusted or identified so that the simulated or estimated frequency response matches the measured response. Compared with simple RoCoF-based methods, these approaches can incorporate richer dynamic information, including generator dynamics, governor response, load-frequency sensitivity, damping effects, and measurement-based modal behavior. They may therefore provide more reliable estimates when the adopted model is well calibrated and the measurement data are of sufficient quality. However, their accuracy depends strongly on model fidelity, parameter availability, disturbance observability, and measurement quality. Their computational burden can also become significant when many operating scenarios or candidate inertia values must be evaluated [65–68].

Online aggregated inertia estimation methods have also been proposed to support real-time situational awareness. These methods use PMU measurements, recursive parameter estimation, Kalman filtering, modal identification, or ambient frequency fluctuations to track changes in system inertia under varying operating conditions [64, 65, 67]. Their main advantage is that they can reveal time-varying inertia caused by renewable dispatch and synchronous generator decommitment. However, they remain primarily system-level monitoring tools. They do not directly indicate how inertia is distributed across network buses, nor do they provide a straightforward way to impose bus-level inertia adequacy constraints in generation scheduling.

Therefore, aggregated inertia estimation remains useful for system-wide frequency-security screening, but it is insufficient when the research objective is to identify local inertia weaknesses and enforce spatially resolved frequency-security constraints.

2.6.2 Spatial and Nodal Inertia Estimation Methods

Spatial and nodal inertia estimation methods were developed to address the limitations of system-wide inertia representation. Instead of describing the system

using a single equivalent inertia value, these methods attempt to capture how inertial support varies across areas, nodes, or buses. This is particularly important in renewable-rich systems where synchronous generation may be displaced unevenly, causing inertia-rich and inertia-weak regions to emerge within the same network [68].

One group of spatial methods uses inertia distribution indices derived from local frequency responses. For example, inertia distribution index approaches compare the frequency deviation at each bus with a reference frequency, such as the center-of-inertia frequency, and use the resulting deviation to infer the relative strength or weakness of inertial support at different locations [30, 31]. These methods are valuable because they demonstrate that inertia effects are not uniform across the network. They provide practical insight into which buses or regions behave closer to the system inertial center and which are more dynamically remote.

The main limitation of index-based methods is that they generally provide relative indicators rather than absolute bus-level inertia values. A bus may be ranked as weak or strong relative to other buses, but the index itself may not be directly usable as a quantitative inertia constraint in an optimization model. In addition, many index-based methods depend on disturbance response data, observation windows, PMU placement, and the selected reference frequency. These dependencies limit their direct applicability in offline planning studies where inertia must be estimated before an actual disturbance occurs.

Another class of spatial methods uses electromechanical wave propagation, modal analysis, dynamic mode decomposition, or measurement-driven identification to infer changes in inertia distribution [51, 69, 70]. These approaches are useful for monitoring and interpreting dynamic behavior in large networks. They can reveal how disturbances propagate and how low-inertia areas influence inter-area oscillations. However, they often require dense, synchronized measurements and careful signal processing. Their outputs may also be sensitive to noise, mode selection, disturbance location, and the quality of measurement data.

Nodal inertia mapping methods provide a more quantitative representation by

attempting to estimate effective inertia at individual buses or nodes. Analytical nodal inertia formulations, such as frequency-divider or reduced-network-based methods, represent an important advancement because they move beyond relative indices and seek to quantify inertia at higher spatial resolution [33]. Such approaches are closer to the needs of inertia-aware scheduling because they recognize that the effective inertia observed at a bus depends on its electrical coupling to synchronous generators.

Despite this improvement, existing nodal inertia estimation methods may still depend on frequency measurements, frequency-divider estimation, disturbance response, or simplified assumptions regarding network coupling. In addition, most of these methods are developed mainly for inertia mapping or monitoring, rather than for repeated integration into multi-objective generation scheduling. This creates a methodological gap between spatial inertia estimation and operational decision-making.

2.6.3 Measurement-Based, Model-Based, and Data-Driven Approaches

From a methodological perspective, inertia estimation techniques can be broadly grouped into measurement-based, model-based, and data-driven approaches. Each category has strengths, but each also has limitations when applied to spatial inertia-aware generation scheduling.

Measurement-based methods estimate inertia using observed frequency, RoCoF, power imbalance, modal response, or ambient system data. Their main advantage is that they are grounded in actual system behavior and can capture operating-point-dependent dynamics. PMU-based methods, in particular, have improved the practical feasibility of inertia monitoring because they provide high-resolution synchronized frequency and voltage-angle measurements [66, 71]. However, measurement-based methods require sufficient measurement coverage, accurate time synchroniza-

tion, noise filtering, and identifiable dynamic excitation. In systems with sparse PMU deployment, weak disturbance signals, or uncertain disturbance magnitudes, the reliability of the estimates may be reduced.

Model-based methods use physical network models, generator parameters, dynamic equations, or reduced network representations to estimate inertia. Their strength lies in their interpretability and their suitability for planning studies, where different operating cases can be evaluated without waiting for real disturbances. Model-based approaches are also useful when the objective is to test how generator commitment, renewable dispatch, or network configuration affects inertia distribution. However, their accuracy depends on the quality of system data and on the validity of modeling assumptions. If generator parameters, network topology, load models, or converter representations are inaccurate, the estimated inertia distribution may not fully match the actual dynamic response.

Model-based inertia estimation methods use physical network models, generator parameters, dynamic equations, or reduced network representations to estimate the effective distribution of inertia across the system [69]. Their main strength is interpretability, since the estimated inertia can be related to generator locations, electrical coupling, network topology, and operating conditions. These methods are particularly suitable for planning studies, where different generator commitment patterns, renewable dispatch levels, and network configurations can be evaluated without relying on the occurrence of real disturbances. They are also useful when the objective is to examine how inertia availability changes across buses, zones, or areas under different operating scenarios [62, 72].

However, the accuracy of model-based approaches depends strongly on the quality of system data and the validity of modeling assumptions. If generator parameters, network topology, load models, controller settings, or converter representations are inaccurate, the estimated inertia distribution may not fully reproduce the actual dynamic response of the system.

Data-driven methods use machine learning, neural networks, covariance analysis,

dynamic mode decomposition, or hybrid cyber-physical models to infer inertia from measured or simulated data [67, 69, 73, 74]. These methods can capture complex nonlinear patterns and may perform well when trained on representative datasets. They are especially attractive for real-time monitoring and for systems where explicit physical modeling is difficult. However, their performance depends on the quality, diversity, and representativeness of training data. They may also suffer from limited interpretability and uncertain generalization when operating conditions differ from those used during training.

For the purpose of this thesis, the most relevant distinction is not only whether a method is measurement-based, model-based, or data-driven, but whether it can support offline and repeated evaluation of bus-level inertia under alternative generation scheduling decisions. A method suitable for this purpose should be quantitative, spatially resolved, computationally efficient, and applicable before a disturbance occurs. Many existing approaches satisfy some of these requirements, but not all simultaneously.

2.6.4 Limitations of Existing Inertia Estimation Methods

The reviewed inertia estimation literature provides important foundations for understanding and monitoring inertia in renewable-rich power systems. Aggregated methods establish the basic relationship between inertia, disturbance size, and RoCoF. Online and measurement-based methods improve real-time situational awareness. Spatial index methods demonstrate that inertia effects vary across the network. Nodal estimation methods move closer to quantitative bus-level representation. Data-driven methods offer powerful tools for extracting inertia-related patterns from large measurement datasets.

However, several limitations remain when these methods are evaluated against the needs of spatial inertia-aware generation scheduling.

First, many inertia estimation methods are aggregated in nature. They estimate

the total system inertia but do not identify how much effective inertial support is available at each bus. This limits their ability to detect local inertia deficiencies and makes them unsuitable for enforcing bus-level inertia constraints.

Second, many spatial methods provide relative indicators rather than absolute inertia values. While relative indices are useful for ranking buses or identifying weak regions, they cannot always be directly translated into minimum inertia requirements for optimization-based scheduling.

Third, several methods are disturbance-dependent. They require a generator outage, load change, or other transient event to excite the frequency response from which inertia is estimated. Such dependence is suitable for post-event analysis, but it is less suitable for offline scheduling because the scheduler must evaluate inertia adequacy before disturbances occur.

Fourth, many methods depend on dense PMU measurements or high-quality dynamic data. This restricts their applicability in systems where measurement infrastructure is limited or where the study is based on planning models rather than real-time operational data.

Fifth, some methods are computationally intensive because they rely on repeated time-domain simulations, modal decomposition, signal processing, or training of data-driven models. This may limit their use inside iterative optimization algorithms, where inertia must be evaluated many times across candidate solutions.

Finally, the connection between inertia estimation and generation scheduling remains underdeveloped. Existing methods often estimate or monitor inertia as a separate analytical task, while scheduling formulations often use simplified aggregated inertia constraints. As a result, the locational contribution of each synchronous generator to each bus is not fully incorporated into the scheduling decision.

These limitations motivate the development of an offline bus-level inertia estimation approach that can quantify the effective inertial support at each bus using network and operating-point information. Such an approach is particularly useful for

planning and scheduling studies because it can be applied repeatedly under different generator commitment, dispatch, and VRES penetration scenarios. In the context of this thesis, bus-level inertia estimation provides the analytical foundation for deriving local minimum inertia thresholds and embedding spatial inertia adequacy constraints into the generation scheduling framework.

2.7 Optimization Methods for Inertia-Aware Scheduling

Inertia-aware generation scheduling is a constrained, nonlinear, and multi-objective optimization problem. The scheduler must determine the commitment and dispatch of synchronous generators, the allowable dispatch of inertia-less VRES, and the corresponding operating condition of the network while satisfying power-balance, generator, voltage, line-flow, and frequency-security constraints. When spatial inertia adequacy is included, the problem becomes more complex because generator scheduling decisions affect not only active-power balance and losses, but also the distribution of effective bus-level inertia across the network.

The optimization problem considered in this research has several characteristics that influence the choice of solution algorithm. First, the decision space contains both continuous variables, such as generator dispatch levels and VRES output, and discrete or integer-like variables, such as generator commitment status. Second, the objective functions are conflicting. Increasing inertia-less VRES dispatch may reduce reliance on synchronous generation, but it can also reduce inertial support and alter transmission losses. Third, the constraints are nonlinear because power-flow equations, voltage limits, transmission losses, and bus-level inertia adequacy are coupled through the network operating point. Fourth, the desired solution is not a single optimum, but a set of Pareto-optimal trade-off solutions that allows the operator to evaluate competing objectives.

For these reasons, multi-objective optimization methods are more suitable than

single-objective optimization methods for this study. A single-objective formulation would require the competing objectives to be combined using fixed weights. Such weighting may obscure important trade-offs and can bias the solution toward one operating preference. In contrast, multi-objective optimization allows the relationship between VRES penetration, power losses, and inertia adequacy to be explored explicitly through Pareto fronts.

2.7.1 Mathematical Programming Approaches

Classical mathematical programming methods, including linear programming, nonlinear programming, mixed-integer linear programming, and mixed-integer nonlinear programming, have been widely applied to unit commitment, economic dispatch, optimal power flow, and frequency-constrained scheduling problems. Their main advantage is that they provide structured formulations with clear optimality conditions. In some cases, especially when the problem can be linearized, mathematical programming methods can provide computationally efficient and reproducible solutions.

Frequency-constrained unit commitment studies commonly use mathematical programming because RoCoF, frequency nadir, and reserve constraints can sometimes be approximated into algebraic or linear forms. For example, aggregated inertia constraints can be expressed as minimum online inertia requirements, making them suitable for mixed-integer scheduling formulations [25, 26, 50]. Such approaches are attractive for large-scale system operation because they can be solved using mature commercial solvers.

However, mathematical programming approaches face limitations when the scheduling problem includes nonlinear AC power flow, mixed discrete-continuous variables, multiple conflicting objectives, and spatially resolved inertia constraints. Linearization can improve tractability, but it may also reduce the physical accuracy of frequency-security representation. Similarly, converting multiple objectives into a weighted single-objective function requires predefined weights, which may not be

known before the trade-off surface is explored. Therefore, although mathematical programming methods are powerful, they are not always convenient when the goal is to explore nonlinear Pareto trade-offs under complex operational constraints.

2.7.2 Metaheuristic and Evolutionary Multi-Objective Algorithms

Metaheuristic and evolutionary algorithms provide an alternative approach for solving nonlinear, non-convex, and multi-objective power system optimization problems. These algorithms do not require the objective functions or constraints to be differentiable, and they can handle discrete and continuous decision variables within the same search process. This makes them useful for problems involving unit commitment, renewable dispatch, power losses, and nonlinear security constraints.

Among evolutionary multi-objective algorithms, the Non-dominated Sorting Genetic Algorithm II (NSGA-II) is one of the most widely used methods. NSGA-II ranks solutions using non-dominated sorting and preserves diversity using crowding distance. Its elitist selection mechanism ensures that high-quality non-dominated solutions are carried forward across generations [75]. These features make NSGA-II suitable for approximating Pareto fronts in problems where the objectives conflict and the feasible region is nonlinear or discontinuous.

The main strength of NSGA-II in the context of inertia-aware scheduling is its ability to generate a diverse set of trade-off solutions without requiring predetermined weights for the objectives. This is important because the relative importance of VRES dispatch maximization, loss minimization, and inertia-security preservation may depend on the operator's planning preference. NSGA-II also accommodates mixed decision variables and can be coupled with external power-flow and inertia-evaluation routines.

However, NSGA-II is not without limitations. Its performance depends on algorithm parameters such as population size, number of generations, crossover probability,

mutation probability, and constraint-handling strategy. It may require a relatively large number of fitness evaluations, especially when each candidate solution must be assessed using power-flow analysis and bus-level inertia calculations. It also does not guarantee global optimality. Therefore, its application should be accompanied by parameter justification, convergence analysis, and repeated simulations where possible.

2.7.3 Comparison of NSGA-II with Alternative Algorithms

Although NSGA-II is appropriate for this study, it is not the only candidate algorithm for multi-objective inertia-aware scheduling. Other algorithms, including SPEA2, MOEA/D, MOPSO, NSGA-III, and MOGWO, could also be considered. Each has specific strengths and limitations depending on problem size, objective count, constraint structure, and desired Pareto-front characteristics [75–80].

The comparison in table 2.1 shows that alternative algorithms could be applied to the inertia-aware scheduling problem, but their use would mainly affect the search process rather than the mathematical formulation of the problem. The objective functions, power-flow constraints, and bus-level inertia adequacy constraints would remain essentially the same. What would change is the way candidate solutions are generated, ranked, diversified, and improved across iterations [75–78, 80]

Table 2.1: Comparison of candidate multi-objective optimization algorithms for inertia-aware generation scheduling

Algorithm	Main Principle	Strengths for Inertia-Aware Scheduling	Limitations / Shortcomings
NSGA-II	Non-dominated sorting with elitism and crowding-distance-based diversity preservation	Well established for nonlinear multi-objective problems; handles mixed discrete-continuous variables; provides diverse Pareto solutions; does not require objective weighting; relatively easy to integrate with power-flow and inertia-evaluation routines	Requires parameter tuning; may need many fitness evaluations; computational time increases when AC power flow and inertia constraints are evaluated repeatedly; does not guarantee global optimality
SPEA2	Strength Pareto ranking with an external archive and density estimation	Maintains an archive of elite non-dominated solutions; can preserve good Pareto solutions across generations; suitable for constrained multi-objective problems	Archive size and density estimation affect performance; may lose diversity if archive truncation is not well managed; computational effort increases with population and archive size
MOEA/D	Decomposes a multi-objective problem into scalar subproblems using weight vectors	Computationally efficient for some multi-objective problems; can distribute search across different regions of the Pareto front; useful when decomposition is meaningful	Performance depends strongly on the choice and distribution of weight vectors; may struggle with irregular, disconnected, or highly non-convex Pareto fronts; constraint handling can be more complex
MOPSO	Particle swarm search extended to multi-objective optimization using an external archive	Simple concept; often converges quickly; suitable for continuous dispatch variables; fewer genetic operators required	Less natural for binary or mixed-integer commitment variables; may suffer from premature convergence; diversity maintenance depends strongly on archive and leader selection mechanisms
NSGA-III	Reference-point-based extension of NSGA-II for many-objective optimization	Suitable when the number of objectives is large; maintains diversity using reference directions; useful for problems with more than three objectives	More complex than NSGA-II; reference-point selection affects results; may be unnecessary for problems with two or three objectives; additional tuning is required
MOGWO	Population-based search inspired by grey wolf leadership hierarchy and hunting behavior	Simple implementation; good exploration capability; can be adapted to multi-objective dispatch problems; useful for nonlinear search spaces	Less theoretically established than NSGA-II for constrained power-system scheduling; may require careful tuning to avoid premature convergence; binary commitment handling may require additional encoding

For example, SPEA2 could be used to maintain an external archive of non-dominated solutions [76], while MOEA/D could decompose the scheduling problem into weighted scalar subproblems [77]. MOPSO could search the dispatch space through swarm movement [78], while NSGA-III would be more attractive if the problem were expanded to include many objectives, such as cost, emissions, VRES penetration, power losses, voltage stability, and inertia adequacy simultaneously [79]. MOGWO could also be considered as an alternative population-based search method [80]. However, these alternatives introduce their own tuning requirements and may not necessarily improve performance for the two- or three-objective structure considered in this thesis.

2.7.4 Justification for Selecting NSGA-II

NSGA-II is selected in this study because it provides a practical balance between solution quality, implementation simplicity, diversity preservation, and compatibility with the structure of the proposed scheduling problem. The problem contains nonlinear constraints, mixed decision variables, and conflicting objectives. NSGA-II can handle these features without requiring gradient information or convexity assumptions.

Another reason for selecting NSGA-II is its ability to produce a Pareto front rather than a single solution. This is important for the present study because the research objective is not merely to find the lowest-loss operating point or the highest VRES dispatch point. Instead, the objective is to examine the trade-off between maximizing inertia-less VRES penetration, minimizing transmission power losses, and maintaining bus-level inertia adequacy. The Pareto front allows these trade-offs to be interpreted explicitly.

NSGA-II is also suitable because the spatial inertia evaluation can be embedded within the fitness-evaluation stage. For each candidate schedule, the algorithm can run power-flow analysis, estimate the bus-level inertia distribution, evaluate minimum inertia constraints, compute objective values, and classify the solution

according to feasibility. This makes NSGA-II compatible with the physics-based evaluation process required by the proposed framework.

Nevertheless, the use of NSGA-II requires careful implementation. The population size, number of generations, crossover rate, mutation rate, and constraint-handling strategy influence convergence and diversity. Therefore, the methodology and results chapters should report the selected parameter values, justify them using either preliminary tuning or literature guidance, and present convergence behavior. This is necessary to demonstrate that the obtained Pareto fronts are not arbitrary outcomes of poorly selected algorithm settings.

2.7.5 Implications for This Study

The optimization method used in this thesis should be understood as the search mechanism for solving the proposed inertia-aware scheduling formulation. If another algorithm were used, the physical meaning of the formulation would not change. The same bus-level inertia estimation method, RoCoF-based inertia thresholds, power-flow constraints, and objective functions would still define the problem. However, the convergence speed, diversity of Pareto solutions, computational burden, and quality of the final trade-off front could differ.

This distinction is important because the main contribution of the thesis lies in the formulation of a spatial inertia-aware scheduling framework, not merely in the use of a particular optimization algorithm. NSGA-II is adopted as a suitable and established algorithm for solving the resulting multi-objective problem. Future work may compare NSGA-II with SPEA2, MOEA/D, MOPSO, NSGA-III, MOGWO, or hybrid algorithms to evaluate whether computational efficiency, convergence robustness, or Pareto-front diversity can be improved for larger systems or real-time applications.

2.8 Synthetic Inertia, Grid-Forming Converters, and BESS Support

The increasing displacement of synchronous generators by inertia-less renewable generation has motivated the development of alternative frequency-support mechanisms. Since solar PV, wind power plants, and battery energy storage systems are connected to the grid through power electronic converters, their contribution to frequency stability depends largely on the adopted converter control strategy. Unlike synchronous generators, these resources do not automatically release stored kinetic energy in response to a frequency disturbance. However, with suitable control design, they can provide fast frequency response, synthetic inertia, damping support, and voltage-frequency regulation services.

Synthetic inertia, grid-forming converter control, and battery energy storage systems are therefore important complementary technologies for low-inertia power systems. They do not eliminate the need to understand spatial inertia distribution, but they can modify the amount, location, and speed of frequency support available to the network. Their inclusion is particularly relevant when VRES penetration is high and the commitment of synchronous generators is reduced. This section reviews these technologies and discusses their relevance to spatial inertia-aware generation scheduling.

2.8.1 Synthetic Inertia from Converter-Interfaced Resources

Synthetic inertia, also referred to as virtual inertia or emulated inertia, is a control-based response that enables converter-interfaced resources to imitate part of the inertial behavior of synchronous machines. Instead of relying on natural electromechanical coupling between rotor speed and grid frequency, synthetic inertia uses measured frequency or RoCoF signals to modify the active-power output of a converter. When frequency declines, the converter temporarily increases active-

power injection. When frequency rises, it may reduce active-power injection or absorb energy, depending on the technology and control objective [81].

The basic concept is that the converter active-power reference is modified according to a frequency-sensitive control law. In a simplified form, the additional active-power command can be expressed conceptually as [82]

$$\Delta P_{\text{SI}} = -K_f \Delta f - K_{\text{RoCoF}} \frac{df}{dt}, \quad (2.10)$$

where ΔP_{SI} is the synthetic inertia or fast frequency-response power command, Δf is the frequency deviation, df/dt is the RoCoF, and K_f and K_{RoCoF} are frequency and RoCoF response gains. The first term provides frequency-droop-like support, while the second term imitates inertial response by reacting to the rate of frequency change.

Wind turbines are commonly discussed as potential providers of synthetic inertia because their rotating blades and generator systems store kinetic energy. However, modern variable-speed wind turbines are usually decoupled from grid frequency by power electronic converters. Therefore, their stored kinetic energy is not naturally released during grid frequency decline unless specific inertial control is implemented [18, 83]. Synthetic inertia from wind turbines must also consider rotor-speed recovery after energy extraction. If excessive kinetic energy is released, the turbine may need a recovery period during which its active-power output is reduced, potentially causing a secondary frequency dip [83].

Solar PV plants do not contain significant rotating mass and therefore cannot provide synthetic inertia from mechanical kinetic energy. However, PV plants can provide fast frequency response if operated with power reserve, for example by deloading from the maximum power point. In such operation, a PV plant intentionally operates below available power so that headroom is available for rapid active-power injection during frequency decline [84–86]. The limitation is that this requires renewable curtailment during normal operation, which creates an economic

trade-off between maximum renewable energy extraction and frequency-support capability.

Battery energy storage systems can also provide synthetic inertia and fast frequency response. Unlike wind and PV, BESS can inject or absorb active power rapidly, subject to state-of-charge, converter rating, and energy capacity constraints. This makes BESS particularly suitable for frequency regulation in low-inertia systems [87]. However, BESS-based synthetic inertia is still control-dependent rather than physical synchronous inertia. Its effectiveness depends on measurement accuracy, control response time, state of charge, power rating, and coordination with other frequency-support resources.

Synthetic inertia is therefore valuable, but it should not be treated as identical to synchronous inertia. Physical inertia responds immediately through electromechanical dynamics and does not require measurement or control action. Synthetic inertia depends on converter control, communication or local measurement, current limits, energy availability, and control tuning. It can improve RoCoF and frequency nadir, but its contribution must be explicitly modelled if it is to be included in scheduling or stability assessment.

2.8.2 Grid-Following and Grid-Forming Converter Control

Converter-interfaced resources may operate under grid-following or grid-forming control modes. Grid-following converters are the most common type in existing renewable energy installations. They synchronize to the grid using a phase-locked loop and inject active and reactive current according to external references. This type of converter assumes that a stable voltage and frequency reference already exists in the network. As a result, grid-following converters can support the grid through controlled current injection, but they do not inherently establish grid frequency or voltage [88].

In low-inertia or weak-grid conditions, heavy reliance on grid-following converters

may create stability challenges. Since these converters depend on grid voltage-angle tracking, their performance can be affected by weak short-circuit strength, voltage disturbances, phase-locked-loop interactions, and current-limiting behavior. They may provide fast active-power support, but they do not naturally replace the synchronizing function of synchronous machines [88, 89].

Grid-forming converters provide a different form of support. Instead of following an externally imposed voltage waveform, they are controlled to establish a voltage magnitude and frequency reference at their terminals. Depending on the control structure, grid-forming converters can emulate synchronous-machine behavior, provide virtual inertia, support voltage regulation, improve damping, and contribute to system strength. Common grid-forming concepts include droop control, virtual synchronous machine control, matching control, and related voltage-source converter strategies [90, 91].

From the perspective of frequency stability, grid-forming converters are important because they can support frequency containment even when synchronous inertia is low [90]. They can respond faster than conventional turbine-governor systems and can be located strategically in weak areas of the network. However, their performance depends on converter rating, control parameters, current limits, dc-side energy availability, and coordination with protection systems. Unlike synchronous inertia, grid-forming support is not defined only by a machine inertia constant. It must be represented through control dynamics and active-power capability.

Therefore, the emergence of grid-forming converters expands the concept of stability support in modern power systems. Stability is no longer supplied only by rotating machines, but by a combination of physical inertia, converter control, fast active-power injection, voltage regulation, and energy storage. This has direct implications for future inertia-aware scheduling models, which may need to co-optimize synchronous inertia, synthetic inertia, and grid-forming converter support.

2.8.3 Battery Energy Storage Systems for Frequency Support

Battery energy storage systems are increasingly deployed to support renewable integration and frequency stability. Their fast response, bidirectional active-power capability, and flexible control make them suitable for several ancillary services, including primary frequency response, fast frequency response, synthetic inertia, reserve provision, peak shaving, and renewable smoothing [87, 92].

In low-inertia systems, BESS can improve frequency stability in three main ways. First, it can provide fast active-power injection immediately after a generation deficit, thereby reducing RoCoF and improving frequency nadir. Second, it can absorb active power during over-frequency events, helping to contain frequency rise. Third, if controlled as a grid-forming resource, BESS can contribute to voltage and frequency reference formation, especially in weak grids or isolated systems [93].

The inclusion of BESS changes the generation scheduling problem. Without BESS, frequency-security constraints are mainly satisfied by retaining enough synchronous inertia and primary frequency response. With BESS, part of the frequency-support requirement can be supplied by fast converter-based active-power response. Therefore, the scheduler may be able to accommodate higher VRES penetration while maintaining frequency security. However, this requires explicit modeling of BESS power limits, energy limits, state of charge, converter rating, response time, and control mode [94, 95].

For scheduling purposes, the frequency-support contribution of BESS can be represented as an additional security resource. A simplified active-power constraint may be expressed as

$$-P_{\text{BESS}}^{\text{ch},\text{max}} \leq P_{\text{BESS}}(t) \leq P_{\text{BESS}}^{\text{dis},\text{max}}, \quad (2.11)$$

where $P_{\text{BESS}}(t)$ is the BESS active-power output, $P_{\text{BESS}}^{\text{dis},\text{max}}$ is the maximum discharge

power, and $P_{\text{BESS}}^{\text{ch,max}}$ is the maximum charging power. The energy state is usually constrained by

$$E_{\text{BESS}}^{\min} \leq E_{\text{BESS}}(t) \leq E_{\text{BESS}}^{\max}, \quad (2.12)$$

where $E_{\text{BESS}}(t)$ is the stored energy. These constraints show that BESS frequency support is not unlimited. It depends on both power capability and energy availability.

If BESS is used to provide synthetic inertia or fast frequency response, an additional reserve headroom requirement may be imposed:

$$P_{\text{BESS}}^{\text{headroom}}(t) \geq P_{\text{FFR}}^{\text{req}}(t), \quad (2.13)$$

where $P_{\text{FFR}}^{\text{req}}(t)$ is the required fast frequency-response capacity. This type of constraint can be integrated into future extensions of inertia-aware scheduling to coordinate synchronous inertia, VRES dispatch, and BESS-based frequency support.

Although BESS can significantly improve frequency security, it does not remove the need for spatial assessment. The location of BESS affects its ability to support local frequency and voltage behavior. A BESS installed near a weak or low-inertia area may provide more effective local support than one located electrically far from the disturbance-prone region. Therefore, BESS deployment should be considered not only in terms of total capacity, but also in terms of network location and interaction with spatial inertia distribution.

2.8.4 Relevance to Spatial Inertia-Aware Scheduling

Synthetic inertia, grid-forming converters, and BESS support are directly relevant to spatial inertia-aware scheduling, but they do not invalidate the need for bus-level inertia assessment. Instead, they extend the set of resources that can contribute

to frequency security. In a system without significant converter-based frequency support, local frequency stability depends heavily on the amount and location of synchronous inertia. In a system with BESS or grid-forming converters, frequency support can also be provided by fast controlled active-power injection. However, this support remains location-dependent and limited by converter and energy constraints.

The framework developed in this thesis focuses on inertia-less VRES penetration and synchronous-generator-based bus-level inertia adequacy. This is appropriate because the study aims to evaluate the frequency-security implications of replacing synchronous generation with renewable generation that does not inherently provide inertia. The adopted modeling scope therefore treats solar PV as an inertia-less resource and does not include detailed converter-control dynamics or BESS operation.

Nevertheless, the reviewed literature indicates that future extensions of the proposed framework could include synthetic inertia and BESS as additional spatial frequency-support resources. In such an extension, the bus-level inertia adequacy constraint could be expanded into a broader bus-level frequency-support adequacy constraint, combining physical synchronous inertia, synthetic inertia, fast frequency response, and grid-forming converter support. Conceptually, this may be expressed as

$$H_j^{\text{SG}}(t) + H_j^{\text{syn}}(t) + H_j^{\text{BESS}}(t) \geq H_j^{\text{min}}(t), \quad (2.14)$$

where $H_j^{\text{SG}}(t)$ represents the effective synchronous inertia at bus j , $H_j^{\text{syn}}(t)$ represents the equivalent synthetic inertia contribution from converter-interfaced resources, $H_j^{\text{BESS}}(t)$ represents the equivalent fast frequency-support contribution from BESS, and $H_j^{\text{min}}(t)$ is the minimum bus-level inertia or frequency-support requirement. The terms $H_j^{\text{syn}}(t)$ and $H_j^{\text{BESS}}(t)$ would require careful definition because converter-based support depends on control response, headroom, energy limits, and location.

This extension would allow the scheduler to determine not only which synchronous generators should remain online, but also where synthetic inertia or BESS support should be deployed to compensate for local inertia deficiencies. It would also allow higher VRES penetration while maintaining bus-level frequency-security constraints. However, such an extension requires detailed converter and storage modeling, which is beyond the present scope of this thesis.

Therefore, synthetic inertia, grid-forming converters, and BESS should be viewed as important future complements to the proposed spatial inertia-aware scheduling framework. The present study establishes the foundation by quantifying spatial synchronous inertia and embedding bus-level inertia adequacy into VRES dispatch optimization. Future work can build on this foundation by incorporating converter-based frequency-support resources into the same spatial scheduling structure.

2.9 Comparative Summary of Related Studies

The preceding sections show that research on low-inertia power systems has developed along two closely related directions. The first direction focuses on estimating or monitoring inertia, either as an aggregated system quantity or as a spatially distributed property. The second direction focuses on incorporating inertia and frequency-security requirements into generation scheduling, unit commitment, and renewable dispatch formulations. Both directions provide important foundations for this thesis.

However, the reviewed literature also shows that existing approaches differ significantly in their spatial resolution, data requirements, modeling assumptions, and suitability for repeated use within optimization. Some methods are physically interpretable but aggregated. Others provide spatial information but rely on PMU measurements, disturbance data, or relative indices. Similarly, several scheduling frameworks incorporate frequency-security constraints, but many represent inertia at system or zonal level rather than as a bus-level operational constraint.

Tables 2.2 and 2.3 summarize representative studies related to inertia estimation, spatial inertia assessment, inertia-constrained scheduling, and converter-based frequency support. The aim is not to present previous work as inadequate, but to identify how each contribution supports the development of the present study and where specific methodological gaps remain.

2.9.1 Comparison of Inertia Estimation and Spatial Inertia Studies

Table 2.2: Comparative summary of selected studies on inertia estimation and spatial inertia assessment

Study	Main Focus	Method / Scale	Strengths	Limitations	Relevance to This Thesis
Chassin et al. [63]	Estimation of system inertia from observed frequency transients	Disturbance-based aggregated inertia estimation	Provides a direct physical link between disturbance size, frequency response, and equivalent inertia; useful for post-event assessment	Requires a sufficiently large disturbance and accurate knowledge of the power imbalance; provides system-level inertia only	Establishes the foundation for RoCoF-based inertia estimation but does not address bus-level inertia distribution
Makolo et al. [64, 64]	Data-driven inertia estimation under high renewable penetration	Frequency-gradient and reduced-order dynamic model approach	Can estimate time-varying inertia using measured system response; useful for renewable-rich systems	Measurement-dependent and mainly system-level; accuracy depends on model identification and quality of frequency data	Supports the need for inertia visibility under VRES penetration, but does not directly provide offline bus-level inertia for scheduling
Yang et al. [65]	Inertia estimation using ambient data and modal identification	Ambient-data-driven aggregated inertia estimation	Avoids reliance on large disturbances; uses normal operating data	Sensitive to measurement quality, excitation level, and modal identification accuracy; limited spatial resolution	Shows the value of measurement-driven inertia estimation, but remains less suitable for deterministic offline optimization
Panda et al. [66]	Online inertia estimation using synchrophasor measurements	PMU-based local and system inertia estimation	Uses high-resolution measurements and can support real-time monitoring	Requires PMU availability and reliable signal processing; disturbance and measurement quality influence accuracy	Demonstrates the feasibility of online inertia tracking, but the thesis requires an offline method usable before dispatch decisions are implemented

Study	Main Focus	Method / Scale	Strengths	Limitations	Relevance to This Thesis
Zeng et al. [71]	Online inertia distribution identification during normal operation	Ambient PMU measurements and system identification	Estimates inertia distribution without requiring severe disturbance events	Depends on measurement coverage, excitation quality, and identification assumptions	Relevant to spatial inertia monitoring, but less convenient for repeated optimization studies without measurement data
Pulgar-Painemal et al. [30]	Inertia distribution, inter-area oscillations, and location of electronically interfaced resources	Inertia Distribution Index based on dynamic frequency behavior	Demonstrates that inertia distribution affects local frequency response and inter-area oscillations	Provides relative spatial indices rather than explicit bus-level inertia values; depends on dynamic response data	Strongly motivates spatial inertia assessment, but does not directly provide quantitative bus-level inertia constraints for scheduling
You et al. [51]	Identification of inertia distribution changes in high-renewable systems	Electromechanical wave propagation using PMU data	Non-invasive and useful for identifying changes in inertia distribution	Requires synchronized measurements and disturbance propagation analysis; provides relative information	Supports the concept that inertia changes spatially, but the method is more suitable for monitoring than offline dispatch optimization
Gayathri and Jena [31]	Practical inertia distribution monitoring in renewable-penetrated grids	Center-of-power-based inertia distribution index	Useful for operational monitoring and identifying dynamically weak areas	Produces an index rather than an explicit inertia value; depends on wide-area measurements	Provides evidence that inertia distribution matters in practical grids, but does not directly enforce bus-level inertia adequacy in scheduling
Pinheiro et al. [33]	Mapping spatial distribution of nodal inertia	Analytical nodal inertia formulation using frequency-divider concepts	Provides a quantitative nodal view of inertia and advances beyond relative indices	Requires estimation of frequency-divider relationships and does not directly integrate nodal inertia into renewable dispatch optimization	Closely related to the thesis objective of bus-level inertia quantification, but the present study extends this toward offline scheduling constraints

Study	Main Focus	Method / Scale	Strengths	Limitations	Relevance to This Thesis
Li et al. [32]	Spatial-related inertia value for high wind integration	Spatial inertia assessment for wind-rich systems	Highlights the importance of inertia location under renewable integration	Focuses mainly on spatial assessment rather than full inertia-constrained dispatch scheduling	Supports the need to consider locational inertia when integrating VRES
Swetala et al. [96]	Offline estimation of power system inertia distribution	Offline bus-level inertia estimation using network reduction, generator current contribution, synchronizing power coefficient, and electrical distance	Provides a measurement-free and planning-oriented approach for estimating effective inertia at each bus	Requires accurate network and generator data; does not by itself solve the generation scheduling problem	Forms the analytical basis for the bus-level inertia constraints embedded in the present thesis framework

Table 2.2 shows that existing inertia estimation methods have made significant progress from aggregated inertia estimation toward spatial and nodal inertia assessment. Aggregated methods are simple and useful for system-wide security screening, but they cannot identify local inertia deficiencies. Measurement-based and data-driven methods improve real-time visibility, but they depend on the availability and quality of dynamic measurements. Spatial index methods reveal the non-uniformity of inertial response, but they often provide relative indicators rather than explicit bus-level inertia values. These observations motivate the need for an offline and quantitative bus-level inertia estimation method that can be repeatedly evaluated under different scheduling scenarios.

2.9.2 Comparison of Inertia-Constrained and VRES Scheduling Studies

Table 2.3: Comparative summary of selected studies on inertia-constrained scheduling and frequency-support resources

Study	Main Focus	Method / Scale	Strengths	Limitations	Relevance to This Thesis
Cai et al. [26]	Renewable penetration planning under inertia constraints	System-level minimum inertia constraint derived from RoCoF limit	Computationally practical and directly links inertia requirement to renewable planning	Uses aggregated system inertia and cannot detect bus-level frequency-security weaknesses	Supports the idea of inertia as a schedulable security constraint, but motivates the need for spatial refinement
Zhang et al. [25]	Frequency-constrained unit commitment for high renewable systems	Analytical frequency constraints based on average system frequency	Incorporates RoCoF and nadir-related security into unit commitment without repeated time-domain simulation	Frequency response is represented at system level; local frequency deviations are not explicitly constrained	Provides an important foundation for frequency-constrained scheduling, but does not address bus-level inertia adequacy
Wu et al. [50]	Unit commitment with enhanced frequency stability	MILP-based frequency-constrained unit commitment	Improves computational tractability by reformulating frequency constraints	Relies on aggregated frequency-security representation; detailed spatial inertia effects are not captured	Shows the benefit of tractable frequency-security constraints, but spatial frequency-security remains limited
Zhang et al. [21]	Short-term scheduling of wind, solar, hydro, and thermal resources	Multi-energy scheduling with dynamic frequency-response constraints	Considers complementary renewable and conventional resources; includes frequency-security indicators	Frequency constraints remain mainly system-oriented and do not explicitly quantify bus-level inertia support	Relevant to VRES-aware scheduling and frequency-security integration
Gu et al. [62]	Zonal inertia-constrained generator dispatch	Area-level minimum inertia constraints considering load-frequency relief	Improves on system-wide inertia constraints by considering area-level adequacy	Zonal aggregation may still hide bus-level variations inside each area	Demonstrates the importance of spatial granularity, motivating bus-level treatment

Study	Main Focus	Method / Scale	Strengths	Limitations	Relevance to This Thesis
Dhara and Rather [61]	Area-wise minimum inertia in renewable-dominated systems	Area-based inertia adequacy and inertia corridors	Recognizes that inertia adequacy should be assessed spatially rather than only system-wide	Operates at area scale; does not fully quantify generator-to-bus inertia contribution for dispatch optimization	Supports the transition from aggregated to spatial inertia constraints
Tuo and Li [52]	Security-constrained unit commitment with locational frequency stability	Locational RoCoF-constrained scheduling using network-coupled swing dynamics	Explicitly considers local RoCoF and represents an important step toward locational frequency-security scheduling	Requires disturbance-specific dynamic RoCoF evaluation and modal/time-window analysis; computational complexity may increase	Closest to the scheduling motivation of this thesis, but the present work focuses on offline bus-level inertia adequacy constraints suitable for iterative multi-objective VRES dispatch
O'Malley et al. [97]	Probabilistic scheduling of under-frequency load shedding in low-inertia systems	Stochastic scheduling of UFLS for credible contingencies	Recognizes uncertainty and protection-related risks in low-inertia operation	Focuses on emergency load shedding rather than preventive bus-level inertia scheduling	Highlights the consequences of insufficient frequency-security margins
Lei et al. [84]	PV power plant frequency support using local inertia information	Adaptive power reserve control for PV plants	Shows that PV plants can contribute to frequency support when operated with reserve headroom	Requires curtailment and converter-control modeling; not a full generation scheduling framework with bus-level inertia constraints	Relevant for future extension of the thesis toward PV-based fast frequency response
Farivar et al. [42]	Grid-connected energy storage technologies	Review of BESS and grid-support functions	Shows that BESS can provide fast response, reserve, and frequency regulation services	Storage support depends on state of charge, converter rating, energy capacity, and control strategy	Supports future extension of spatial inertia-aware scheduling to include BESS-based frequency support

Study	Main Focus	Method / Scale	Strengths	Limitations	Relevance to This Thesis
Shim et al. [92]	Integration of fast energy storage into frequency-control reserves	Energy-storage-based frequency reserve coordination	Demonstrates the value of fast-acting storage in high-renewable grids	Requires coordinated reserve modeling and storage constraints; not focused on spatial inertia estimation	Relevant to future incorporation of BESS into bus-level frequency-support adequacy constraints
Panagi and Aristidou [46]	Fast frequency response sizing for low-inertia systems	Data-driven sizing of fast frequency response reserves	Addresses frequency security under low inertia using fast response resources	Focuses on reserve sizing rather than generator scheduling with spatial inertia distribution	Supports the need to coordinate inertia and fast frequency-response resources in future work
Swetala et al. [98]	Optimal inertia-less VRES penetration and power loss minimization considering inertia distribution constraints	Multi-objective optimization with bus-level inertia distribution constraint	Integrates spatial inertia adequacy into VRES dispatch and loss minimization	Based on the modeling assumptions and benchmark systems adopted in the study; future work can extend to BESS, converter controls, and uncertainty	Represents the direct contribution developed and expanded in this thesis

Table 2.3 shows that frequency-constrained and inertia-constrained scheduling methods have advanced significantly beyond conventional economic dispatch. Existing studies have demonstrated how RoCoF, frequency nadir, reserve, and minimum inertia constraints can be incorporated into scheduling models. However, many of these methods rely on aggregated or zonal representations of inertia. Such representations are useful for reducing computational complexity but may not be sufficient when local frequency-security risks arise from non-uniform inertia distribution.

The reviewed studies also show that BESS, synthetic inertia, and fast frequency-response resources can improve frequency security in low-inertia systems. Nevertheless, their contribution depends on converter controls, available headroom, energy limits, and network location. These technologies therefore complement, rather than replace, the need for spatially resolved frequency-security assessment.

Overall, the comparative review indicates that the remaining research gap lies at the interface between spatial inertia estimation and operational scheduling. There is still a need for a framework that quantifies effective bus-level inertia under different generator commitment and VRES dispatch conditions, derives local inertia adequacy requirements from RoCoF security criteria, and embeds these requirements directly into a multi-objective generation scheduling problem.

2.10 Research Gaps

The reviewed literature shows that significant progress has been made in understanding frequency stability challenges in low-inertia power systems, estimating inertia under high penetration of inertia-less VRES, and incorporating frequency-security requirements into generation scheduling. Existing studies have established the relationship between inertia, RoCoF, frequency nadir, and renewable integration, and have proposed several system-level, area-level, and measurement-based approaches for inertia assessment and frequency-constrained scheduling. However,

the comparative review also shows that important methodological gaps remain at the interface between spatial inertia estimation and operational generation scheduling.

2.10.1 Limited Spatial Resolution in Inertia-Constrained Scheduling

Most inertia-constrained scheduling frameworks represent inertia as an aggregated system quantity derived from the center-of-inertia frequency or average system frequency. These approaches are computationally attractive and provide a practical first-level frequency-security criterion because they can convert RoCoF limits into minimum system inertia requirements [25, 26, 50]. However, they implicitly assume that inertial support is uniformly available across the network or that local frequency deviations follow the system average frequency closely.

This assumption becomes weak in systems with high and geographically non-uniform VRES penetration. When synchronous generators are displaced unevenly, inertia becomes concentrated around the remaining online synchronous units, while renewable-rich or electrically remote areas may experience reduced inertial support. Under such conditions, a system may satisfy an aggregate inertia requirement while some buses still experience excessive local RoCoF. Therefore, system-level inertia constraints may overestimate frequency-security margins at weak or low-inertia buses.

Area-level methods improve this representation by enforcing inertia adequacy within zones or regions [61, 62]. However, zonal aggregation can still mask bus-level differences inside each area. Buses within the same zone may have different electrical distances from synchronous generators and may therefore experience different local frequency responses. This indicates the need for a finer bus-level representation of inertia adequacy in generation scheduling.

2.10.2 Separation Between Spatial Inertia Estimation and Scheduling

Spatial inertia estimation methods have shown that inertial response is not uniformly distributed across power networks. Index-based, PMU-based, modal, and nodal inertia estimation methods have provided useful tools for identifying inertia-rich and inertia-weak regions [30, 31, 33, 51]. These methods have improved understanding of local frequency dynamics and have demonstrated that bus frequency behavior may differ from the center-of-inertia response.

However, many of these methods are developed mainly for monitoring, post-disturbance analysis, or spatial ranking. Several approaches require disturbance data, PMU measurements, modal decomposition, observation windows, or frequency-divider estimation. Others provide relative indices rather than explicit bus-level inertia values. Although these methods are valuable for situational awareness, they are not always directly suitable for iterative generation scheduling, where inertia must be evaluated repeatedly before a disturbance occurs and under many possible commitment and dispatch conditions.

There is therefore a gap between spatial inertia estimation and operational scheduling. Existing spatial methods reveal where inertia may be weak, while existing scheduling methods often continue to enforce frequency security using aggregate inertia constraints. A unified framework is needed in which bus-level inertia is quantified under each candidate dispatch condition and then used directly as a constraint in the scheduling problem.

2.10.3 Insufficient Representation of Generator-to-Bus Inertial Contribution

In a spatially non-uniform power system, the inertial support observed at a bus depends not only on the inertia constants of online synchronous generators, but

also on their electrical coupling to that bus. A generator that is electrically close to a bus is expected to provide stronger inertial influence than a generator that is electrically remote or weakly connected. Therefore, local inertia adequacy should consider generator location, network topology, electrical distance, synchronizing strength, and operating condition.

Existing aggregated and zonal scheduling approaches do not explicitly quantify how each committed synchronous generator contributes to the effective inertia at each bus. Even some locational frequency-constrained formulations focus on local RoCoF expressions without directly establishing a reusable bus-level inertia adequacy measure based on generator-to-bus inertial contribution [52]. As a result, the scheduling problem may not fully capture how changing synchronous generator commitment and VRES dispatch alters the spatial pattern of inertial support.

This gap is important because generation scheduling decisions directly change the sources, magnitude, and location of inertia. When VRES dispatch increases, some synchronous generators may be unloaded or decommitted, and the resulting reduction in local inertial support may not be uniform across the network. A scheduling framework that aims to maximize inertia-less VRES penetration must therefore account for how each generator contributes to the inertia available at each bus.

2.10.4 Limited Integration of Bus-Level RoCoF-Based Inertia Requirements

RoCoF-based inertia requirements are widely used in frequency-security studies because they provide a direct link between disturbance size, allowable RoCoF, and minimum inertia. In most existing formulations, this requirement is derived at system level or area level. While this is useful for broad security screening, it does not directly indicate the minimum inertia required at individual buses.

For spatial frequency-security assessment, minimum inertia requirements should

be determined locally, especially in systems where local RoCoF may deviate from the center-of-inertia RoCoF. Without bus-level thresholds, it is difficult to determine whether a specific bus has adequate effective inertia to withstand a credible disturbance. This creates a gap between local frequency-security needs and the constraints imposed in generation scheduling.

A bus-level RoCoF-based inertia threshold can provide a clearer operational criterion. It allows each bus to be assessed according to whether its effective inertia is sufficient to maintain the local RoCoF within an acceptable limit. Embedding such thresholds into scheduling would ensure that renewable dispatch decisions do not compromise local frequency stability.

2.10.5 Limited Multi-Objective Treatment of VRES Dispatch, Losses, and Spatial Inertia Adequacy

High VRES penetration creates a multi-objective scheduling problem. Increasing inertia-less VRES dispatch supports decarbonization and reduces dependence on conventional generation, but it may also displace synchronous inertia and change transmission power flows. At the same time, minimizing transmission losses may favor dispatch patterns that are not always aligned with maximum renewable utilization or inertia adequacy.

Existing scheduling studies often focus on one or two aspects of this problem, such as cost minimization, renewable penetration, system-level inertia adequacy, or frequency-response constraints. However, fewer studies explicitly examine the trade-off among inertia-less VRES dispatch, network losses, and bus-level inertia adequacy within one optimization framework. This limitation reduces the ability of system operators to evaluate how much renewable generation can be safely accommodated without creating local frequency-security weaknesses.

A multi-objective spatial inertia-aware formulation is therefore needed to expose these trade-offs. Such a framework should not only identify a single operating point,

but should provide a set of Pareto-optimal solutions that show the relationship between renewable dispatch, loss performance, and frequency-security adequacy.

2.10.6 Summary of the Identified Research Gap

From the reviewed literature, the main research gap can be summarized as follows:

Existing inertia estimation and inertia-constrained scheduling approaches do not fully provide an offline, bus-level, and optimization-ready framework that quantifies the effective contribution of synchronous-generator inertia to each network bus, derives local RoCoF-based inertia adequacy thresholds, and embeds these thresholds into a multi-objective generation scheduling problem for maximizing inertia-less VRES dispatch while maintaining spatial frequency stability.

This gap directly motivates the present thesis. The study addresses it by developing and validating a spatial inertia-aware generation scheduling optimization framework with three linked components. First, an offline method is developed to estimate bus-level inertia distribution by quantifying the effective contribution of each synchronous generator to each network bus. Second, minimum bus-level inertia thresholds are analytically derived from RoCoF-based frequency-security requirements under credible disturbances. Third, these bus-level inertia constraints are embedded into a multi-objective NSGA-II-based scheduling framework that maximizes inertia-less VRES dispatch while considering network loss performance and local frequency-stability adequacy.

By linking spatial inertia estimation, bus-level inertia adequacy, and VRES-aware generation scheduling, the thesis extends existing inertia-constrained scheduling approaches from system-level or zonal frequency-security assessment toward bus-level spatial frequency-security enforcement.

2.11 Chapter Summary

This chapter reviewed the literature relevant to spatial inertia-aware generation scheduling in power systems with high penetration of inertia-less variable renewable energy sources. The review first established that the increasing integration of inertia-less VRES, particularly solar PV and wind generation, has changed the technical basis of frequency stability. While VRES support decarbonization and reduce dependence on conventional generation, their variability, uncertainty, and inertia-less VRES operation reduce the natural inertial contribution traditionally supplied by synchronous generators. This creates a scheduling challenge in which maximizing renewable dispatch must be balanced against the need to preserve adequate frequency-support capability.

The chapter then discussed power system stability in low-inertia and converter-dominated grids. Classical frequency stability concepts remain important because inertia, primary frequency response, damping, and reserve continue to shape post-disturbance frequency behavior. However, modern converter-dominated systems introduce additional stability mechanisms associated with converter controls, grid-following operation, grid-forming operation, fast frequency response, and synthetic inertia. These developments show that frequency stability in renewable-rich systems is no longer determined only by total synchronous inertia, but also by the location, control characteristics, and dynamic availability of frequency-support resources.

The review also examined inertia representation and spatial frequency dynamics. Aggregated inertia remains useful for system-level frequency-security screening because it provides a simple link between disturbance size, RoCoF, and minimum inertia requirements. Nevertheless, aggregated inertia can mask local frequency vulnerabilities when synchronous generators are displaced non-uniformly or when renewable generation is concentrated in electrically weak areas. Under such conditions, local bus frequencies may deviate from the center-of-inertia frequency, and local RoCoF may exceed acceptable limits even when system-wide inertia appears

adequate. This motivates the treatment of inertia as a spatially distributed network property rather than only as a system-wide scalar quantity.

Inertia estimation techniques were reviewed in terms of their spatial resolution, data requirements, and suitability for operational planning. Aggregated inertia estimation methods are valuable for system-wide monitoring and post-event assessment, but they do not identify local inertia deficiencies. Measurement-based, PMU-based, modal, and data-driven methods improve inertia visibility, but many depend on disturbance data, dense measurement coverage, or signal-processing assumptions. Spatial and nodal inertia estimation methods provide greater locational insight, but some produce relative indices rather than explicit bus-level inertia values. These limitations indicate the need for an offline, quantitative, and optimization-ready bus-level inertia estimation approach.

The chapter further reviewed generation scheduling under high VRES penetration. Conventional scheduling methods focus mainly on economic dispatch, unit commitment, reserve requirements, and network constraints. VRES-aware scheduling extends this by considering renewable variability and dispatchability, but frequency stability is often treated indirectly. Frequency-constrained and inertia-constrained scheduling approaches improve this by incorporating RoCoF, frequency nadir, reserve, or minimum inertia constraints. However, most existing formulations use system-level or zonal inertia representations. As a result, they may not fully capture how generator commitment and VRES dispatch affect inertia adequacy at individual buses.

Optimization methods for inertia-aware scheduling were also examined. Mathematical programming approaches are suitable when the problem can be linearized or expressed through tractable algebraic constraints, but they may become restrictive when the formulation includes nonlinear power-flow constraints, mixed decision variables, and conflicting objectives. Evolutionary and metaheuristic methods, particularly NSGA-II, provide a practical alternative for multi-objective scheduling problems because they can approximate Pareto fronts without requiring objec-

tive weighting or convexity assumptions. A comparison with SPEA2, MOEA/D, MOPSO, NSGA-III, and MOGWO showed that these algorithms are possible alternatives, but NSGA-II provides a suitable balance between implementation simplicity, diversity preservation, and compatibility with the physics-based evaluation required in this study.

The chapter also reviewed synthetic inertia, grid-forming converters, and BESS-based frequency support. These technologies can improve frequency stability in low-inertia systems by providing fast active-power response, virtual inertia, voltage-frequency support, and reserve services. However, their contribution is control-dependent and limited by converter rating, available headroom, state of charge, energy capacity, and location. Therefore, they complement rather than eliminate the need for spatial inertia assessment. Their inclusion represents an important future extension of spatial inertia-aware scheduling frameworks.

Finally, the comparative review of related studies showed that the main research gap lies at the interface between spatial inertia estimation and operational generation scheduling. Existing studies provide important foundations in inertia estimation, nodal inertia mapping, frequency-constrained scheduling, and converter-based support. However, there remains a need for an offline framework that quantifies effective bus-level inertia, derives local RoCoF-based inertia adequacy thresholds, and embeds these thresholds directly into a multi-objective generation scheduling problem that maximizes inertia-less VRES dispatch while preserving local frequency stability.

This research addresses the identified gap by developing a spatial inertia-aware generation scheduling optimization framework. The methodology presented in Chapter 3 builds on the reviewed literature by formulating an offline bus-level inertia estimation method, deriving minimum bus-level inertia thresholds, and integrating these constraints into an NSGA-II-based multi-objective scheduling framework for frequency-stable VRES dispatch.

CHAPTER 3

METHODOLOGY

This chapter presents the methodological framework developed to achieve the objectives of this research. The proposed framework establishes an analytical-to-operational workflow for enabling inertia-aware integration of inertia-less VRES while preserving frequency stability under low-inertia conditions. The organization of this chapter follows the logical progression of the framework: (i) modeling foundations, (ii) offline estimation of spatial inertia, (iii) derivation of minimum bus-level inertia thresholds, (iv) formulation and solution of the inertia-constrained optimization problem, and (v) validation strategy and case-study implementation details.

3.1 Overall Research Framework

The methodological framework developed in this chapter directly responds to the research gaps identified in Chapter 2 and the objectives stated in Chapter 1. The reviewed literature showed that most inertia-aware scheduling approaches rely on aggregated or area-level inertia constraints, while limited attention has been given to bus-level inertia distribution and its integration into generation scheduling under high inertia-less VRES penetration. In addition, existing inertia estimation methods are often separated from operational dispatch optimization, making it difficult to evaluate how generator commitment and VRES dispatch decisions affect local frequency-security margins. Therefore, the methodology in this chapter is structured to close this gap by first estimating spatial bus-level inertia, then deriving RoCoF-based minimum inertia requirements, and finally embedding these

requirements into a multi-objective generation scheduling framework.

The overall research framework is illustrated in Figure 3.1.

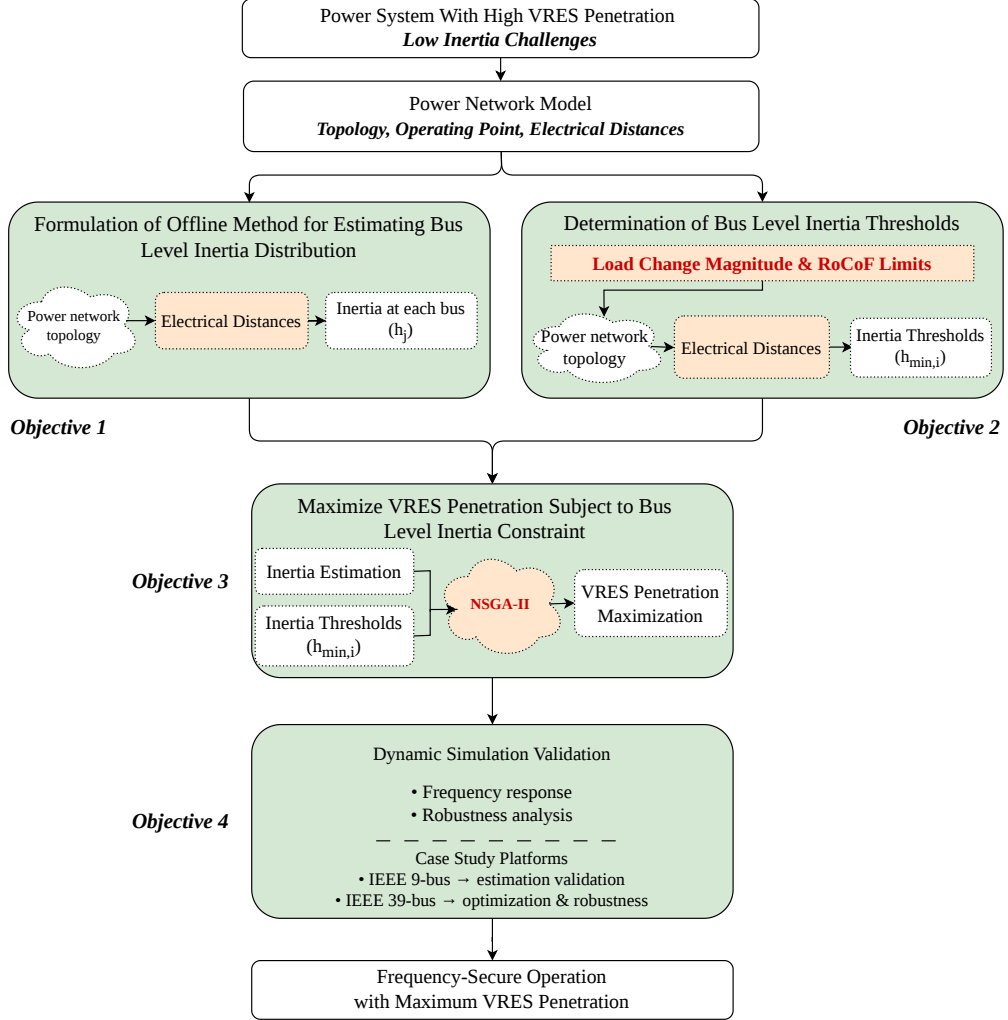


Figure 3.1: Conceptual research framework.

The framework is organized into three sequential methodological layers. First, a power-network modeling foundation is established using network topology, operating conditions, and electrical-distance relationships to formulate an offline method for estimating the spatial distribution of inertia at the bus level. This analytical formulation yields the effective inertia associated with each bus and forms the basis for frequency security assessment. Second, minimum bus-level inertia thresholds are derived using representative active-power disturbance scenarios caused by load change events and RoCoF security limits, thereby translating

frequency-stability requirements into spatially distributed inertia constraints. Third, these inertia requirements are embedded within an inertia-aware multi-objective optimization framework that maximizes inertia-less VRES dispatch subject to network-operational constraints and bus-level inertia constraints. The optimization problem is solved using the NSGA-II evolutionary algorithm.

To maintain a direct mapping to the research objectives, the remainder of this chapter details: power-system modeling foundations, the proposed offline bus-level inertia estimation method, RoCoF-based minimum inertia threshold determination under load change disturbances, formulation and solution of the inertia-aware optimization problem, and the validation strategy used to verify analytical consistency and robustness.

3.2 Formulation of the Offline Method for Estimating Bus-Level Inertia Distribution

This section formulates an offline analytical–computational method for estimating the spatial distribution of equivalent inertia across network buses. The method maps physical generator inertia constants $\{H_g\}$ to bus-level equivalent inertia values $\{h_j\}$ through: (i) an operating-point reduction that yields generator–bus coupling elements Y_{gj}^{red} , and (ii) closed-form weighting factors that encode electrical proximity and synchronizing stiffness. The resulting $\{h_j\}$ are computed without requiring dynamic measurements and are subsequently used for inertia comparison, sensitivity analysis, and time-domain interpretation via RoCoF and coherency patterns.

3.2.1 Generator Dynamic Model

Synchronous generators constitute the primary physical source of inertia in conventional power systems. The rotating masses of their turbine–generators store kinetic

energy that is exchanged with the electrical network following an active-power imbalance. When a disturbance occurs, any mismatch between mechanical input power and electrical output power produces rotor acceleration or deceleration, thereby opposing rapid frequency deviations and limiting the initial RoCoF.

The electromechanical interaction governing this inertial response is described by the classical swing equation. The nonlinear incremental swing equation for synchronous generator g is expressed as

$$2H_g \frac{d\Delta f_g}{dt} + D\Delta f_g = \Delta P_{m,g} - \Delta P_{e,g}, \quad (3.1)$$

where H_g is the inertia constant, Δf_g denotes the deviation of generator frequency from the nominal value, D is the damping coefficient representing aggregate frequency-dependent load and damping effects, $\Delta P_{m,g}$ is the incremental mechanical input power, and $\Delta P_{e,g}$ is the incremental electrical output power.

Immediately following a disturbance, the system response is dominated by inertial effects occurring over a short time interval preceding primary frequency-control action. During this initial inertial interval, the mechanical input power is assumed constant ($\Delta P_{m,g} \approx 0$) and damping effects are neglected in order to isolate inertial dynamics. Under these assumptions, the swing equation reduces to

$$2H_g \frac{df_g}{dt} = -\Delta P_g, \quad (3.2)$$

where df_g/dt denotes generator RoCoF and $\Delta P_g \triangleq \Delta P_{e,g}$ represents the incremental electrical power change acting on generator g under the sign convention in (3.2).

Equation (3.2) forms the basis for relating local incremental power and RoCoF, and is subsequently used to derive an equivalent inertia representation at network buses.

The simplified generator representation is adopted intentionally because the proposed formulation is concerned with the initial inertial response rather than the complete frequency-control process. During this short time interval, governor ac-

tion, automatic generation control, and turbine mechanical-power adjustment have not yet produced a significant response. Therefore, assuming $\Delta P_{m,g} \approx 0$ is appropriate for deriving the initial RoCoF-based inertia relationship. In addition, AVR and excitation-system dynamics are not explicitly included because the proposed bus-level inertia estimator is based primarily on active-power imbalance, generator inertial constants, synchronizing coupling, and reduced-network electrical-distance relationships.

3.2.2 Steady-State Network Model, Directed Connectivity, and Kron Reduction

While synchronous generators provide the physical source of rotational inertia, non-generator buses do not possess inherent kinetic energy. Instead, they receive equivalent inertial support through electromechanical coupling with the online synchronous generators. The magnitude and distribution of this support depend on network topology and on the operating-point pattern of real-power transfer. The proposed method therefore extracts generator–bus coupling from a reduced network representation that is conditioned on the solved AC power flow.

3.2.2.1 Operating-point network model

The steady-state network is represented by the nodal admittance model

$$\mathbf{I} = \mathbf{Y}_{\text{bus}} \mathbf{V}, \quad (3.3)$$

where $\mathbf{Y}_{\text{bus}} \in \mathbb{C}^{n \times n}$ is the bus admittance matrix formed from branch and shunt data, and $\mathbf{I}, \mathbf{V} \in \mathbb{C}^n$ are current and voltage phasor vectors evaluated at the operating point obtained from the AC power-flow solution. The operating point also provides branch real-power flows $\{P_{ij}\}$ for each network element (i, j) .

3.2.2.2 Directed graph of real-power transfer and generator reachability

A directed graph $\mathcal{G}_d = (\mathcal{B}, \mathcal{E}_d)$ is constructed from the solved operating point to encode real-power transfer orientation. For each physical branch (i, j) , define the directed edge

$$i \rightarrow j \quad \text{if} \quad P_{ij} > \varepsilon_P,$$

and $j \rightarrow i$ if $P_{ij} < -\varepsilon_P$, where $\varepsilon_P > 0$ is a small numerical tolerance used to avoid direction flips due to near-zero flows. Branches with $|P_{ij}| \leq \varepsilon_P$ are treated consistently across all operating points.

For each bus j , define the set of reachable generators

$$\mathcal{G}(j) \triangleq \{g \in \{1, \dots, N_G\} : \exists \text{ a directed path } g \rightarrow \dots \rightarrow j \text{ in } \mathcal{G}_d\}.$$

This reachability set formalizes generator dominance and is used to reproduce the zero electrical-distance entries reported in the results: when $g \notin \mathcal{G}(j)$, generator g contributes no inertial influence to bus j under the proposed flow-aligned coupling interpretation.

3.2.2.3 Kron reduction

To obtain effective coupling relationships among retained nodes, Kron reduction is applied. Partition the bus set into retained nodes $\mathcal{J}$ and eliminated nodes $\mathcal{K}$:

$$\begin{bmatrix} \mathbf{I}_{\mathcal{J}} \\ \mathbf{I}_{\mathcal{K}} \end{bmatrix} = \begin{bmatrix} \mathbf{Y}_{\mathcal{J}\mathcal{J}} & \mathbf{Y}_{\mathcal{J}\mathcal{K}} \\ \mathbf{Y}_{\mathcal{K}\mathcal{J}} & \mathbf{Y}_{\mathcal{K}\mathcal{K}} \end{bmatrix} \begin{bmatrix} \mathbf{V}_{\mathcal{J}} \\ \mathbf{V}_{\mathcal{K}} \end{bmatrix}. \quad (3.4)$$

Assuming no external current injections at eliminated buses during the reduction ($\mathbf{I}_{\mathcal{K}} = \mathbf{0}$), eliminated-bus voltages are

$$\mathbf{V}_{\mathcal{K}} = -\mathbf{Y}_{\mathcal{K}\mathcal{K}}^{-1} \mathbf{Y}_{\mathcal{K}\mathcal{J}} \mathbf{V}_{\mathcal{J}}. \quad (3.5)$$

Substituting (3.5) into the retained-bus equation yields

$$\mathbf{I}_{\mathcal{J}} = \mathbf{Y}_{\text{red}} \mathbf{V}_{\mathcal{J}}, \quad (3.6)$$

with the Kron-reduced admittance matrix

$$\mathbf{Y}_{\text{red}} = \mathbf{Y}_{\mathcal{J}\mathcal{J}} - \mathbf{Y}_{\mathcal{J}\mathcal{K}} \mathbf{Y}_{\mathcal{K}\mathcal{K}}^{-1} \mathbf{Y}_{\mathcal{K}\mathcal{J}}. \quad (3.7)$$

Reduction sets used in this thesis. For each operating point, the reduction retains the generator internal buses and the bus of interest. In implementation, a single reduction is performed with the retained set

$$\mathcal{J} \triangleq \mathcal{G} \cup \mathcal{B}_{\text{load}},$$

where $\mathcal{G}$ denotes the set of generator internal buses and $\mathcal{B}_{\text{load}}$ denotes the set of buses for which bus-level inertia is evaluated. The reduced off-diagonal entries provide the generator–bus coupling elements Y_{gj}^{red} required for the subsequent weighting calculations.

To enforce the flow-aligned coupling interpretation used in the results, reachability is imposed as follows: for each bus j , if $g \notin \mathcal{G}(j)$ then Y_{gj}^{red} is treated as zero when forming normalized weights. This step reproduces the “radial buses have exactly one nonzero electrical distance equal to unity” behavior observed in the results.

3.2.3 Linearization and Simplification Conditions

The derivation of the proposed bus-level inertia estimator is based on an operating-point linearization of the electromechanical coupling between synchronous generators and network buses. These assumptions are introduced to obtain a tractable offline formulation for estimating inertia distribution. The main linearization and simplification conditions are summarized as follows.

1. The formulation is evaluated around a solved pre-disturbance AC power-flow operating point. Therefore, bus-voltage magnitudes, voltage angles, branch power-flow directions, and reduced-network admittance terms are obtained from the steady-state operating condition.
2. Voltage-angle deviations during the initial inertial response are assumed to be small. Hence, the approximations

$$\sin \Delta_{gj} \approx \Delta_{gj}, \quad \cos \Delta_{gj} \approx 1$$

are used when deriving the reduced-network angle relationship.

3. Transmission lines are assumed to be predominantly reactive in the reduced-network interpretation [99]. Therefore, line resistance effects are neglected in the leading-order angle-coupling expression. This assumption is consistent with high-voltage transmission networks where electromechanical coupling is mainly governed by synchronizing power interactions.
4. During the inertial-response interval, network topology, voltage magnitudes, generator internal voltage magnitudes, and reduced-network admittance parameters are assumed approximately constant. The dominant dynamic effect during this interval is the acceleration or deceleration of synchronous-generator rotors.
5. The initial post-disturbance redistribution of incremental electrical power is approximated using linearized power-angle sensitivities evaluated at the pre-disturbance operating point. Therefore, the disturbance power-sharing ratio is represented as

$$\frac{\Delta P_g}{\Delta P_j} \approx \frac{\partial P_g}{\partial P_j}.$$

6. The synchronizing power coefficient is used as a linearized measure of how strongly generator g electrically counteracts an angular deviation at bus j .
7. Governor, AGC, AVR, excitation-system, and converter-control dynamics are

not explicitly included in the analytical inertia-allocation formulation. Their effects are outside the initial RoCoF-focused formulation and are considered through dynamic validation and stated limitations.

Under these assumptions, the proposed formulation provides an operating-point-dependent estimate of how synchronous-generator inertia is electrically reflected at each bus. The validity of these assumptions is assessed in Chapter 4 through RoCoF-based dynamic interpretation, sensitivity analysis, and comparison with existing inertia-distribution methods.

3.2.4 Reduced-Network Angle Coupling

Based on the linearization and simplification conditions stated in subsection 3.2.3, the reduced-network angle coupling is derived as follows. Using the reduced network representation, Kirchhoff's Current Law (KCL) at bus j is written as

$$\sum_{g=1}^{N_G} (E_g e^{j\delta_g} - V_j e^{j\delta_j}) Y_{gj}^{\text{red}} = \frac{V_j e^{j\delta_j}}{Z_L}, \quad (3.8)$$

where the load admittance is defined as $Y_L = 1/Z_L = G_L + jB_L$, and Y_{gj}^{red} denotes the reduced admittance between generator internal node g and bus j , written in polar form as

$$Y_{gj}^{\text{red}} = |Y_{gj}^{\text{red}}| e^{j\theta_{gj}},$$

where θ_{gj} is the admittance phase angle.

To obtain an explicit relationship in terms of relative angle differences, the KCL equation is rewritten in a rotating reference aligned with bus j . Dividing equation (3.8) by $e^{j\delta_j}$ and defining $\Delta_{gj} = \delta_g - \delta_j$ yields

$$\sum_{g=1}^{N_G} |Y_{gj}^{\text{red}}| (E_g e^{j(\Delta_{gj} + \theta_{gj})} - V_j e^{j\theta_{gj}}) = V_j (G_L + jB_L).$$

Taking real parts and applying the small-angle approximation $\sin \Delta_{gj} \approx \Delta_{gj}$,

$\cos \Delta_{gj} \approx 1$, gives

$$\sum_{g=1}^{N_G} |Y_{gj}^{\text{red}}| [(E_g - V_j) \cos \theta_{gj} - E_g \sin \theta_{gj} (\delta_g - \delta_j)] = V_j G_L.$$

Rearranging and solving explicitly for δ_j yields

$$\delta_j = \frac{\sum_{g=1}^{N_G} |Y_{gj}^{\text{red}}| E_g \sin \theta_{gj} \delta_g - \sum_{g=1}^{N_G} |Y_{gj}^{\text{red}}| E_g \cos \theta_{gj} + V_j \left(G_L + \sum_{g=1}^{N_G} |Y_{gj}^{\text{red}}| \cos \theta_{gj} \right)}{\sum_{g=1}^{N_G} |Y_{gj}^{\text{red}}| E_g \sin \theta_{gj}}. \quad (3.9)$$

For the analytical derivation of spatial inertia distribution, transmission lines are assumed to be predominantly reactive and line resistances are neglected in the reduced-network interpretation. Under this assumption, $|\cos \theta_{gj}| \ll |\sin \theta_{gj}|$, and the leading-order angle mapping reduces to

$$\delta_j = \frac{\sum_{g=1}^{N_G} |Y_{gj}^{\text{red}}| E_g \delta_g}{\sum_{m=1}^{N_G} |Y_{mj}^{\text{red}}| E_m} + \frac{V_j G_L}{\sum_{m=1}^{N_G} |Y_{mj}^{\text{red}}| E_m}. \quad (3.10)$$

Equation (3.10) shows that the bus voltage angle is governed by two contributions: (i) a weighted aggregation of generator internal angles determined by the reduced admittance couplings, and (ii) an additive offset arising from the conductive component of the local load at the operating point. The second term is constant over the inertial-response interval and therefore vanishes upon time differentiation.

3.2.5 Frequency Coupling and Bus-Level Inertia Representation

Electrical frequency at a particular bus is proportional to the time derivative of bus voltage angle,

$$f_j = f_0 + \frac{1}{2\pi} \dot{\delta}_j, \quad \frac{df_j}{dt} = \frac{1}{2\pi} \ddot{\delta}_j.$$

During the inertial-response interval, voltage magnitudes and reduced admittance parameters are assumed constant. Differentiating (3.10) twice therefore yields a

linear RoCoF coupling relation of the form

$$\frac{df_j}{dt} = \sum_{g=1}^{N_G} \hat{I}_{gj} \frac{df_g}{dt}, \quad (3.11)$$

where the normalized current coupling component $\hat{I}_{gj}$ is defined as

$$\hat{I}_{gj} = \frac{|Y_{gj}^{\text{red}}| E_g}{\sum_{m=1}^{N_G} |Y_{mj}^{\text{red}}| E_m}. \quad (3.12)$$

In implementation, the normalization in (3.12) is evaluated over the reachable generator set $\mathcal{G}(j)$; for $g \notin \mathcal{G}(j)$, $\hat{I}_{gj} = 0$.

Equation (3.11) demonstrates that the RoCoF at a non-generator bus arises from the inertial responses of synchronous generators, weighted by electrically derived normalized current coupling components obtained from the reduced network.

Defining an equivalent swing equation at bus j ,

$$2h_j \frac{df_j}{dt} = -\Delta P_j, \quad (3.13)$$

and substituting (3.2) and (3.11) yields

$$\frac{1}{h_j} = \sum_{g=1}^{N_G} \frac{\hat{I}_{gj}}{H_g} \frac{\Delta P_g}{\Delta P_j}. \quad (3.14)$$

Equation (3.14) expresses the inverse equivalent inertia at bus j as a weighted aggregation of generator inertia constants scaled by both normalized current coupling components and relative disturbance participation factors.

3.2.6 Power Contribution Approximation and Electrical Distance

The weighting in (3.14) depends on the disturbance power-sharing ratio $\Delta P_g / \Delta P_j$. To obtain a tractable offline estimator, the initial post-disturbance redistribution of

incremental electrical power is approximated using linearized power–angle sensitivities at the pre-disturbance operating point. Under this small-signal approximation,

$$\frac{\Delta P_g}{\Delta P_j} \approx \frac{\partial P_g}{\partial P_j}. \quad (3.15)$$

Applying the chain rule gives

$$\frac{\partial P_g}{\partial P_j} = \frac{\partial P_g}{\partial \delta_j} \left(\frac{\partial P_j}{\partial \delta_j} \right)^{-1}, \quad (3.16)$$

where δ_j denotes the voltage angle at bus j . Since the incremental injected power at bus j is supplied collectively by the generators participating in inertial response, the total sensitivity satisfies

$$\frac{\partial P_j}{\partial \delta_j} = \sum_{m=1}^{N_G} \frac{\partial P_m}{\partial \delta_j}. \quad (3.17)$$

Substituting (3.17) into (3.16) yields a normalized participation approximation:

$$\frac{\Delta P_g}{\Delta P_j} \approx \frac{\frac{\partial P_g}{\partial \delta_j}}{\sum_{m=1}^{N_G} \frac{\partial P_m}{\partial \delta_j}}. \quad (3.18)$$

The sensitivity term $\partial P_g / \partial \delta_j$ corresponds to the synchronizing power coefficient, which quantifies how strongly generator g electrically counteracts angular deviations occurring at bus j . Using the reduced-network power transfer relation, the synchronizing coefficient is defined as

$$K_{gj}^{\text{syn}} = \frac{\partial P_g}{\partial \delta_j} = E_g V_j |Y_{gj}^{\text{red}}| \sin(\Delta_{gj} + \theta_{gj}), \quad (3.19)$$

where $\Delta_{gj} = \delta_g - \delta_j$ and θ_{gj} is the admittance angle of the reduced transfer path.

Substituting (3.19) into (3.18) produces a normalized synchronizing power coefficient

cient,

$$\hat{K}_{gj} = \frac{K_{gj}^{\text{syn}}}{\sum_{m=1}^{N_G} K_{mj}^{\text{syn}}}, \quad (3.20)$$

which characterizes the relative contribution of each generator to supplying the disturbance observed at bus j . Consistent with the directed reachability logic, the normalization in (3.20) is evaluated over $\mathcal{G}(j)$; for $g \notin \mathcal{G}(j)$, $\hat{K}_{gj} = 0$.

Substituting the approximation $\Delta P_g / \Delta P_j \approx \hat{K}_{gj}$ into (3.14) yields the closed-form inverse inertia estimator

$$\frac{1}{h_j} = \sum_{g=1}^{N_G} \frac{\hat{I}_{gj} \hat{K}_{gj}}{H_g}. \quad (3.21)$$

Motivated by the complementary roles of electrical proximity ($\hat{I}_{gj}$) and electromechanical stiffness ($\hat{K}_{gj}$), an electrical-distance factor is introduced as

$$d_{gj} = (\hat{I}_{gj} \hat{K}_{gj})^{1/2}. \quad (3.22)$$

The equivalent bus inertia can therefore be written compactly as

$$\frac{1}{h_j} = \sum_{g=1}^{N_G} \frac{d_{gj}^2}{H_g}, \quad (3.23)$$

yielding the final analytical expression for spatial bus-level inertia:

$$h_j = \left(\sum_{g=1}^{N_G} \frac{d_{gj}^2}{H_g} \right)^{-1}. \quad (3.24)$$

Equation (3.24) is used to compute the bus inertia values reported in 4. For buses supplied by multiple generators, the harmonic-mean structure of (3.24) produces composite inertia values governed by both the electrical distances and the relative generator inertia constants.

3.2.7 Applicability Limits of the Steady-State-Based Inertia Allocation

The proposed formulation is primarily steady-state and analytical because it is designed to estimate the spatial allocation of synchronous-generator inertia at a specified operating point. It should therefore be interpreted as an offline inertia-allocation method rather than a complete nonlinear dynamic frequency-response model. The dynamic source of inertia remains the rotating mass of the online synchronous generators, represented by the inertia constants H_g , while the steady-state AC power-flow solution is used to determine how this inertia is electrically reflected at each bus through reduced-network coupling, synchronizing power coefficients, and electrical-distance factors.

This interpretation is valid within the inertial-response interval following a disturbance. During this short interval, generator mechanical input powers, voltage magnitudes, network topology, and reduced-network coupling terms are assumed to remain approximately constant, while the dominant dynamic effect is the acceleration or deceleration of synchronous-generator rotors. Under this assumption, the pre-disturbance operating point provides a suitable basis for estimating generator-to-bus inertial coupling. The formulation does not explicitly model the full post-disturbance frequency trajectory, frequency nadir, damping evolution, governor action, automatic generation control, excitation-system dynamics, or converter-control dynamics.

The applicability of the steady-state-derived inertia allocation is therefore verified through dynamic validation in Chapter 4, the offline coupling weights are used to reconstruct bus RoCoF from simulated generator RoCoF trajectories obtained from time-domain simulations in subsection 4.1.6. The reconstructed bus RoCoF is then compared with the simulated bus RoCoF to assess whether the steady-state-derived coupling factors capture the dominant spatial inertial influence during the inertial frequency-response period. In addition, the proposed bus inertia

ranking is compared with existing analytical nodal-inertia and time-domain inertia-distribution methods in subsection subsection 4.1.7. These validation steps provide evidence that the proposed formulation is suitable for offline planning optimization studies.

3.2.8 Computational Workflow for Reproducing the Offline Inertia Results

This subsection specifies the computational workflow used to reproduce the offline inertia results, including: (i) base-case electrical distances and bus inertia, (ii) load-variation-induced redistribution, (iii) sensitivity to generator inertia scaling, (iv) scalability to the IEEE 39-bus system, and (v) the interface used to interpret time-domain RoCoF and coherency patterns. All the studies except (iv), were conducted using the IEEE 9 bus test system. The full network topology and data are presented in section 3.6.1.

3.2.8.1 Base-case electrical distances and bus inertia

For a fixed test system and operating point, the following steps are executed.

1. *Solve the AC power-flow problem.* Using the Appendix A base-case injections and demands, solve the steady-state AC power flow to obtain the bus-voltage magnitudes and angles $\{V_i, \delta_i\}_{i \in \mathcal{B}}$, together with branch real-power flows $\{P_{ij}\}_{(i,j) \in \mathcal{L}}$.
2. *Construct the directed real-power flow graph.* Build a directed graph $\mathcal{G}_d = (\mathcal{B}, \mathcal{A})$ whose node set is the bus set $\mathcal{B}$. For each physical line $(i, j) \in \mathcal{L}$, compute the real-power flow P_{ij} from the AC power-flow solution and assign a directed arc according to

$$i \rightarrow j \quad \text{if} \quad P_{ij} > \varepsilon_P, \quad j \rightarrow i \quad \text{if} \quad P_{ij} < -\varepsilon_P, \quad (3.25)$$

where ε_P is a small tolerance introduced to prevent spurious direction changes due to numerical noise and near-zero flows. If $|P_{ij}| \leq \varepsilon_P$, the branch is treated as directionally ambiguous for that operating point and is excluded from the directed set $\mathcal{A}$ (equivalently, it contributes no directed path for reachability determination).

Let $\mathcal{G} \subset \mathcal{B}$ denote the set of generator terminal buses. For each bus j , compute the *reachability set*

$$\mathcal{G}(j) = \{g \in \mathcal{G} \mid \exists \text{ a directed path } g \rightarrow j \text{ in } \mathcal{G}_d\}, \quad (3.26)$$

3. *Compute reduced coupling admittances.* Form the bus-admittance matrix $\mathbf{Y}_{\text{bus}}$ for the IEEE 9-bus system using network line parameters. Apply Kron reduction using (3.7) to obtain the reduced admittance matrix $\mathbf{Y}_{\text{red}}$ over the retained boundary buses. Extract the reduced coupling elements $\{Y_{gj}^{\text{red}}\}$ required for all generator–bus pairs (g, j) under study.

4. *Enforce reachability in coupling interpretation.* The reduced admittance elements $\{Y_{gj}^{\text{red}}\}$ represent electrical coupling in the reduced network. To ensure that the spatial inertia aggregation reflects the base-case real-power transfer structure, reachability in the directed graph is enforced as a feasibility filter:

$$Y_{gj}^{\text{red}} \leftarrow 0, \quad \text{if } g \notin \mathcal{G}(j), \quad (3.27)$$

prior to forming the normalized weights. This step guarantees that generators with no directed real-power path to bus j do not contribute to the electrical-distance and inertia allocation at that bus.

5. *Compute normalized weights and electrical distances.* For each bus $j \in \mathcal{B}$, evaluate the weighting terms using only the reachable generators $g \in \mathcal{G}(j)$.

(a) *Normalized current coupling.* Compute $\hat{I}_{gj}$ using (3.12) with the summation restricted to reachable generators.

- (b) *Synchronizing participation.* Evaluate the synchronizing coefficient K_{gj}^{syn} at the operating point using (3.19).
 - (c) *Electrical distance.* Compute the electrical-distance coefficient using (3.22).
6. *Compute bus-level inertia.* Using generator inertia constants $\{H_g\}$ provided in Table 3.2, compute the base-case bus-level inertia values for all buses using (3.24). The SG inertias are expressed to common base system before being applied.

The outputs of this stage are: (i) the electrical-distance matrix $\{d_{gj}\}$, which is tabulated for the IEEE 9-bus base case and used to generate the electrical-distance heatmaps under load variation; and (ii) the bus inertia vector $\{h_j\}$, which is used to produce the base-case spatial inertia distribution plots (including bar and surface visualizations) and to support the subsequent sensitivity analyses to load changes and generator inertia scaling.

3.2.8.2 Parametric Load-Variation Study and Flow-Reversal Regimes

To evaluate the dependence of the electrical-distance formulation and the resulting bus-level inertia estimates on operating conditions, a structured parametric load-variation study is conducted on the IEEE 9-bus system.

Load-variation definition. The active power demand at Bus 5 is progressively reduced from its base-case value. Let P_{D5}^0 denote the base-case active load at Bus 5. The modified load is defined as

$$P_{D5} = P_{D5}^0 - \Delta P_{D5}, \quad (3.28)$$

where ΔP_{D5} is varied over the admissible range (p.u. on the 100 MVA study base).

Generation balancing mechanism. The reduction in active-power demand introduces a generation–load imbalance that must be absorbed by the generators during the power-flow solution. In this study, the system slack generator is allowed to absorb the mismatch resulting from the load variation. Consequently, the active-power output of the slack generator adjusts automatically during the AC power-flow computation, while the scheduled outputs of the remaining generators remain fixed at their base-case values.

This implementation isolates the effect of load redistribution on network power-flow patterns and electrical connectivity without introducing additional degrees of freedom associated with re-dispatch among multiple generators.

For each value of ΔP_{D5} , the following procedure is executed.

1. **Modify operating point and solve AC power flow.**

The active load at Bus 5 is reduced according to (3.28), while all other system parameters remain unchanged. An AC power-flow solution is computed to obtain updated bus voltages $\{V_i, \delta_i\}$ and branch real-power flows $\{P_{ij}\}$.

2. **Detect flow reversals and identify operating regimes.**

The sign of each branch real-power flow P_{ij} is compared with the corresponding base-case value. A flow reversal is recorded if

$$\text{sign}(P_{ij}^{\text{new}}) \neq \text{sign}(P_{ij}^{\text{base}}), \quad (3.29)$$

subject to tolerance ε_P .

The regime boundary is determined through a systematic parametric sweep of ΔP_{D5} . The load reduction is increased incrementally and an AC power-flow solution is computed for each operating point. The smallest value of ΔP_{D5} at which a branch real-power flow reverses direction relative to the base case is recorded as the transition point between operating regimes.

For the IEEE 9-bus system, two distinct regimes are identified:

- *Case I (small load reduction):*

$$0 < \Delta P_{D5} \leq 0.46 \text{ p.u.}$$

In this regime, all branch real-power flow directions remain identical to the base-case solution. The directed graph $\mathcal{G}_d$ and reachability sets $\mathcal{G}(j)$ are therefore unchanged.

- *Case II (large load reduction):*

$$0.46 < \Delta P_{D5} \leq 1.25 \text{ p.u.}$$

At $\Delta P_{D5} = 0.46 \text{ p.u.}$, the first flow reversal occurs on the branch connecting Buses 4 and 5. Beyond this point, at least one branch undergoes direction change, altering generator–bus reachability relationships.

The upper bound of 1.25 p.u. corresponds to the largest load reduction for which a feasible and convergent AC power-flow solution is obtained. Beyond this point the load at Bus 5 becomes negative, which would represent net power injection rather than consumption and is therefore excluded from the study.

3. Reconstruct directed graph and reachability.

For each operating point, construct the directed graph $\mathcal{G}_d$ using the updated branch flows and recompute the reachability sets $\mathcal{G}(j)$ according to (3.26). In Case I, $\mathcal{G}(j)$ is identical to the base case; in Case II, reachability may change due to flow reversals.

4. Recompute electrical distances and bus inertia.

Using the updated operating point and reachability structure, repeat the procedure described in Subsection 3.2.8.1:

- Compute reduced admittance elements Y_{gj}^{red} .

- Compute normalized weights $\hat{I}_{gj}$ and $\hat{K}_{gj}$.
- Compute electrical-distance coefficients d_{gj} .
- Convert generator inertia constants to the common 100 MVA base and compute updated bus inertias h_j .

To quantify deviation from the base case, electrical-distance differences are computed elementwise as

$$d_{gj}^{(\Delta)} = d_{gj}^{(\text{case})} - d_{gj}^{(\text{base})}, \quad (3.30)$$

and similarly for bus inertia

$$h_j^{(\Delta)} = h_j^{(\text{case})} - h_j^{(\text{base})}. \quad (3.31)$$

This parametric experiment enables systematic assessment of how operating-point variation and topology-induced changes in directed connectivity influence spatial inertia allocation. In particular, the emergence of Case II demonstrates that electrical-distance structure is not invariant to load redistribution when power-flow reversal alters generator reachability, thereby directly affecting bus-level inertia estimation.

3.2.8.3 Sensitivity of bus to generator inertia constants

Another parametric sensitivity study is conducted to examine how variations in synchronous generator inertias influence the spatial distribution of bus-level inertia. The objective of this experiment is to verify that the proposed electrical-distance formulation correctly captures the propagation of inertial influence through network connectivity and to quantify how localized changes in generator inertia redistribute effective inertia across the system.

The study is performed using the IEEE 9-bus test system, which provides a controlled environment in which generator–bus coupling can be clearly interpreted.

The analysis considers independent variation of individual generator inertias while maintaining identical network topology and operating conditions.

Experimental setup.

Let H_g^0 denote the nominal inertia constant of generator g . For each generator, the inertia constant is scaled independently using a multiplicative factor

$$H_g = \alpha H_g^0, \quad (3.32)$$

where the scaling parameter is selected from the set

$$\alpha \in \{0.50, 0.75, 1.00, 1.25, 1.50\}.$$

During each experiment, only one generator inertia constant is modified at a time, while all remaining generators retain their nominal inertia values. This isolation ensures that the observed changes in bus-level inertia can be attributed exclusively to the perturbed generator.

Computation procedure.

For each scaling factor α and each generator g , the following steps are executed:

1. Solve the AC power flow to obtain the base operating point.
2. Perform Kron reduction using equation (3.7) to obtain the reduced admittance matrix.
3. Compute electrical distances d_{gj} using equation (3.22).
4. Recalculate bus-level inertia values h_j using equation (3.24) with the modified inertia constant.
5. Evaluate the relative inertia variation at each bus:

$$\Delta h_j^{\text{rel}} = \frac{h_j(\alpha) - h_j(1)}{h_j(1)}. \quad (3.33)$$

Evaluation objective.

The sensitivity analysis is designed to verify three expected physical properties:

1. Spatial locality: inertia variations should primarily affect buses electrically close to the perturbed generator.
2. Electrical-distance attenuation: sensitivity magnitude should decrease with decreasing electrical proximity.
3. Structural consistency: buses without directed power-flow paths from the perturbed generator should exhibit negligible inertia change.

Confirmation of these properties provides independent validation that the electrical-distance formulation correctly represents how synchronous-machine inertia propagates through the network.

3.2.8.4 RoCoF-Based Dynamic Interpretation

The offline spatial inertia method yields two key structural outputs:

- (i) A generator-dominance structure defined by the reachability sets $\mathcal{G}(j)$ and electrical-distance coefficients d_{gj} .
- (ii) RoCoF reconstruction weights derived from the normalized current-coupling coefficients $\hat{I}_{gj}$ (equation (3.12)).

To verify that these offline structural indicators are dynamically meaningful, a time-domain disturbance experiment is conducted on the IEEE 9-bus system.

Dynamic disturbance design. A 25 MW step load increase is applied simultaneously at load buses 5, 6, and 8 at $t = 0.5$ s. The disturbance magnitude is selected to remain within the small-signal regime while producing clearly observable inertial dynamics.

Time-domain simulations are performed in DIgSILENT PowerFactory v15.1.7. Bus-frequency and generator-frequency trajectories are exported and post-processed in MATLAB R2023 for RoCoF extraction and reconstruction analysis.

RoCoF extraction from dynamic simulation. Let $f_g(t)$ and $f_j(t)$ denote the generator and bus frequency trajectories, respectively. RoCoF is computed using linear regression over a fixed post-disturbance window

$$t \in [0.5, 1.0] \text{ s},$$

corresponding to the first 500 ms after disturbance initiation.

For each trajectory, the estimated RoCoF is

$$\dot{f}(t) = \left. \frac{df(t)}{dt} \right|_{t \in [0.5, 1.0]}, \quad (3.34)$$

which provides a noise-robust estimate of the initial inertial slope.

Offline weight computation at the corresponding operating point. The normalized current-coupling coefficients $\hat{I}_{gj}$ are computed using the pre-disturbance AC power-flow solution corresponding to the simulated case. These weights reflect the relative electrical coupling between generator g and bus j at that operating point and are independent of dynamic state evolution.

Weighted RoCoF reconstruction. For each bus j , the predicted bus RoCoF is reconstructed as a weighted combination of generator RoCoFs:

$$\hat{\dot{f}}_j(t) = \sum_{g \in \mathcal{G}(j)} \hat{I}_{gj} \dot{f}_g(t). \quad (3.35)$$

The reconstructed RoCoF $\hat{\dot{f}}_j(t)$ is then compared against the simulated bus RoCoF $\dot{f}_j(t)$. Agreement between the two confirms that the offline electrical-distance

weights capture the effective inertia distribution governing the initial frequency response.

Dynamic interpretation of generator dominance. Equation (3.35) implies that buses dominated by a particular generator (or generator pair) should exhibit RoCoF magnitudes and oscillatory characteristics aligned with those generators.

To further assess this, frequency differences between selected generator–bus and bus–bus pairs are examined:

$$\Delta f_{a,b}(t) = f_a(t) - f_b(t). \quad (3.36)$$

Buses predicted to be dominated by the same generator (based on large d_{gj} or dominant $\hat{I}_{gj}$ weights) are expected to exhibit phase-aligned oscillations and small $\Delta f_{a,b}(t)$, indicating frequency coherency.

Conversely, buses with distinct dominant generators may exhibit larger transient frequency separations during the initial inertial period.

Physical interpretation. The initial RoCoF at a bus is inversely proportional to the effective inertia supporting it (equation (3.13)). Therefore, buses with lower estimated inertia h_j are expected to display larger-magnitude RoCoF following the disturbance. Consistency between:

- the offline inertia estimates h_j ,
- the reconstructed RoCoF $\hat{\dot{f}}_j(t)$, and
- the simulated RoCoF $\dot{f}_j(t)$

provides dynamic validation of the spatial inertia formulation and confirms that the electrical-distance matrix encodes physically meaningful generator–bus influence relationships.

3.2.8.5 Comparative Analysis with the existing methods

To evaluate the accuracy of the proposed spatial inertia formulation, the estimated bus-level inertia values are benchmarked against two representative approaches from the literature. All methods are applied to the same IEEE 9-bus operating point and on the common 100 MVA system base.

1) Proposed spatial inertia formulation. Bus-level inertia values $\{h_j\}$ are computed using the analytical expression in (3.24). Buses are then ranked in ascending order of h_j , such that buses with lower inertia (higher frequency vulnerability) receive higher priority in the ranking.

2) Analytical nodal-inertia estimator [33]. For the same operating point, the nodal inertia estimator proposed in [33] is implemented. The bus-level inertia is estimated as

$$h_j^{\text{Pineiro}} = \frac{\sum_{g=1}^{N_G} K_{g,j}}{\sum_{g=1}^{N_G} \frac{D_{g,j}}{H_g} K_{g,j}}, \quad (3.37)$$

where $K_{g,j}$ denotes the synchronizing power coefficient between generator g and bus j , $D_{g,j}$ represents the frequency divider term defined in [33], and H_g is the generator inertia constant expressed on the same 100 MVA system base used in this study.

The resulting values $\{h_j^{\text{Pineiro}}\}$ are ranked from lowest to highest to produce a comparative bus ordering.

3) Time-domain index-based method [30]. Using the same 25 MW disturbance applied at buses 5, 6, and 8, the time-domain index proposed in [30] is computed from dynamic simulation trajectories.

The inertia-related index at bus j is defined as

$$d_j = \int_{t_0}^{t_0+T} (f_j(t) - f_{\text{COI}}(t))^2 dt, \quad (3.38)$$

where the center-of-inertia (COI) frequency is

$$f_{\text{COI}}(t) = \frac{\sum_{g=1}^{N_G} H_g f_g(t)}{\sum_{g=1}^{N_G} H_g}. \quad (3.39)$$

The integration window is selected as

$$t \in [0.5, 2.0] \text{ s},$$

capturing both the initial inertial response and the first swing.

Larger values of d_j indicate stronger deviation from the coherent system response and therefore lower effective inertia support. Buses are ranked in descending order of d_j to obtain the index-based inertia ranking.

Ranking comparison. The three resulting rankings:

- Proposed spatial inertia ranking $\{h_j\}$,
- Analytical nodal inertia ranking $\{h_j^{\text{Pinheiro}}\}$,
- Time-domain index ranking $\{d_j\}$,

are compared to evaluate consistency in computing bus inertia of the proposed method with the existing ones.

Agreement between the proposed method and the time-domain index-based ranking confirms that the electrical-distance formulation captures the spatial distribution of inertial support without requiring full dynamic simulation.

3.3 Determination of Minimum Bus-Level Inertia Thresholds

This section derives spatially resolved minimum inertia thresholds from the RoCoF security constraint under a representative active-power disturbance caused by a sudden load change event. The objective is to convert a disturbance defined at a reference location into bus-wise inertia requirements $\{h_{\min,i}\}$ that can be enforced as feasibility constraints in the inertia-aware optimization model (Section 3.4). The derivation is restricted to the initial inertial-response interval, during which governor action and secondary control have not yet produced a material response. The procedure comprises two steps: (i) map the disturbance magnitude at bus c to an equivalent propagated imbalance at each bus using reduced-network electrical coupling, and (ii) enforce local RoCoF limits to obtain the minimum inertia requirement at each bus.

3.3.1 Disturbance-to-Bus Mapping

Minimum inertia requirements are evaluated with respect to a disturbance that produces a sudden active-power imbalance of magnitude $|\Delta P_c|$ at bus c due to a load variation event. Immediately after the event, electromechanical frequency dynamics do not develop uniformly across the network; the initial response exhibits spatial dependence due to non-uniform coupling, particularly under low-inertia conditions [100].

3.3.1.1 Reduced-network coupling model.

Consider the steady-state network linearized around the pre-disturbance operating point. Over the inertial window, represent the disturbance as an incremental excitation at bus c associated with the net active-power imbalance $|\Delta P_c|$. Eliminating passive buses via Kron reduction yields a reduced admittance matrix $\mathbf{Y}_{\text{red}}$ relating

currents and voltages at retained boundary nodes. The (i, c) entry quantifies the current response at node i due to an excitation applied at node c under the reduced boundary conditions [101]. Hence, the magnitude $|Y_{ic}|$ provides an admittance-based indicator of how strongly bus i is electrically coupled to the disturbance location, and therefore how strongly the initial imbalance may influence bus i during the early inertial interval.

3.3.1.2 Normalized propagated disturbance allocation.

Motivated by the physical interpretation of $\mathbf{Y}_{\text{red}}$, the propagated disturbance at bus i is approximated as a normalized allocation of the disturbance magnitude proportional to coupling strength:

$$\Delta P_i = \frac{|Y_{ic}|}{\sum_{j=1}^n |Y_{jc}|} \Delta P_c, \quad (3.40)$$

where Y_{ic} denotes the reduced admittance between buses i and c and n is the number of buses. This allocation is not intended to reproduce the full spatiotemporal electromechanical wave dynamics; rather, it provides an analytically transparent proxy for relative bus-wise exposure to the disturbance across the network based on reduced-network coupling.

3.3.1.3 Implementation.

To reproduce the propagated disturbance distribution, the following operating-point-consistent steps are performed:

1. Solve the base-case AC power flow for the study system to obtain the pre-disturbance operating point.
2. Select the disturbance location c and magnitude $|\Delta P_c|$. The disturbance magnitude is defined as $|\Delta P_c|$, corresponding to the largest load in the system.

The disturbance location c is therefore the bus where this load is connected. In this study, the disturbance is modeled as a sudden change in this load, representing either a load disconnection or an equivalent increase in demand, which produces an instantaneous active-power imbalance in the system.

3. Form $\mathbf{Y}_{\text{bus}}$ at the operating point and compute a Kron-reduced matrix $\mathbf{Y}_{\text{red}}$ for the retained boundary buses that include all buses for which ΔP_i is to be evaluated. The reduction uses the standard expression in (3.7).
4. Compute the propagated disturbances for all buses using the normalized allocation rule in (3.40).
5. Plot the resulting bus-wise magnitudes $\{|\Delta P_i|\}$ to obtain the spatial disturbance distribution.

It should be noted that different types of power system disturbances influence frequency dynamics in distinct ways. In particular, the loss of a generating unit introduces two simultaneous effects: a sudden active-power deficit and the removal of the inertia associated with the tripped machine. In contrast, variations in load demand (load disconnection or load increase) produce an active-power imbalance without directly altering the amount of synchronous inertia available in the system.

The objective of the present formulation is to quantify the minimum inertia required to restrain the initial RoCoF for a specified disturbance magnitude rather than to explicitly model the detailed dynamics of generator outage events. Accordingly, the disturbance is represented as an active-power imbalance while the inertia distribution is evaluated separately using the spatial inertia estimation procedure described in Section 3.2.

Because the RoCoF relationship in (3.41) depends on the magnitude of the disturbance $|\Delta P_i|$, the minimum inertia thresholds derived in (3.43) are governed by disturbance magnitude rather than the direction of the frequency excursion. Consequently, the proposed framework remains applicable to both load increases

and load decreases provided that the disturbance magnitude used in the analysis reflects the studied operating scenario.

Because (3.40) is linear in $|\Delta P_c|$, the propagated disturbances scale proportionally with disturbance magnitude under fixed operating-point coupling. This property implies that if the thresholds derived for a selected worst-case disturbance magnitude $|\Delta P_c|$ satisfy the RoCoF criterion, then any smaller imbalance $|\Delta P| < |\Delta P_c|$ will produce a proportionally smaller initial RoCoF within the same linear inertial-response regime.

3.3.2 RoCoF Constraint and Minimum Inertia Requirement

Immediately following the load change event, primary and secondary frequency controls are assumed inactive over the inertial-response window, and frequency dynamics are dominated by the release or absorption of kinetic energy from online synchronous machines. Under the same inertial-window assumptions used in the spatial inertia formulation, the local frequency behavior at each bus is represented by the equivalent bus swing equation (3.13). Expressing frequency in per-unit with respect to the nominal frequency f_0 yields the RoCoF–inertia relation over the inertial interval:

$$|\dot{f}_i| = \left| \frac{df_i}{dt} \right| = \frac{f_0}{2h_i} |\Delta P_i|. \quad (3.41)$$

Secure operation requires that bus-wise RoCoF does not exceed a prescribed limit:

$$|\dot{f}_i| \leq |\dot{f}_{\max,i}|. \quad (3.42)$$

To satisfy the frequency security requirement, a uniform RoCoF limit is adopted across all buses, i.e. $|\dot{f}_{\max,i}| = |\dot{f}_{\max}|$ for all $i \in \mathcal{B}$. Although the RoCoF limit is uniform, the resulting inertia requirements remain spatially non-uniform because ΔP_i varies according to the electrical coupling to the disturbance location.

3.3.2.1 Closed-form minimum inertia threshold.

Substituting (3.40) into (3.41) and enforcing (3.42) yields a minimum inertia condition at each bus. Rearranging gives the minimum bus-level inertia threshold:

$$h_{\min,i} = \frac{f_0}{2|\dot{f}_{\max,i}|} \frac{|Y_{ic}|}{\sum_{j=1}^n |Y_{jc}|} |\Delta P_c| \quad (3.43)$$

Equation (3.43) shows that the required inertia increases with disturbance magnitude $|\Delta P_c|$ and with coupling strength $|Y_{ic}|$ to the disturbance location, reflecting greater exposure of electrically closer buses during the initial response. Conversely, stricter RoCoF limits (smaller allowable $|\dot{f}_{\max,i}|$) require higher inertia levels to restrain the initial frequency gradient.

3.3.2.2 Computation of $\{h_{\min,i}\}$ and security margins.

The procedure for quantifying minimum inertia thresholds and the associated security margin is summarized in the flowchart in Figure 3.2.

The resulting set $h_{\min,i}$ defines a spatially resolved RoCoF-security envelope for the selected load change disturbance magnitude under the specified RoCoF criterion. These thresholds are directly enforceable within the inertia-aware optimization framework as minimum bus-level inertia constraints.

3.4 Formulation of the Inertia-Constrained Multi-Objective Optimization Problem

This section formulates the inertia-constrained, multi-objective dispatch problem used to generate the Pareto-optimal operating points. The optimization co-determines maximum inertia-less VRES dispatch, synchronous generator (SG) commitment, and SG active-power dispatch, while enforcing steady-state AC

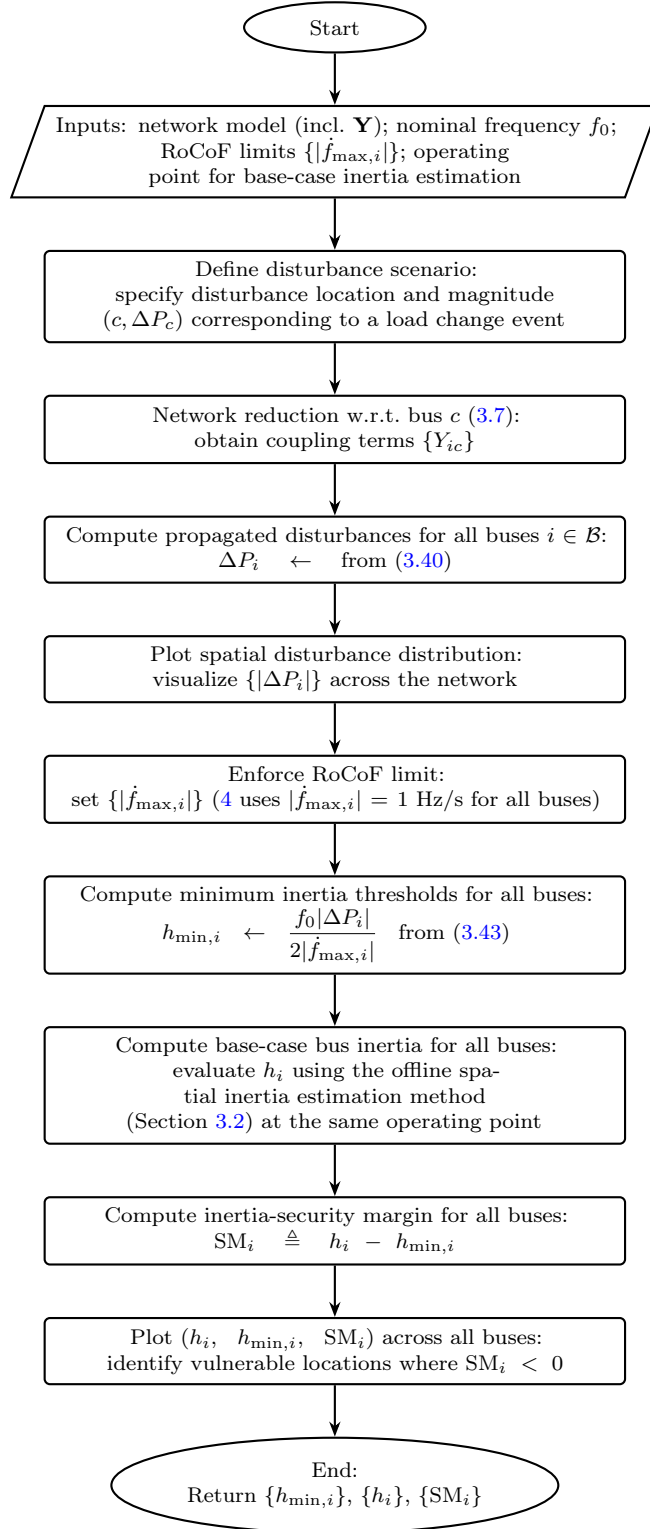


Figure 3.2: Flowchart of the minimum bus-level inertia threshold calculation and inertia-security margin evaluation for a load change disturbance.

feasibility and bus-level inertia adequacy constraint.

The minimum bus-level inertia thresholds $\{h_{\min,i}\}$ are pre-computed from a specified load change disturbance magnitude and RoCoF limit (Section 3.3) and then enforced as a feasibility constraint during scheduling. Bus-level available inertia $\{h_i\}$ is computed consistently with the committed SG set using the offline spatial inertia estimator (Section 3.2), evaluated at the operating point of each candidate schedule.

3.4.1 Deterministic Formulation and Treatment of Uncertainty

The inertia-constrained multi-objective optimization problem formulated in this section is deterministic. For each optimization run, load demand, PV availability, generator limits, network parameters, and minimum bus-level inertia thresholds are treated as known inputs. This modeling choice allows the proposed framework to focus on the relationship between inertia-less VRES dispatch, synchronous generator commitment, transmission losses, and spatial bus-level inertia adequacy under a specified operating condition.

The formulation does not explicitly include uncertainty variables, probability distributions, chance constraints, or robust uncertainty sets within the optimization problem itself. Therefore, the proposed NSGA-II formulation should not be interpreted as a full stochastic optimization or robust optimization model. Instead, it is a deterministic multi-objective scheduling formulation solved for defined operating scenarios.

However, uncertainty in load demand and PV availability is not ignored. It is addressed through scenario-based and stochastic validation, as described in subsection 3.7.1. In that validation stage, Monte Carlo operating scenarios are generated from variations in normalized load demand and PV availability. The proposed spatial inertia-aware scheduling framework and the benchmark method are then evaluated under identical stochastic operating conditions and fixed PV

penetration levels. The resulting operating points are assessed using dynamic frequency-security indicators, including RoCoF and frequency nadir.

This approach provides a practical compromise between modeling detail and computational tractability. Embedding uncertainty directly inside the optimization would require probability-weighted objective functions, scenario-dependent constraints, chance constraints, or robust feasibility conditions for multiple load and PV realizations. Such a full stochastic or robust optimization formulation is beyond the present scope. Nevertheless, the stochastic validation in subsection 3.7.1 provides evidence on whether the deterministic schedules produced by the proposed framework remain consistent with acceptable frequency-security performance under uncertain load and PV operating conditions.

3.4.2 Sets, Decision Variables, and Dependent States

Consider a power network defined by the bus set $\mathcal{B}$, line set $\mathcal{L}$, synchronous generator set $\mathcal{G}$, and inertia-less VRES bus set $\mathcal{R}$. The decision variables are

- $P_{G,g}$: active power output of synchronous generator $g \in \mathcal{G}$;
- $P_{R,r}$: active power dispatched from VRES unit $r \in \mathcal{R}$;
- $u_g \in \{0, 1\}$: commitment status of synchronous generator g ($u_g = 1$ online, $u_g = 0$ offline).

For each candidate decision vector, the steady-state AC operating point is obtained from an AC power-flow (PF) solution.

3.4.3 Objective Vector

The scheduling problem is posed as a three-objective minimization:

$$\min_{\mathbf{x}} [f_1(\mathbf{x}), f_2(\mathbf{x}), f_3(\mathbf{x})], \quad \mathbf{x} = \{P_{G,g}, P_{R,r}, u_g\}. \quad (3.44)$$

- (O1) **Maximize inertia-less VRES penetration.** Because NSGA-II is minimization-based, renewable penetration is represented by minimizing the negative penetration ratio:

$$f_1 = -\frac{\sum_{r \in \mathcal{R}} P_{R,r}}{\sum_{g \in \mathcal{G}} P_{G,g} + \sum_{r \in \mathcal{R}} P_{R,r}}. \quad (3.45)$$

- (O2) **Minimize transmission active-power losses.** Total real-power losses are computed from the converged AC PF operating point as

$$f_2^{\text{pu}} = \sum_{(i,j) \in \mathcal{L}} G_{ij} (V_i^2 + V_j^2 - 2V_i V_j \cos(\delta_i - \delta_j)), \quad (3.46)$$

where G_{ij} is the line conductance and (V_i, δ_i) are PF outputs. When reporting losses in MW in Chapter 4, the conversion is

$$f_2^{\text{MW}} = f_2^{\text{pu}} S_{\text{base}}.$$

- (O3) **Minimize the number of committed synchronous generators.**

$$f_3 = \sum_{g \in \mathcal{G}} u_g. \quad (3.47)$$

3.4.4 Operational and Network Constraints

Each chromosome must satisfy steady-state operating feasibility, equipment operating limits, and the spatial inertia adequacy requirements. In the optimization model, power adequacy is enforced at the system level, while bus-level quantities are retained only for network security variables that are inherently location-dependent, namely voltage magnitudes and bus-level inertia.

Let

$$P_D^{\text{tot}} = \sum_{i \in \mathcal{B}} P_{D,i}, \quad Q_D^{\text{tot}} = \sum_{i \in \mathcal{B}} Q_{D,i} \quad (3.48)$$

denote the total active and reactive system demands, respectively. Similarly, let

$$P_G^{\text{tot}} = \sum_{g \in \mathcal{G}} P_{G,g}, \quad Q_G^{\text{tot}} = \sum_{g \in \mathcal{G}} Q_{G,g}, \quad P_R^{\text{tot}} = \sum_{r \in \mathcal{R}} P_{R,r} \quad (3.49)$$

denote the total synchronous-generator active/reactive output and the total VRES active-power injection.

3.4.4.1 System-wide power balance constraints

For each chromosome, the scheduled total generation must satisfy the overall system demand after accounting for network losses. Thus, the active-power balance is enforced as

$$P_G^{\text{tot}} + P_R^{\text{tot}} = P_D^{\text{tot}} + P_{\text{loss}}, \quad (3.50)$$

where P_{loss} is the total active-power transmission loss obtained from the AC power-flow solution.

Similarly, the reactive-power balance must satisfy

$$Q_G^{\text{tot}} = Q_D^{\text{tot}} + Q_{\text{loss}}, \quad (3.51)$$

where Q_{loss} represents the net reactive-power requirement of the network as determined from the AC power-flow solution.

3.4.4.2 SG and VRES active-power limits

The active-power output of each synchronous generator is bounded by its technical operating range and is further conditioned on its binary commitment status. Similarly, each VRES unit can only be dispatched within the available renewable power under the corresponding operating scenario. These bounds ensure that the chromosome respects source-specific generation capabilities and resource availability.

$$u_g P_{G,g}^{\min} \leq P_{G,g} \leq u_g P_{G,g}^{\max}, \quad \forall g \in \mathcal{G}, \quad (3.52)$$

$$0 \leq P_{R,r} \leq P_{R,r}^{\text{available}}, \quad \forall r \in \mathcal{R}. \quad (3.53)$$

3.4.4.3 Bus voltage magnitude limits

Voltage security remains location-dependent and is therefore enforced at every bus:

$$V_i^{\min} \leq V_i \leq V_i^{\max}, \quad \forall i \in \mathcal{B}. \quad (3.54)$$

3.4.4.4 Spatial inertia adequacy constraint

Minimum bus-level inertia thresholds $h_{\min,i}$ are derived from the RoCoF-based criterion of Section 3.3 for a specified load change disturbance $(c, \Delta P_c)$ and a specified RoCoF limit $|\dot{f}_{\max}|$. For a given study scenario, $h_{\min,i}$ are computed *once* from the base-case operating point and treated as fixed constraints during each NSGA-II run. This threshold set is then used to define feasible and infeasible operating points.

Since inertia distribution is inherently spatial, the inertia adequacy condition is enforced at every bus:

$$h_i \geq h_{\min,i}, \quad \forall i \in \mathcal{B}. \quad (3.55)$$

Bus inertia evaluation under commitment decisions.

Let $\mathcal{G}_{\text{on}} = \{g \in \mathcal{G} \mid u_g = 1\}$ denote the set of online SGs. For each chromosome with a converged power-flow solution, bus-level inertia h_i is computed using the closed-form spatial inertia estimator in equation (3.24), with the summation restricted to $\mathcal{G}_{\text{on}}$. Decommitted units are excluded from the bus inertia computation so that h_i reflects the inertia physically available under the candidate commitment.

To preserve consistency with the offline inertia estimation method, the electrical-

distance terms d_{gi} used in equation (3.24) are evaluated from the power-flow operating point associated with the chromosome, using the network reduction and coupling workflow in Section 3.2. Therefore, when $\mathcal{G}_{\text{on}}$ changes, the reduced-network coupling and the resulting d_{gi} and $\{h_i\}$ are recomputed for that chromosome.

3.5 NSGA-II Optimization Implementation and Solution Workflow

The optimization problem in Section 3.4 is nonlinear and multi-objective with mixed-integer variables. Spatial inertia adequacy constraint introduces strong non-convexities and discontinuities. NSGA-II is therefore adopted due to its ability to approximate non-convex Pareto sets while accommodating discrete and continuous variables [75].

In this study, NSGA-II is integrated with physics-based evaluation. Each candidate solution is assessed via AC power flow, network reduction, electrical-distance computation, and bus-level inertia estimation. The overall workflow is shown in Figure 3.3.

3.5.1 Constraint Handling and Feasibility Rules

In the NSGA-II implementation, each candidate decision vector $\mathbf{x}$ is encoded as a chromosome Ω , whose genes correspond to the variables $\{P_{G,g}\}_{g \in \mathcal{G}}$, $\{P_{R,r}\}_{r \in \mathcal{R}}$, and $\{u_g\}_{g \in \mathcal{G}}$. Each chromosome is decoded into a candidate operating point and evaluated using an AC power-flow (PF) solution in MATPOWER. Because the optimization contains both continuous dispatch variables and binary commitment variables, not all generated chromosomes are physically feasible. Feasibility is therefore enforced through a constrained-domination strategy, while violation magnitudes are quantified through penalty indices to rank infeasible solutions consistently.

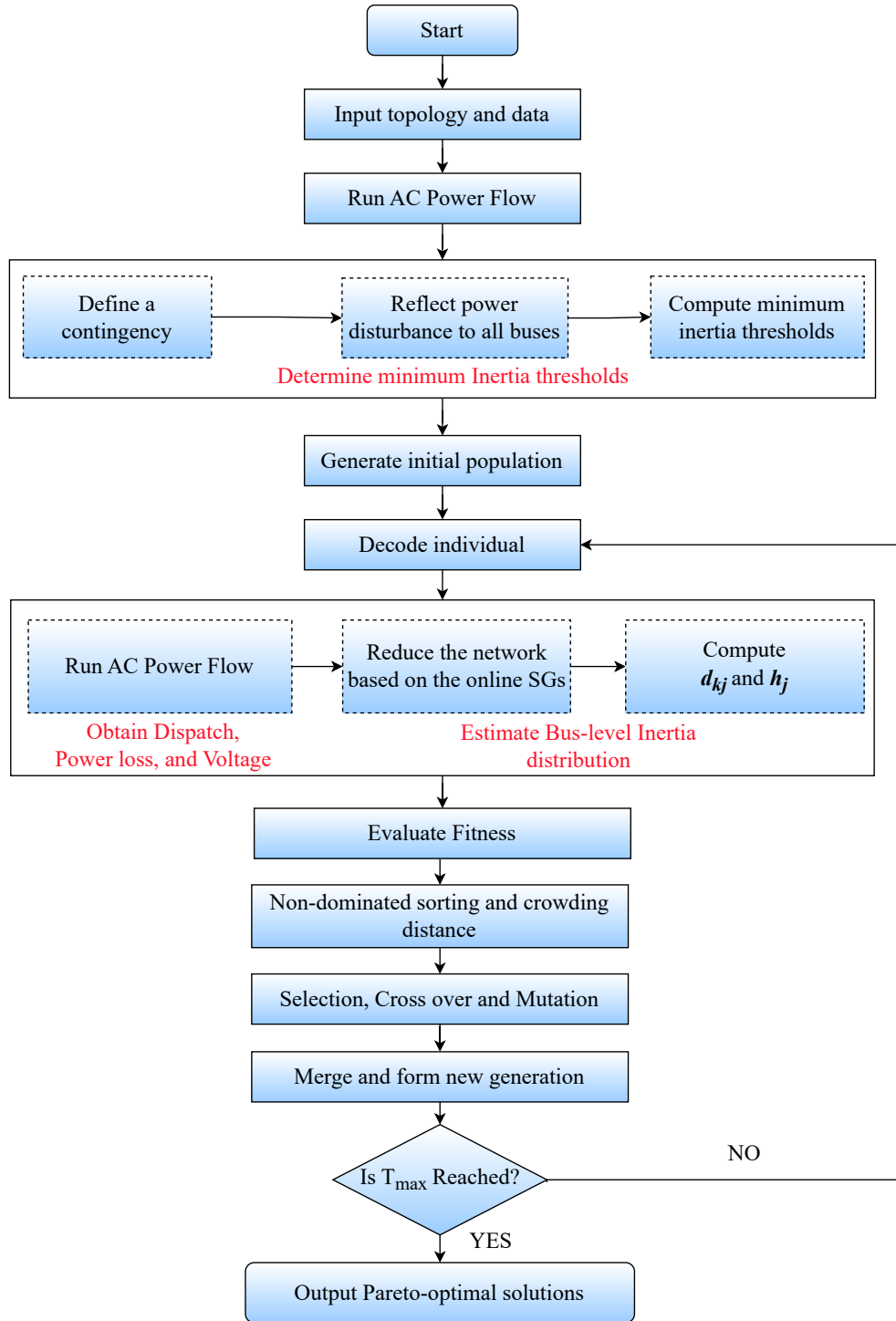


Figure 3.3: Overall workflow of the inertia-aware optimization framework.

Let δ_{conv} denote the PF convergence indicator:

$$\delta_{\text{conv}} = \begin{cases} 1, & \text{if the AC PF converges,} \\ 0, & \text{if the AC PF does not converge.} \end{cases} \quad (3.56)$$

A non-convergent PF solution is assigned the penalty

$$V_{\text{PF}} = \lambda_{\text{PF}} (1 - \delta_{\text{conv}}), \quad (3.57)$$

where λ_{PF} is the non-convergence penalty factor.

If the PF converges, bus-voltage violations are evaluated using the limits in (3.54).

The corresponding voltage-violation penalty is defined as

$$V_{\text{volt}} = \lambda_V \sum_{i \in \mathcal{B}} [\max(0, V_i - V_i^{\max}) + \max(0, V_i^{\min} - V_i)], \quad (3.58)$$

where λ_V is the voltage-violation penalty factor.

After obtaining a converged PF solution, the bus-level inertia values $\{h_i\}_{i \in \mathcal{B}}$ are computed using the inertia-evaluation procedure described in Section 3.2. The inertia-adequacy violation is then quantified by

$$V_h = \lambda_h \sum_{i \in \mathcal{B}} \max(0, h_{\min,i} - h_i), \quad (3.59)$$

where λ_h is the inertia-violation penalty factor.

The total violation index used to compare infeasible chromosomes is

$$V_{\text{tot}} = V_{\text{PF}} + V_{\text{volt}} + V_h. \quad (3.60)$$

For non-convergent PF cases, only V_{PF} is evaluated, and the remaining violation terms are set to zero. Thus, non-convergent chromosomes satisfy

$$V_{\text{tot}} = V_{\text{PF}}.$$

Constraint handling during ranking follows Deb’s constrained-domination principle [75]. For any two chromosomes, a feasible chromosome is always preferred over an infeasible chromosome. If both chromosomes are infeasible, the chromosome with the smaller total violation index V_{tot} is preferred. If both chromosomes are feasible, comparison is performed using Pareto rank and crowding distance only. This strategy ensures that feasibility is prioritized while still preserving a meaningful search direction within infeasible regions.

3.5.2 Initialization and NSGA-II Parameter Settings

Each chromosome contains the synchronous-generator dispatch variables $\{P_{G,g}\}_{g \in \mathcal{G}}$, VRES dispatch variables $\{P_{R,r}\}_{r \in \mathcal{R}}$, and synchronous-generator commitment variables $\{u_g\}_{g \in \mathcal{G}}$, as defined in Section 3.4.2. The initial population is generated randomly within the admissible variable bounds of (3.52)–(3.53), with binary commitment genes initialized in $\{0, 1\}$ and continuous genes sampled within their corresponding lower and upper limits.

The NSGA-II control parameters adopted in this study are summarized in Table 3.1. These values were selected based on standard practice in evolutionary multi-objective optimization and were further checked through preliminary simulation runs to ensure stable convergence behavior and adequate Pareto-front spread for the IEEE 39-bus case study.

The penalty coefficients were chosen sufficiently large to ensure that infeasible chromosomes are consistently deprioritized under constrained domination, while avoiding excessive numerical scaling in the violation index.

3.5.2.1 Justification of parameter settings.

The NSGA-II parameters in Table 3.1 were selected to balance Pareto-front quality, feasibility discovery, and computational burden. Since each chromosome evaluation requires AC power-flow solution, reduced-network computation, electrical-distance

Table 3.1: NSGA-II control parameters and algorithmic settings

Notation	Parameter and Description	Value
Population Setup		
N_p	Population size	50
$T_{\max}$	Maximum number of generations	300
Variation Parameters		
P_c	Crossover probability	0.9
P_m	Mutation probability	0.05
p_m	Bit-flip mutation probability for binary genes	$1/ \mathcal{G} $
η_c	SBX distribution index	20
η_m	Polynomial-mutation distribution index	20
Penalty Factors		
λ_V	Voltage magnitude violation penalty factor	1000
λ_{PF}	Power-flow non-convergence penalty factor	1000
λ_h	Bus-level inertia inadequacy penalty factor	1000

evaluation, and bus-level inertia adequacy checking, excessively large population sizes or generation numbers substantially increase computational cost. Conversely, very small population sizes may reduce solution diversity and may fail to adequately explore the mixed continuous–binary search space.

The selected population size of $N_p = 50$ was adopted as a compromise between search diversity and computational tractability. Preliminary runs with smaller populations produced limited Pareto-front spread and fewer feasible non-dominated solutions, particularly because the inertia constraint restricts the feasible search region. Larger populations improved diversity only marginally but increased runtime because the number of candidate evaluations increases approximately in proportion to $N_p T_{\max}$.

The maximum generation number of $T_{\max} = 300$ was selected after monitoring the evolution of the feasible non-dominated set and the hypervolume indicator. Stabilization of the hypervolume was interpreted as evidence that further iterations produced limited improvement relative to the additional computational cost.

The crossover probability was set to $P_c = 0.9$ to encourage recombination of promising dispatch and commitment patterns. The mutation probability was set to $P_m = 0.05$ for continuous variables, while binary commitment genes were mutated using a bit-flip probability of $1/|\mathcal{G}|$. This mutation setting provides a low but nonzero probability of changing synchronous-generator commitment status, which is important because commitment decisions strongly influence inertia feasibility.

The SBX and polynomial mutation distribution indices were both set to 20. These values generate moderate local perturbations around parent solutions, allowing the algorithm to refine promising operating points without causing excessive disruption to feasible dispatch patterns.

3.5.3 Fitness Evaluation Procedure

For each chromosome, fitness evaluation consists of a sequential feasibility and objective-assessment procedure. This computational pipeline produces the quantities required for non-dominated sorting, including the objective values, violation indices, Pareto ranks, and crowding distances.

For a chromosome

$$\Omega = \left\{ \{P_{G,g}\}_{g \in \mathcal{G}}, \{P_{R,r}\}_{r \in \mathcal{R}}, \{u_g\}_{g \in \mathcal{G}} \right\}, \quad (3.61)$$

the evaluation steps are as follows.

1. *AC power-flow solution.* The chromosome variables are imposed on the network model and an AC PF is solved in MATPOWER to obtain the bus-voltage magnitudes and angles, generator reactive outputs, and branch-flow quantities. If the PF does not converge, the chromosome is marked infeasible through (3.57), and no voltage or inertia checks are performed.
2. *Voltage-feasibility assessment.* If the PF converges, the bus-voltage limits in (3.54) are checked and the voltage-violation penalty V_{volt} is computed using

(3.58).

3. *Loss evaluation.* The transmission-loss objective is computed from the converged AC PF solution using (3.46). In the optimization stage, this objective is evaluated in per-unit form, that is, $f_2 \equiv f_2^{\text{pu}}$.
4. *Spatial inertia evaluation.* Using the converged operating point and the committed generator set $\mathcal{G}_{\text{on}}$, the bus-level inertia values $\{h_i\}$ are computed using the offline inertia-estimation framework of Section 3.2, restricted to the generators that are online in the chromosome under evaluation. The inertia constraints in (3.55) are then checked and the associated penalty V_h is evaluated using (3.59).
5. *Objective-vector construction.* The multi-objective performance of the chromosome is represented by the objective vector

$$\mathbf{f}(\Omega) = [f_1(\Omega), f_2(\Omega), f_3(\Omega)],$$

where f_1 , f_2 , and f_3 are defined by (3.45)–(3.47).

6. *Normalization for diversity calculation.* To prevent objective-scale disparity from biasing the crowding-distance calculation, per-generation min–max normalization is applied as

$$f_i^{\text{norm}} = \frac{f_i - f_i^{\min}}{f_i^{\max} - f_i^{\min}}, \quad i \in \{1, 2, 3\}, \quad (3.62)$$

where $f_i^{\min}$ and $f_i^{\max}$ are the minimum and maximum values of objective i within the population being ranked. If $f_i^{\max} = f_i^{\min}$, the normalized value is assigned as $f_i^{\text{norm}} = 0$ for all chromosomes in that objective.

Pareto dominance is applied to the original objective values, whereas the normalized objectives are used only in the crowding-distance calculation.

3.5.4 Non-Dominated Sorting and Crowding Distance

After fitness evaluation, chromosomes are ranked using constrained non-dominated sorting. Let $\mathbf{x}_p$ and $\mathbf{x}_q$ denote two chromosomes with objective vectors

$$\mathbf{f}(\mathbf{x}) = [f_1(\mathbf{x}), f_2(\mathbf{x}), f_3(\mathbf{x})].$$

If both chromosomes are feasible, $\mathbf{x}_p$ is said to Pareto-dominate $\mathbf{x}_q$ if

$$\begin{cases} f_i(\mathbf{x}_p) \leq f_i(\mathbf{x}_q), & \forall i \in \{1, 2, 3\}, \\ \exists j \in \{1, 2, 3\} : f_j(\mathbf{x}_p) < f_j(\mathbf{x}_q). \end{cases} \quad (3.63)$$

Non-dominated sorting partitions the population into successive fronts

$$\mathcal{F}_1, \mathcal{F}_2, \dots, \mathcal{F}_K,$$

where $\mathcal{F}_1$ contains the non-dominated chromosomes, $\mathcal{F}_2$ contains chromosomes dominated only by those in $\mathcal{F}_1$, and so forth.

To preserve diversity within each front, the crowding distance is computed using the normalized objectives. For a chromosome indexed by p within a given front, the crowding distance is defined as

$$D_p = \sum_{i=1}^3 (f_{i,p+1}^{\text{norm}} - f_{i,p-1}^{\text{norm}}), \quad (3.64)$$

where the subscripts $p-1$ and $p+1$ denote the neighbouring chromosomes after sorting the front in ascending order of the i th normalized objective. Boundary chromosomes are assigned an infinite crowding distance to ensure preservation of extreme Pareto-front solutions.

3.5.5 Selection, Crossover, and Mutation

After ranking and crowding-distance assignment, the offspring population is generated from the parent population through binary tournament selection followed by crossover and mutation. Since the chromosome contains both continuous dispatch genes and binary commitment genes, the variation operators are applied in a type-consistent manner.

Let the parent population at generation t be

$$\mathcal{P}_t = \{\Omega_1, \Omega_2, \dots, \Omega_{N_p}\},$$

where each chromosome has the form

$$\Omega = \left\{ \{P_{G,g}\}_{g \in \mathcal{G}}, \{P_{R,r}\}_{r \in \mathcal{R}}, \{u_g\}_{g \in \mathcal{G}} \right\}.$$

Selection. Parent chromosomes are selected using binary tournament selection. Given two candidate chromosomes Ω_p and Ω_q , preference is assigned according to the following hierarchy:

1. if one chromosome is feasible and the other is infeasible, the feasible chromosome is selected;
2. if both chromosomes are infeasible, the chromosome with the smaller total violation index V_{tot} is selected;
3. if both chromosomes are feasible, the chromosome with the lower Pareto rank is selected;
4. if both chromosomes have the same Pareto rank, the chromosome with the larger crowding distance is selected.

This comparison rule is denoted compactly by

$$\Omega_p \succ \Omega_q,$$

meaning that Ω_p is preferred over Ω_q in tournament selection.

Crossover. Once two parent chromosomes, denoted $\Omega^{(p_1)}$ and $\Omega^{(p_2)}$, are selected, crossover is applied with probability P_c .

For the continuous genes corresponding to synchronous-generator dispatch and VRES dispatch, simulated binary crossover (SBX) with distribution index η_c is used. For a representative pair of parent genes x_1 and x_2 , the offspring genes c_1 and c_2 are computed as

$$c_1 = \frac{1}{2} [(1 + \beta)x_1 + (1 - \beta)x_2], \quad c_2 = \frac{1}{2} [(1 - \beta)x_1 + (1 + \beta)x_2], \quad (3.65)$$

where the spread factor β is defined by

$$\beta = \begin{cases} (2u)^{\frac{1}{\eta_c+1}}, & u \leq 0.5, \\ \left(\frac{1}{2(1-u)}\right)^{\frac{1}{\eta_c+1}}, & u > 0.5, \end{cases} \quad (3.66)$$

with $u \sim \mathcal{U}(0, 1)$. This operator is applied component-wise to all continuous genes in $\{P_{G,g}\}$ and $\{P_{R,r}\}$.

For the binary commitment genes $\{u_g\}$, uniform discrete recombination is used. For each generator $g \in \mathcal{G}$, the offspring inherits the commitment state from one of the two parents:

$$u_g^{(c)} = \begin{cases} u_g^{(p_1)}, & \text{with probability 0.5,} \\ u_g^{(p_2)}, & \text{with probability 0.5.} \end{cases} \quad (3.67)$$

This preserves the binary nature of the commitment variables throughout crossover.

Mutation. After crossover, mutation is applied with probability P_m .

For the continuous dispatch genes, polynomial mutation with distribution index η_m is used. For a continuous decision variable $x \in [x^L, x^U]$, the mutated value is

$$x' = x + \delta_q(x^U - x^L), \quad (3.68)$$

where

$$\delta_q = \begin{cases} [2u + (1 - 2u)(1 - \Delta_1)^{\eta_m+1}]^{\frac{1}{\eta_m+1}} - 1, & u \leq 0.5, \\ 1 - [2(1 - u) + 2(u - 0.5)(1 - \Delta_2)^{\eta_m+1}]^{\frac{1}{\eta_m+1}}, & u > 0.5, \end{cases} \quad (3.69)$$

with

$$\Delta_1 = \frac{x - x^L}{x^U - x^L}, \quad \Delta_2 = \frac{x^U - x}{x^U - x^L},$$

and $u \sim \mathcal{U}(0, 1)$.

For the binary commitment genes, bit-flip mutation is applied independently to each commitment variable:

$$u_g \leftarrow 1 - u_g \quad \text{with probability} \quad p_m = \frac{1}{|\mathcal{G}|}, \quad (3.70)$$

for each $g \in \mathcal{G}$. This operator enables exploration of alternative commitment schedules by switching individual synchronous generators between online and offline states.

Repair and bound restoration. After crossover and mutation, all offspring genes are repaired to satisfy their admissible domains. Continuous VRES-dispatch genes are projected onto

$$0 \leq P_{R,r} \leq P_{R,r}^{\text{available}}, \quad \forall r \in \mathcal{R}.$$

For synchronous-generator dispatch genes, commitment consistency is enforced explicitly:

$$P_{G,g} = \begin{cases} 0, & u_g = 0, \\ \text{clip}(P_{G,g}, P_{G,g}^{\min}, P_{G,g}^{\max}), & u_g = 1, \end{cases} \quad \forall g \in \mathcal{G}, \quad (3.71)$$

where $\text{clip}(\cdot)$ denotes projection onto the stated interval. Binary genes are finally enforced to satisfy

$$u_g \in \{0, 1\}, \quad \forall g \in \mathcal{G}.$$

The resulting set of offspring chromosomes forms the offspring population

$$\mathcal{Q}_t = \text{Variation}(\mathcal{P}_t), \quad (3.72)$$

which is then evaluated using the procedure described in [Section 3.5.3](#).

3.5.6 Parent-Offspring Merging and Generation Update

NSGA-II uses an elitist replacement mechanism in which the current parent population and the newly generated offspring population are merged before survivor selection. At generation t , the combined population is

$$\mathcal{R}_t = \mathcal{P}_t \cup \mathcal{Q}_t, \quad (3.73)$$

so that

$$|\mathcal{R}_t| = 2N_p.$$

The chromosomes in $\mathcal{R}_t$ are ranked again using constrained non-dominated sorting, producing an ordered sequence of fronts

$$\mathcal{F}_1, \mathcal{F}_2, \dots, \mathcal{F}_K,$$

where $\mathcal{F}_1$ is the best front and larger front indices correspond to progressively less preferred chromosomes. Crowding distances are recomputed within each front.

The next parent population $\mathcal{P}_{t+1}$ is then constructed by filling chromosomes from the ranked fronts in ascending order of front index until N_p chromosomes have been selected. If the next complete front cannot be fully accommodated, the remaining positions are filled by choosing the chromosomes with the largest crowding distances from that partially admitted front.

Let m denote the first front for which full inclusion would exceed the population limit. Then

$$\sum_{k=1}^{m-1} |\mathcal{F}_k| < N_p, \quad \sum_{k=1}^m |\mathcal{F}_k| \geq N_p.$$

Accordingly, the next parent population is formed as

$$\mathcal{P}_{t+1} = \left(\bigcup_{k=1}^{m-1} \mathcal{F}_k \right) \cup \mathcal{F}_m^*, \quad (3.74)$$

where $\mathcal{F}_m^* \subseteq \mathcal{F}_m$ contains the subset of chromosomes with the largest crowding distances required to satisfy

$$|\mathcal{P}_{t+1}| = N_p.$$

This merge-sort-truncate procedure constitutes the elitist generation-update mechanism of NSGA-II. It preserves the best non-dominated feasible chromosomes found so far while maintaining diversity across the Pareto front through crowding-distance-based survivor selection.

The iterative cycle of offspring generation, fitness evaluation, ranking, and population update is repeated until the stopping criterion

$$t = T_{\max}$$

is reached. For the present study, the maximum number of generations is set to $T_{\max} = 300$.

3.5.7 Post-processing to Generate Results for Evaluation

This subsection defines the deterministic post-processing steps used to construct the reported Pareto points, dispatch tables, inertia plots, and loss metrics.

3.5.7.1 Pareto Front Extraction and Representative Operating Points

After the NSGA-II algorithm reaches the termination criterion of $T_{\max}$ generations, the final population of chromosomes is evaluated and filtered to retain only feasible non-dominated solutions. These solutions form the Pareto set $\mathcal{P}$, representing the achievable trade-off between inertia-less VRES penetration and transmission losses under the spatial inertia constraints.

Each feasible solution corresponds to a chromosome

$$\Omega_{\text{Chromosome}} = \{P_G^1, \dots, P_G^{|\mathcal{G}|}, P_R^1, \dots, P_R^{|\mathcal{R}|}, u_1, \dots, u_{|\mathcal{G}|}\},$$

which uniquely determines both the generator dispatch and the synchronous generator commitment pattern.

From the binary commitment genes $\{u_g\}$, the online generator set

$$\mathcal{G}_{\text{on}} = \{g \in \mathcal{G} \mid u_g = 1\}$$

is obtained. This set determines the available synchronous inertia and therefore the feasible VRES penetration level. The continuous dispatch genes $\{P_G^g, P_R^r\}$ determine the spatial pattern of active power injections that governs the resulting transmission losses through the AC power flow solution.

For each chromosome in the Pareto set $\mathcal{P}$, three objective values are recorded:

- the inertia-less VRES penetration level π_k derived from the renewable dispatch variables $\{P_R^r\}$,
- the corresponding network transmission loss ℓ_k obtained from the AC power

flow.

- set of SGs committed.

The Pareto solutions are ordered according to increasing penetration π_k to reveal the progression of feasible operating states as the system moves from low to high renewable utilization. Because the commitment variables $\{u_g\}$ are discrete, changes in generator commitment produce distinct operating regimes along the Pareto front.

To facilitate interpretation of the Pareto structure, a set of representative operating points is selected from $\mathcal{P}$. Each representative point corresponds to a chromosome whose commitment vector $\{u_g\}$ defines a particular synchronous generator set $\mathcal{G}_{\text{on}}$. The associated dispatch variables $\{P_G^g, P_R^r\}$ then determine the corresponding power-flow solution and system losses.

Knee point identification Among the feasible Pareto solutions, a knee point is identified to represent the operating state where the trade-off between VRES penetration and transmission losses is most balanced. Let (π_k, ℓ_k) denote the penetration and loss values associated with solution k . These quantities are first normalized to the interval $[0, 1]$ across the Pareto set $\mathcal{P}$.

The knee point is determined as the solution that maximizes the perpendicular distance to the line connecting the two extreme Pareto points in the (π, ℓ) objective space. This geometric criterion identifies the operating point where a marginal improvement in one objective would lead to a comparatively larger degradation in the other.

The resulting chromosome is selected as the representative knee operating point. Its commitment vector $\{u_g\}$ and dispatch variables $\{P_G^g, P_R^r\}$ are subsequently used in Chapter 4 to analyze the structure of the Pareto front and the evolution of synchronous generator commitment patterns across increasing VRES penetration levels.

3.5.7.2 Dispatch tables

For a selected operating point, SG dispatch values $\{P_{G,g}\}$ and statuses $\{u_g\}$ are taken from the chromosome, and the corresponding loading percentages are computed as

$$\text{Loading}_g(\%) = 100 \frac{P_{G,g}}{P_{G,g}^{\max}} \quad \text{for } u_g = 1.$$

VRES plant dispatch and loading are computed as

$$\text{Loading}_r(\%) = 100 \frac{P_{R,r}}{P_{R,r}^{\text{available}}}.$$

3.5.7.3 Inertia distribution and bottleneck identification

For each selected operating point, the optimized bus inertias $\{h_i\}$ are computed under its committed SG set, and compared against $\{h_{\min,i}\}$ to generate the inertia distribution plot. The inertia security margin is

$$\text{SM}_i = h_i - h_{\min,i}.$$

The bottleneck region is identified as the set of buses with the smallest positive margins (i.e., buses where SM_i is within a small tolerance), and is used to highlight inertia-constrained areas.

3.5.7.4 Branch loss ratio (LRR)

Let $P_{\ell,ij}^{\text{base}}$ denote the real-power loss on branch (i,j) in the pre-optimization base case (e.g., 0% VRES dispatch), and $P_{\ell,ij}^{\text{opt}}$ the loss under an optimized operating point. The loss ratio on each branch is

$$\text{LRR}_{ij} = \frac{P_{\ell,ij}^{\text{opt}}}{P_{\ell,ij}^{\text{base}}}. \quad (3.75)$$

Branches are separated into the sets $\{\text{LRR}_{ij} \leq 1\}$ and $\{\text{LRR}_{ij} > 1\}$ to generate

the two-panel loss comparison plot.

3.5.7.5 Sensitivity Analysis with Respect to RoCoF Limits

To investigate the influence of frequency stability requirements on optimal dispatch decisions, a parametric sensitivity analysis is conducted with respect to the allowable RoCoF limit. The RoCoF constraint directly determines the minimum bus-level inertia thresholds derived in Section 3.3, and therefore governs the minimum number and spatial distribution of synchronous generators that must remain online.

RoCoF Limit Definition

Let $\dot{f}_{\max}$ denote the allowable magnitude of RoCoF following the specified load change disturbance. The minimum inertia requirement at each bus is computed using equation (3.43). Because $h_{\min,i}$ is inversely proportional to $\dot{f}_{\max}$, tightening the RoCoF limit increases the inertia requirement, while relaxing the limit reduces the minimum inertia constraint.

Sensitivity Study Design

The optimization problem defined in Section 3.4 is solved repeatedly under different RoCoF limits:

- Strict limit: $\dot{f}_{\max} = 1 \text{ Hz/s}$,
- Moderate limit: $\dot{f}_{\max} = 2 \text{ Hz/s}$,
- Relaxed limit: $\dot{f}_{\max} = 3 \text{ Hz/s}$.

For each RoCoF setting:

1. Bus-level inertia thresholds $h_{\min,i}$ are computed using equation (3.43).
2. The NSGA-II multi-objective optimization problem is solved to obtain Pareto-optimal solutions.

3. All other system parameters, including network topology, the specified load change disturbance definition, and load conditions, are held constant to isolate the effect of the RoCoF constraint.

Evaluation Metrics

For each RoCoF limit, the resulting Pareto fronts are analyzed in terms of:

1. Maximum feasible VRES penetration,
2. Associated transmission losses,
3. Implicit synchronous generator commitment patterns required to satisfy inertia constraints.

This parametric analysis enables systematic assessment of how the strictness of frequency stability requirements influences:

- Inertia availability,
- Spatial generation distribution,
- Network loading conditions,
- Renewable integration capability.

The corresponding Pareto fronts for the selected RoCoF limits are presented in Chapter 4 (Figure 4.22).

3.5.7.6 Performance Assessment Under Time-Varying Load and VRES Profiles at RoCoF limit of 1Hz/s

To evaluate the performance of the proposed inertia-aware scheduling framework under realistic seasonal variability, a deterministic 24-hour analysis is conducted. The objective is to examine how time-varying load demand and solar PV availability

influence optimal dispatch decisions, inertia feasibility, and network operating conditions across representative winter operating hours.

Data Source and Profile Construction

Hourly load demand and solar PV availability data for the year 2018 are obtained from the Open Energy Data Initiative (OEDI) database [102, 103]. The raw time-series data are normalized to produce per-unit scaling factors in the interval $[0, 1]$.

The normalized load profile is denoted by $\alpha_{\text{load}}(t)$ and the normalized PV availability profile by $\gamma_{\text{PV}}(t)$, where $t \in \{1, \dots, 24\}$ represents the hour of the day.

Mapping to the IEEE 39-Bus System

The normalized profiles are mapped to the IEEE 39-bus system by scaling the base-case load and installed PV capacities as follows:

$$P_{D,i}(t) = \alpha_{\text{load}}(t) P_{D,i}^0, \quad Q_{D,i}(t) = \alpha_{\text{load}}(t) Q_{D,i}^0, \quad (3.76)$$

where $P_{D,i}^0$ and $Q_{D,i}^0$ denote the base-case real and reactive demands at bus i .

Available PV generation at bus j is computed as

$$P_{\text{PV-available},j}(t) = \gamma_{\text{PV}}(t) P_{\text{PV},j}^{\text{rated}}, \quad (3.77)$$

where $P_{\text{PV},j}^{\text{rated}}$ is the installed PV capacity at that bus.

This mapping preserves the spatial distribution of demand and generation while introducing realistic temporal variability.

Operating Scenario Definition

For each hour t :

1. The scaled load and PV availability are computed using Eqs. (3.76)–(3.77).
2. An AC power-flow solution is obtained to establish the operating state and

network losses.

3. The inertia-constrained multi-objective optimization problem (Section 3.4) is solved using NSGA-II, enforcing the minimum bus-level inertia thresholds derived in Section 3.3.

This procedure yields a set of Pareto-optimal dispatch solutions for each hour, capturing the trade-off between network losses and VRES penetration subject to spatial inertia constraints.

Representative Operating Points

To illustrate system behavior under distinct seasonal conditions, three representative hours are selected:

- Hour 8: Morning peak demand with low PV availability.
- Hour 13: Solar peak with moderate load.
- Hour 18: Late-afternoon period with declining PV and reduced load.

These operating points represent structurally different inertia-demand interactions:

- High-load dominated operation,
- High-PV dominated operation,
- Low-load low-generation operation.

For each representative hour, the resulting Pareto fronts are analyzed to assess:

1. Feasibility of inertia constraints,
2. Sensitivity of dispatch decisions to PV availability,
3. Impact of loading conditions on network losses.

The resulting 24-hour load and PV profiles used in this analysis are shown in Figure 4.23. The corresponding Pareto fronts for the selected representative hours are presented and discussed in section 4.3.4.2.

3.5.7.7 Convergence metric (hypervolume)

To assess convergence, the hypervolume indicator is computed per generation using the set of feasible non-dominated solutions in normalized objective space. A fixed reference point in $[0, 1]^3$ is used, and the hypervolume is tracked across generations to quantify the growth and stabilization of the attained Pareto set.

3.5.7.8 Computational timing

Runtime is measured over a complete NSGA-II run from initial population evaluation through termination, including all PF and inertia evaluations. The number of evaluated candidate solutions equals $N_p \times T_{\max}$, and PF evaluation time is recorded separately to quantify its share of the overall runtime.

3.6 Test Systems and Case Study Definitions

A progressive validation pathway is adopted. The IEEE 9-bus system is used as a controlled environment for analytical verification and interpretability of generator–bus electromechanical coupling and spatial inertia mapping. The modified IEEE 39-bus system is used as the primary platform for scalability assessment, inertia-threshold computation, optimization studies, and robustness validation under time-varying operating conditions.

All studies are performed on a common MVA base, $S_B = 100$ MVA and nominal frequency, $f_0 = 60$ Hz to ensure consistency between inertia estimation, frequency security constraint, optimization, and dynamic-response assessments. Generator inertia constants provided on individual machine bases are converted to the common

system base before performing inertia-related calculations using

$$H_g^{\text{sys}} = H_g^{\text{mach}} \frac{S_g^{\text{rated}}}{S_B}, \quad (3.78)$$

where H_g^{sys} is the inertia constant of generator g expressed on the system MVA base, H_g^{mach} is the inertia constant on the individual machine's own MVA base, S_g^{rated} denotes the rated apparent power of generator g , and S_B is the system base MVA. This conversion ensures a consistent inertia representation across all generators.

3.6.1 IEEE 9-Bus Test System

The IEEE 9-bus system (WSCC 3-machine system) is a widely adopted benchmark for dynamic and inertia-related investigations [99, 104, 105]. Its simplicity supports transparent interpretation of electrical coupling and inertia propagation. The system comprises of 9 buses, 3 synchronous generators, 3 loads, and 6 transmission lines, on a base of $S_B = 100$ MVA and $f_0 = 60$ Hz. The system's single-line diagram is shown in Figure 3.4.

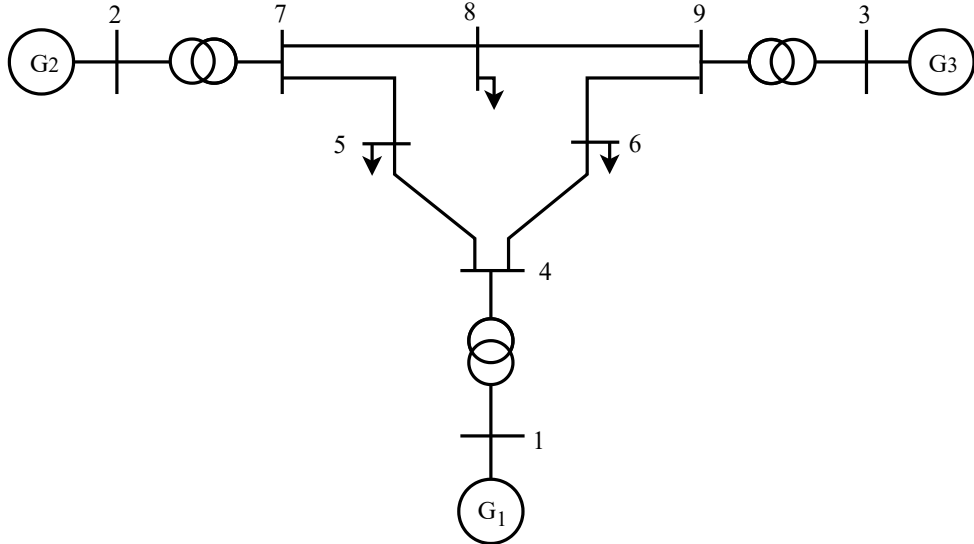


Figure 3.4: IEEE 9-bus test system

The system model data were adopted from the benchmark model provided in

PowerFactory version 15.1.7, which is based on the formulations and test system parameters described by Anderson and Fouad [99]. Table 3.2 summarizes the key parameters of the synchronous generators, while additional network and load data are provided in Appendix A for completeness.

Generator	Bus Number	MVA rating	Inertia H [s] on SG rating
G_1	1	247.5	4.775
G_2	2	192.0	1.770
G_3	3	128.0	1.175

Table 3.2: IEEE 9-bus generator data

3.6.2 Modified IEEE 39-Bus Test System

All optimization and robustness-validation studies were conducted on the modified IEEE 39-bus (New England) test system shown in Figure 3.5. The model represents a simplified transmission-network equivalent of the New England region [106] and serves as the primary platform for inertia-threshold computation, optimization analysis, and probabilistic validation.

The large MVA rating is introduced to ensure that the machine can represent the required kinetic energy without artificially constraining the inertia representation through its rating. Since the kinetic energy stored in a synchronous machine is proportional to the product HS_{rated} , a large base rating allows the inertia constant to reflect the desired inertial contribution without affecting system power balances.

The system consists of 39 buses, 46 transmission lines, 12 transformers, 19 aggregated loads, and 10 synchronous generators G_1 – G_{10} connected at buses 30–39. Generator G_1 represents an equivalent external grid with rated apparent power set to 10,000 MVA following the PowerFactory benchmark model [67, 107]; however, its active-power limits are restricted to avoid dominance in steady-state power-flow solutions. The large MVA rating is introduced to ensure that the machine can represent the large kinetic energy of the external grid representation through its rating.

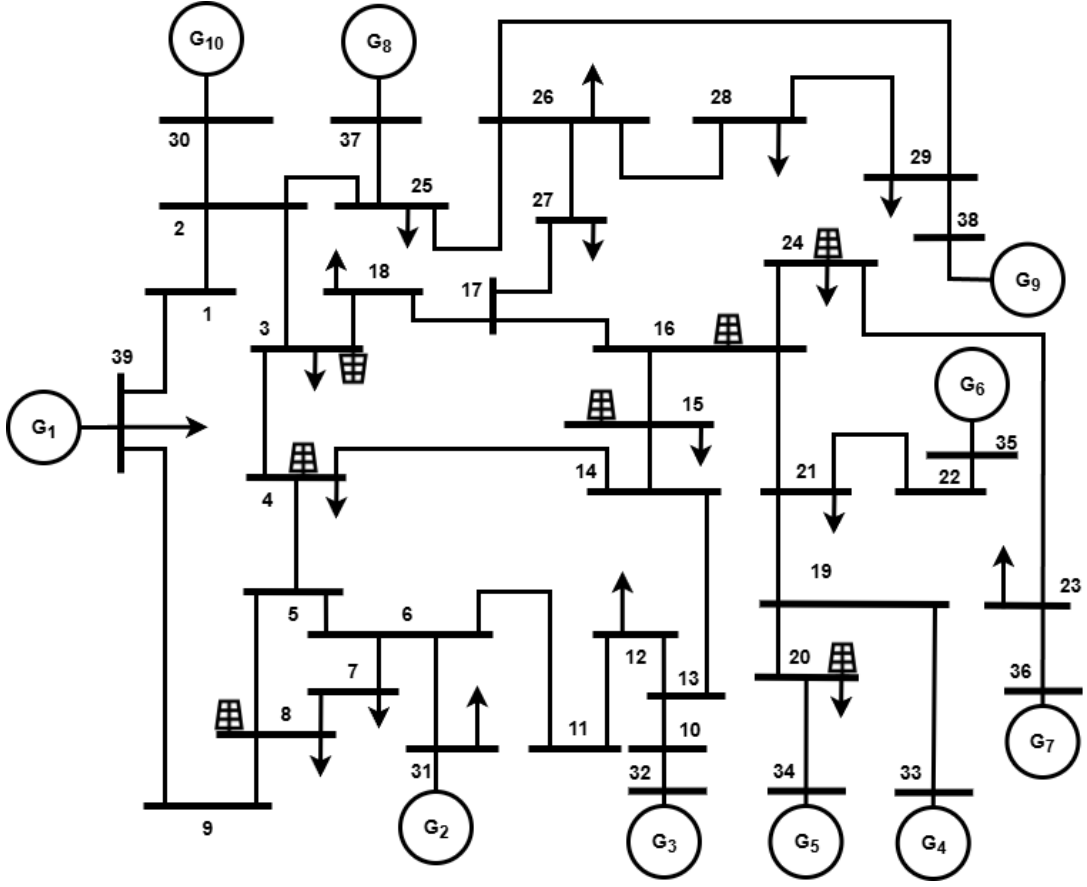


Figure 3.5: Modified IEEE 39-bus test system.

To ensure steady-state feasibility across candidate operating points, bus voltage magnitudes are constrained as

$$0.95 \text{ p.u.} \leq V_i \leq 1.05 \text{ p.u.}, \quad \forall i \in \mathcal{B}. \quad (3.79)$$

Table 3.3 summarizes the synchronous generator parameters used in this study, including inertia constants and active-power limits. Generator inertia constants are expressed on machine bases and converted to the system base when required during inertia estimation. Minimum active-power limits are set to 20% of the corresponding maximum outputs.

Utility-scale photovoltaic (PV) generation is incorporated in the modified IEEE 39-bus system to represent inertia-less variable renewable energy sources (VRES). The locations and capacities of the PV plants are determined using the optimal

Table 3.3: Synchronous generator data for the modified IEEE 39-bus system

Bus	ID	S_{rated} (MVA)	H_g (s)	P_{min} (MW)	P_{max} (MW)
39	G1	10000	5.00	220.0	1100.0
31	G2	700	4.33	129.2	646.0
32	G3	800	4.48	145.0	725.0
33	G4	800	3.58	130.4	652.0
34	G5	600	4.33	101.6	508.0
35	G6	800	4.35	137.4	687.0
36	G7	700	3.77	116.0	580.0
37	G8	700	3.47	112.8	564.0
38	G9	1000	3.45	173.0	865.0
30	G10	1200	4.20	208.0	1040.0

siting and sizing methodology proposed in [108].

The adopted method combines loss sensitivity analysis with an optimization procedure to identify buses where PV injections most effectively reduce network active-power losses. First, loss sensitivity factors (LSFs) are computed for all buses using the base-case AC power-flow solution. These indices quantify the sensitivity of total network losses to incremental active-power injections at each bus and are therefore used to identify candidate buses for PV installation. Buses exhibiting higher sensitivity values are considered more effective locations for PV integration.

After selecting candidate buses based on the LSF ranking, an optimization procedure is applied to determine the appropriate PV capacities. The optimization objective is the minimization of total network active-power losses subject to standard power-system operating constraints, including AC power-flow equations, bus voltage limits, and generation capacity limits. During the optimization process, PV units are represented as active-power injections connected at the selected buses, and the power-flow solution is updated iteratively while the algorithm adjusts the PV capacities to achieve minimum-loss operation.

The resulting PV-integrated configuration is used to construct a renewable-rich version of the IEEE 39-bus test system, which serves as the base network for the

proposed optimization framework presented in Chapter 4.

3.7 Inertia-constrained optimal VRES generation dispatch validation

Using the benchmark systems defined in Section 3.6, this section defines the validation methodology adopted to verify the inertia-aware optimization framework. Complementary validation layers are used to establish (i) analytical consistency and (ii) probabilistic robustness under time-varying operating conditions.

Validation is conducted at two complementary levels:

- (i) Stochastic Validation of Frequency Security Under Fixed PV Penetration.
- (ii) Analytical consistency with a kinetic-energy (KE) based frequency security benchmark.

3.7.1 Stochastic Validation of Frequency Security Under Fixed PV Penetration

The deterministic analysis presented in Section 3.5.7.6 demonstrates the operational behavior of the proposed inertia-aware scheduling framework under time-varying load and solar PV conditions across representative operating hours. However, real power system operation is inherently stochastic due to the variability of renewable generation and load demand. Consequently, a stochastic validation procedure is conducted to assess whether the proposed spatial inertia-aware optimization remains consistent with desirable frequency stability outcomes under uncertain operating conditions.

The validation specifically addresses the following question:

Under identical stochastic operating conditions and equal PV penetration

levels, does the proposed spatial inertia-aware optimization produce operating points that are more strongly aligned with acceptable frequency stability indices, particularly RoCoF and frequency nadir, than the benchmark methodology?

The PV penetration levels are fixed during the validation process such that, both the proposed framework and the benchmark methodology are required to accommodate the same PV injection under each stochastic operating scenario. The resulting operating points are subsequently evaluated using time-domain dynamic simulation.

3.7.1.1 Stochastic Operating Scenario Generation

The stochastic operating scenarios are constructed using the same normalized load and PV availability datasets introduced in Section 3.5.7.6. The hourly scaling factors $\alpha_{\text{load}}(t)$ and $\gamma_{\text{PV}}(t)$ are mapped to the IEEE 39-bus system using equations (3.76)–(3.77). These equations define the scaled bus demands $P_{D,i}(t)$, $Q_{D,i}(t)$, and the available PV generation $P_{\text{PV-available},j}(t)$, while preserving the spatial distribution of load and generation in the network.

To capture the stochastic variability of the operating conditions, a Monte Carlo sampling procedure is applied to the joint distribution of PV availability and load demand derived from the historical dataset. For each hour of the day, a set of stochastic realizations is generated in terms of the normalized factors

$$\alpha_{\text{load}}^{(s)}(t), \quad \gamma_{\text{PV}}^{(s)}(t), \quad (3.80)$$

where s denotes the Monte Carlo sample index. Each realization defines a unique operating condition of the system in terms of bus loads and available PV generation.

3.7.1.2 PV Penetration Conditioning

To ensure a consistent comparison between the proposed and benchmark approaches, the total PV penetration level is prescribed for each Monte Carlo scenario. Let

$$P_D^{(s)} = \sum_i P_{D,i}^{(s)} \quad (3.81)$$

denote the total system demand under scenario s . The total PV injection accommodated in the optimization is constrained as

$$\sum_j P_{PV,j}^{(s)} = \rho_{PV} P_D^{(s)}, \quad (3.82)$$

where ρ_{PV} is the specified PV penetration level.

Both the proposed framework and the benchmark methodology are therefore evaluated under identical renewable uptake and stochastic operating conditions.

3.7.1.3 Dispatch Optimization Under Competing Frameworks

For each Monte Carlo realization, two independent dispatch optimizations are performed.

Proposed Framework The proposed inertia-aware scheduling framework described in Section 3.4 is solved using NSGA-II. The optimization enforces the spatial inertia constraints derived from the minimum bus-level inertia thresholds presented in Section 3.3. The resulting dispatch solution determines synchronous generator commitments, generator output levels, and spatial inertia distribution across buses.

Benchmark Methodology Under the same operating scenario and fixed PV penetration, the benchmark approach based on the methodology of Saleem et al. [109]

is applied to independently determine the inertia support required to maintain frequency reliability. The benchmark relies on aggregated inertia representation and frequency stability indices derived from the frequency response model described in [109]. The resulting dispatch therefore reflects the inertia requirements implied by the benchmark methodology.

3.7.1.4 Disturbance Definition

To evaluate the dynamic frequency response of the resulting operating points, two complementary load change disturbance scenarios are considered for each Monte Carlo sample.

Load Disconnection Event The first disturbance corresponds to the sudden disconnection of the largest load in the system. This event produces a rapid generation surplus and results in a positive frequency excursion. The disturbance magnitude is defined as

$$\Delta P_c = -P_{D,c}, \quad (3.83)$$

where $P_{D,c}$ is the active demand of the largest load bus under the given operating scenario.

Equivalent Load Increase Event The second disturbance represents a sudden increase in system demand with magnitude equal to that of the largest load disconnection event. This disturbance produces a generation deficit and leads to a negative frequency excursion. The disturbance magnitude is therefore

$$\Delta P_c = +P_{D,c}. \quad (3.84)$$

Considering both disturbances enables evaluation of frequency dynamics under both deficit and surplus power imbalance conditions while maintaining a consistent

disturbance magnitude across scenarios.

3.7.1.5 Dynamic Frequency Response Evaluation

The optimized operating points obtained from both frameworks are evaluated through time-domain simulation using the disturbance scenarios defined above. For each simulation, RoCoF and the frequency nadir indices are extracted.

The RoCoF limit used throughout the study is 1 Hz/s, consistent with the constraint applied in Section 3.5.7.6. The frequency nadir is computed as the minimum value reached by the system frequency trajectory following the disturbance.

3.7.1.6 Statistical Comparison of Frameworks

The above procedure is repeated for all Monte Carlo operating scenarios. For each scenario, paired observations of RoCoF and frequency nadir are obtained for both the proposed and benchmark frameworks. The resulting distributions are then compared using statistical metrics including, the mean RoCoF magnitude, standard deviation of the RoCoF, mean frequency nadir, and standard deviation of the frequency nadir.

Quantitative benchmarking metrics. To express the stochastic validation results using standardized quantitative indicators, the comparison between the proposed framework and the benchmark method is performed using mean RoCoF magnitude, RoCoF standard deviation, mean frequency nadir, nadir standard deviation, percentage RoCoF improvement, nadir improvement, and violation count.

The percentage improvement in mean RoCoF magnitude is computed as

$$\text{RoCoF Improvement}(\%) = \frac{|\text{RoCoF}_{\text{bench}}| - |\text{RoCoF}_{\text{prop}}|}{|\text{RoCoF}_{\text{bench}}|} \times 100, \quad (3.85)$$

where $\text{RoCoF}_{\text{bench}}$ and $\text{RoCoF}_{\text{prop}}$ denote the mean RoCoF magnitudes obtained from the benchmark and proposed frameworks, respectively.

The reduction in variability of a frequency-security index is evaluated as

$$\text{Std. Reduction}(\%) = \frac{\sigma_{\text{bench}} - \sigma_{\text{prop}}}{\sigma_{\text{bench}}} \times 100, \quad (3.86)$$

where σ_{bench} and σ_{prop} are the corresponding standard deviations.

For the frequency nadir, the improvement is expressed both as an absolute increase in the minimum frequency and as a reduction in the frequency drop from the nominal frequency:

$$\text{Nadir Drop Reduction}(\%) = \frac{(f_0 - f_{\text{nadir,bench}}) - (f_0 - f_{\text{nadir,prop}})}{f_0 - f_{\text{nadir,bench}}} \times 100. \quad (3.87)$$

The stochastic security violation count is evaluated from the individual Monte Carlo samples as

$$N_{\text{viol}} = \sum_{s=1}^{N_s} \mathbb{I} \left(|\text{RoCoF}^{(s)}| > 1 \text{ Hz/s} \right), \quad (3.88)$$

where N_s is the total number of Monte Carlo samples and $\mathbb{I}(\cdot)$ is an indicator function equal to one when the RoCoF limit is violated and zero otherwise. The corresponding violation rate is computed as

$$\text{Violation Rate}(\%) = \frac{N_{\text{viol}}}{N_s} \times 100. \quad (3.89)$$

These metrics are used in Chapter 4 to quantify the improvement of the proposed spatial inertia-aware optimization relative to the benchmark methodology, particularly in terms of whether the resulting dispatch solutions remain more strongly bounded by acceptable frequency stability limits under identical stochastic

operating conditions and fixed PV penetration levels.

3.7.2 Analytical Consistency with Kinetic-Energy-Based Methods

To evaluate the analytical validity of the proposed spatial inertia framework, the resulting frequency-security indices are compared against the kinetic-energy-based frequency assessment methodology proposed by Rakhshani *et al.* [110]. This approach provides an analytical mapping between the available system kinetic energy and the expected frequency stability indices following a disturbance. The comparison therefore serves as an independent benchmark for assessing whether the inertia allocation produced by the proposed optimization framework yields frequency-security estimates consistent with established kinetic-energy-based theory.

The validation procedure is implemented through the following steps.

Step 1: Selection of optimized operating point.

The NSGA-II optimization framework described in Sections 3.4–3.5 produces a set of Pareto-optimal inertia allocations. From this set, a representative operating point is selected using the curvature-based selection criterion described in Section 3.5.7. The selected operating point defines the spatial distribution of bus inertias $\{h_j\}_{j \in \mathcal{B}}$ and the corresponding generation dispatch configuration.

Step 2: Computation of equivalent bus kinetic energy.

For the selected operating point, the equivalent kinetic energy associated with each bus is computed from the optimized bus inertia values. The equivalent kinetic energy at bus i is expressed as

$$KE_i = h_i S_B \quad (3.90)$$

where KE_i denotes the equivalent kinetic energy at bus i , h_i represents the

optimized inertia constant assigned to that bus, and S_B is the system base power. This quantity represents the rotational energy stored in the synchronous machines that is electrically reflected at bus i under the selected dispatch configuration.

Step 3: Disturbance specification and reflected disturbance mapping.

A disturbance scenario is defined according to the procedure described in Section 3.3. The disturbance magnitude and location are first specified in terms of an active power imbalance. Using the equation (3.40), the disturbance is then mapped to each bus in the system to obtain the corresponding reflected disturbance magnitude ΔP_i .

For each bus i , the pair $(KE_i, \Delta P_i)$ therefore characterizes the local inertia level and the effective disturbance magnitude experienced at that bus.

Step 4: KE-based estimation of frequency-stability indices.

Using the analytical framework of [110], the RoCoF and frequency nadir associated with each $(KE_i, \Delta P_i)$ pair are obtained from the corresponding lookup curves. These curves establish the relationship between available kinetic energy and disturbance magnitude under the assumption of negligible load damping. The resulting estimates provide analytical predictions of the bus-level RoCoF and frequency nadir.

Step 5: Dynamic simulation using the optimized operating point.

To obtain the corresponding dynamic responses, time-domain simulations are performed for the same operating point. For each bus i , the reflected disturbance ΔP_i is applied and the resulting frequency trajectory $f_i(t)$ is recorded. This procedure is repeated sequentially for all buses in the system.

The simulation horizon is 15 s with a fixed time step of 1 ms. Load damping is set to zero so that the response is dominated by inertial dynamics, ensuring consistency with the assumptions underlying the KE-based analytical framework.

From the simulated frequency trajectories, the RoCoF is computed from the initial slope of the frequency response:

$$\text{RoCoF}_{\text{dyn}} = \left. \frac{df(t)}{dt} \right|_{t \in [0, 0.2]}. \quad (3.91)$$

The frequency nadir is defined as

$$f_{\text{nadir}} = \min_{t \in [0, 15]} f(t). \quad (3.92)$$

The resulting data set therefore contains, for each bus, the reflected disturbance magnitude, the optimized bus inertia, the KE-based estimates of RoCoF and nadir, and the corresponding indices obtained from time-domain simulation.

Step 6: Statistical consistency assessment.

Finally, the KE-based analytical estimates are compared with the dynamic simulation results across all buses. For each frequency-security index, a least-squares regression model is fitted:

$$y_{\text{dyn}} = a y_{\text{KE}} + b + \varepsilon \quad (3.93)$$

where y denotes either RoCoF or frequency nadir, a and b are regression parameters, and ε represents the residual error.

3.8 Chapter Summary

This chapter defined the methodological framework used to achieve the research objectives. First, power-system dynamic and network modeling foundations were established to support spatial inertia representation. Second, an offline analytical method was developed to estimate bus-level inertia distribution using Kron-reduced electrical coupling and an electrical-distance formulation. Third, RoCoF-based minimum inertia thresholds were derived as spatial security constraint under a credible disturbance. Fourth, these thresholds were embedded into a multi-objective

inertia-aware scheduling formulation solved using NSGA-II, with physics-based fitness evaluation and constraint handling. Finally, a validation strategy was defined to verify the offline inertia estimates and to assess optimization consistency and robustness under time-varying and probabilistic operating conditions. The outcomes of these procedures are presented and discussed in [Chapter 4](#).

CHAPTER 4

RESULTS AND DISCUSSION

This chapter presents the results obtained from the methodological framework developed in Chapter 3 and evaluates the proposed approaches with respect to the research objectives. The results are presented as follows: first, the offline spatial inertia estimation method is validated using the IEEE 9-bus test system to provide physical interpretation and analytical verification of the electrical-distance formulation; the method is then extended to the IEEE 39-bus system, where minimum bus-level inertia thresholds are derived using the RoCoF-based framework and subsequently incorporated into the NSGA-II optimization model for inertia-aware scheduling of inertia-less VRES. Finally, dynamic simulations, analytical benchmarking, and probabilistic studies are used to validate the optimization framework under different operating conditions. Accordingly, the IEEE 9-bus system is employed primarily for methodological validation, while the IEEE 39-bus system serves as the main test system for bus inertia threshold determination, optimization analysis, and robustness assessment.

4.1 Offline Inertia Estimation

This section, presents the results for the first specific objective. The IEEE 9-bus test system is primarily employed to test the formulated method for estimating spatial inertia distribution. In this context, the section presents how the electrical-distances formulation replicates the generator–bus influence on inertia distribution, how inertia redistributes under load variations, and how generator inertia contributes to bus inertia. Later, the IEEE 39 bus test system is used for testing the scalability

of the proposed inertia estimation technique.

4.1.1 Base-Case Inertia Distribution: IEEE 9-Bus System

After running the steady-state AC load-flow simulation for the IEEE 9-bus system according to the procedure described in Section 3.2.8.1, the active-power flow directions on each transmission line were extracted and used to construct a directed graphical representation of generator–bus connectivity. Figure 4.1 illustrates the directed graph representation of the IEEE 9-bus network, where branch directions are assigned according to the active power flows obtained from the load-flow solution. The solved load flow reveals a clear separation between radially supplied buses and buses located in the meshed interior of the network.

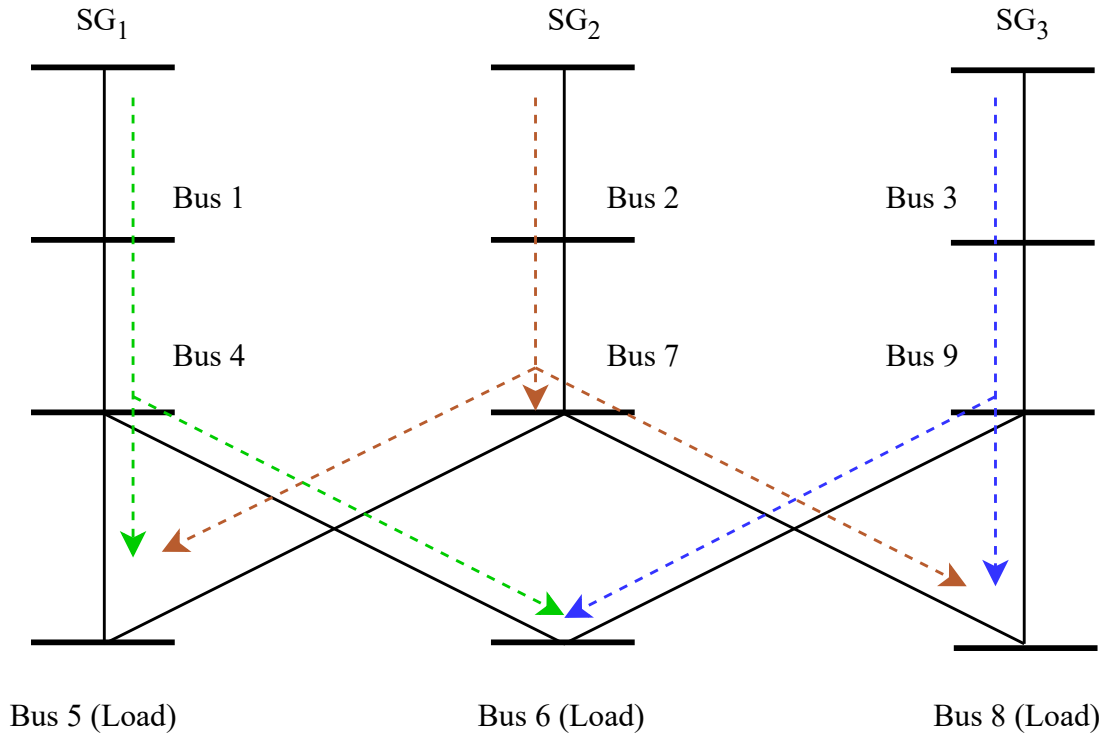


Figure 4.1: Graphical representation of the line power flow direction (IEEE 9—Bus system)

Buses 4, 7, and 9 lie along simple radial branches supplied exclusively by generators SG_1 , SG_2 , and SG_3 , respectively. Since no other generator delivers real power

to these buses, their electrical distances to all non-supplying generators must be zero. By contrast, buses 5, 6, and 8 are positioned in the meshed region where their power injections are shared between two different generators each. Bus 5 receives power jointly from SG_1 and SG_2 , bus 6 from SG_1 and SG_3 , and bus 8 from SG_2 and SG_3 . This suggests that inertia at bus 5, 6, and 8 is contributed by pairs of SGs and their frequency dynamic behavior will reflect blended contributions from respective SGs pair.

Following the identification of generator–bus connectivity, the network is reduced through Kron reduction using the formulation in equation (3.7). The reduced admittance matrix $\mathbf{Y}_{\text{red}}$ is used for computing the normalized current contributions and synchronizing-power coefficients defined in section 3.2, and these in turn determine the electrical distances d_{kj} via equation (3.22). The computed electrical distances for the IEEE 9-bus system are presented in Table 4.1.

Table 4.1: Computed electrical distances

Generation Unit	Bus 4	Bus 5	Bus 6	Bus 7	Bus 8	Bus 9
SG_1	1.000	0.649	0.482	0	0	0
SG_2	0	0.351	0	1.000	0.424	0
SG_3	0	0	0.518	0	0.576	1.000

The numerical distances in Table 4.1 reflect the quantitative connectivity inferred from the power-flow directions. For the radially supplied buses 4, 7, and 9, exactly one electrical distance is equal to 1, indicating highest electrical proximity to their respective upstream generator, while all other distances vanish. This indicates that these buses are dynamically dominated by a single generator. For the meshed buses 5, 6, and 8, two nonzero electrical distances appear in each row, corresponding to the two generators supplying real power to each bus. This distribution demonstrates how buses in meshed regions naturally acquire blended inertia contributions from multiple SGs.

Using the electrical distances in Table 4.1, the bus-level inertia values are computed using equation (3.24). The resulting inertia distribution is illustrated in Figure 4.2,

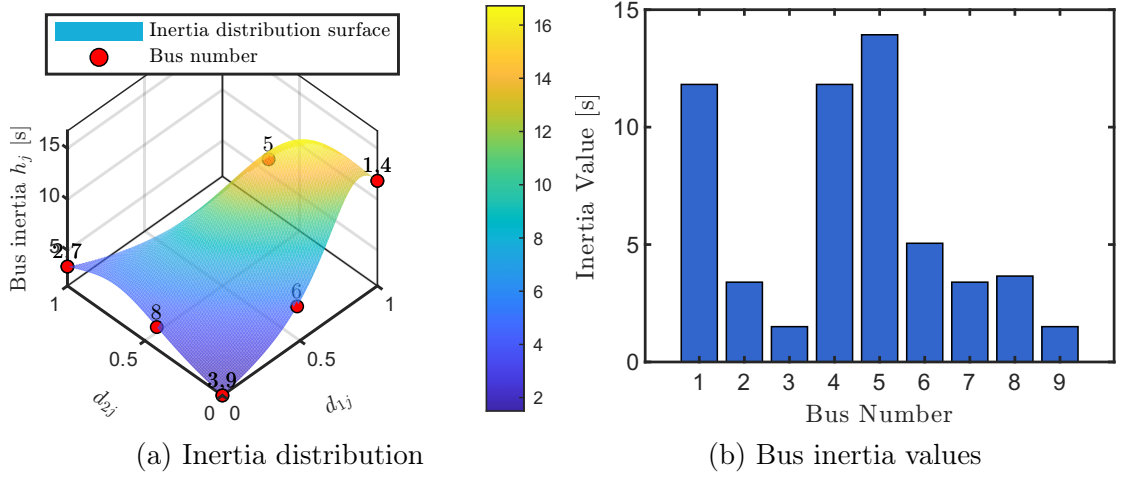


Figure 4.2: Inertia distribution and bus inertia values for the IEEE 9-bus system.

where Figure 4.2a presents the spatial inertia surface and Figure 4.2b shows the corresponding bus-level inertia values.

The generator terminal buses reproduce the exact inertia of their respective generators, which can be directly observed in Figure 4.2b, where the inertia values at buses 1, 2, and 3 coincide with the inertia constants of SG_1 , SG_2 , and SG_3 , respectively. This occurs because each terminal bus has an electrical distance of unity to its own generator and zero to all others.

A similar behavior is observed for the radial buses 4, 7, and 9. As shown in Figure 4.2b, the inertia values at these buses match those of their upstream generators. Bus 4 inherits the inertia of SG_1 , bus 7 inherits that of SG_2 , and bus 9 inherits that of SG_3 . This arises because only one electrical distance is nonzero for each of these buses. The same pattern is also visible in the inertia surface of Figure 4.2a, where the inertia values along the radial directions extending from the generators remain constant. Dynamically, these buses therefore behave as direct extensions of their respective generators.

In contrast, the meshed buses 5, 6, and 8 exhibit inertia values that reflect the simultaneous influence of two nonzero electrical distances. These contributions merge through the harmonic-mean structure of the bus-level inertia expression (equation (3.24)). For instance, bus 5 blends contributions from SG_1 and SG_2 ,

bus 6 blends contributions from SG_1 and SG_3 , and bus 8 blends contributions from SG_2 and SG_3 . This suggests that, meshed buses do not follow the inertial response of a single generator; instead, their local frequency response evolves as a weighted combination of the inertial dynamics of multiple generators.

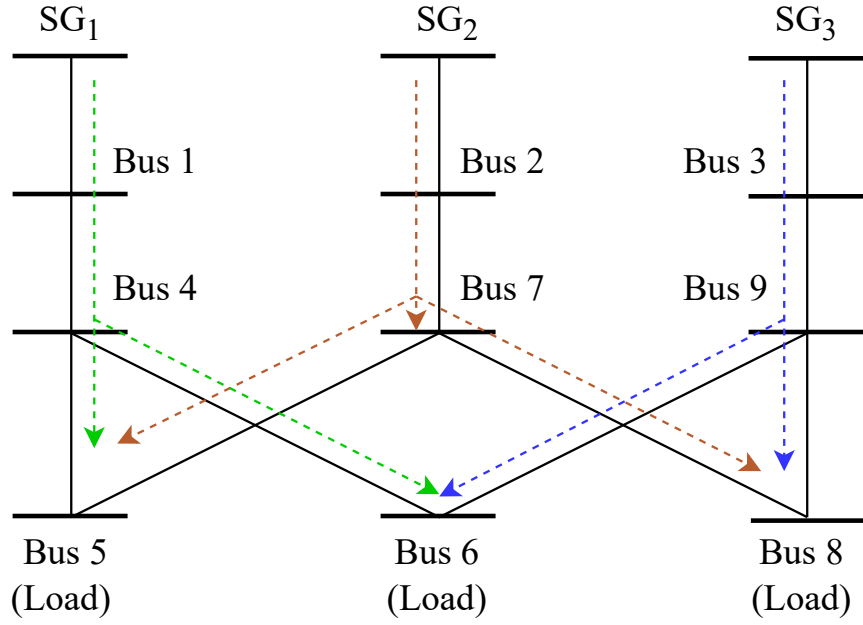
Overall, the patterns observed in Table 4.1 and Figure 4.2 highlight the inherently spatial and non-uniform nature of inertia distribution in power networks. The inertia distribution is governed not only by generator placement, but also by electrical proximity, topology, and direction of real-power flow.

4.1.2 Effect of Load Variations on Electrical Distances and Bus-Level Inertia

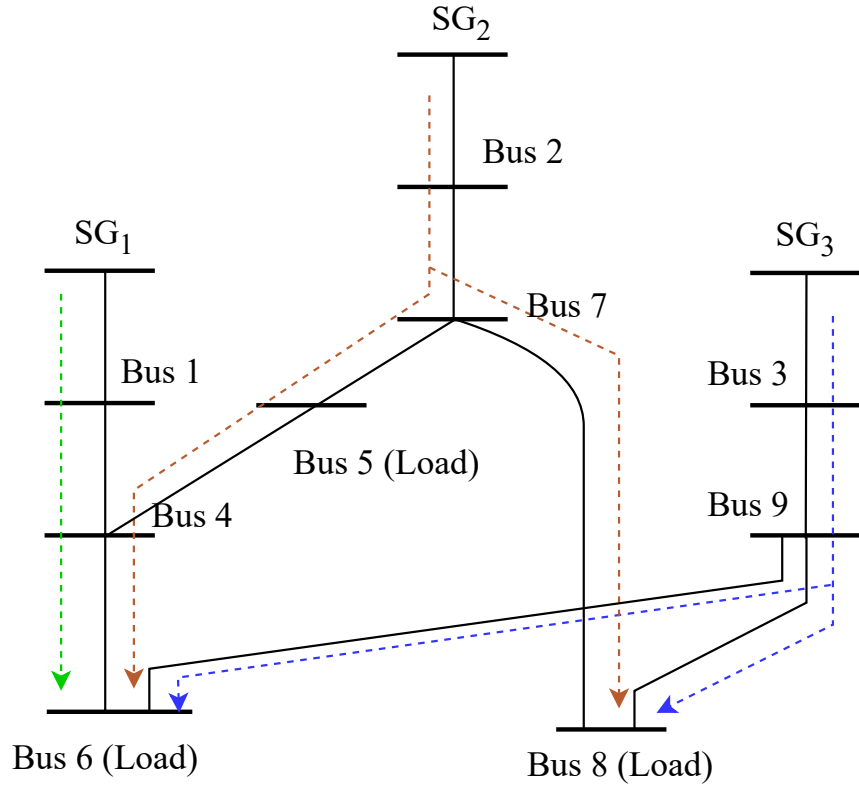
This subsection examines how variations in load at Bus 5 influence electrical distances and the resulting bus-level inertia distribution. The analysis considers two operating regimes identified through the parametric load-variation procedure described in Subsection 3.2.8.2: (i) small load reductions that preserve the base-case power-flow pattern, and (ii) larger reductions that induce branch flow reversals.

Figures 4.3a and 4.3b present the directed graphs modeled to represent power flow directions for a small and large load reduction at Bus 5 respectively. A small load reduction ($0 < \Delta P_{D5} \leq 0.46$ p.u.) does not alter the direction of the line flows relative to the base case.

Figure 4.3b illustrates the line flow pattern resulted due to larger load reduction at Bus 5 ($0.46 < \Delta P_{D5} \leq 1.25$ p.u.). Once the flow direction changes, the set of generators that are reachable from a particular bus—defined as generators connected to that bus through a directed path that follows the orientation of real-power flows in the network—is reconfigured. Consequently, a generator that previously contributed to that bus may lose its directed path entirely, while the electrical distance associated with the remaining reachable generator increases accordingly.



(a) $0 < \Delta P_{D5} \leq 0.46$ p.u.



(b) $0.46 \text{ p.u.} \leq \Delta P_{D5} \leq 1.25$ p.u.

Figure 4.3: Directed graph representation of the IEEE 9 bus test system for different load-reduction levels at bus 5.

The base-case map in Figure 4.4a reproduces the electrical-distance structure reported in Table 4.1. This behavior can be directly observed from the color distribution across the generator columns. Radial buses (B4, B7, and B9) exhibit a single dominant bright cell (value close to 1) associated with one generator while the remaining entries in the row are close to zero, indicating that their inertia contribution originates almost entirely from a single upstream generator. In contrast, buses located in the meshed portion of the network (B5, B6, and B8) display multiple intermediate color intensities across generator columns, indicating that their equivalent inertia is composed of contributions from more than one generator. This visual pattern confirms that electrical coupling in the meshed region results in blended generator influence, whereas radial buses inherit inertia predominantly from a single electrically connected generator. Under a small load reduction at bus 5 (Case I), the heatmap in Figure 4.4b remains effectively identical to the base case, since no generator–bus paths changed.

A large load reduction (Case II) produces a qualitatively different response due to power-flow reversals in some branches. Specifically, the branch $4 \rightarrow 5$ reverses to $5 \rightarrow 4$, generator SG_1 loses reachability to bus 5, generator SG_2 gains a new directed path to bus 6 via $2 \rightarrow 7 \rightarrow 5 \rightarrow 4 \rightarrow 6$, and the reachability set at bus 5 collapses from $\{SG_1, SG_2\}$ to $\{SG_2\}$. Generator SG_3 remains unaffected. These directed-connectivity changes induce structural modifications in the admissible generator–bus paths, therefore, producing localized changes in the electrical distances. As shown in Figure 4.4c, only the buses whose reachability changes—buses 4, 5, and 6—exhibit significant deviations from the base case, and the difference map in Figure 4.4d confirms this behavior.

The impact of these connectivity changes on the bus-level inertia is shown in Figure 4.5, where the yellow bars corresponding to the large-load-reduction case exhibit pronounced departures from the base-case (blue) and negligible variation (red) profiles. The inertia deviations align exactly with the buses whose electrical distances were structurally modified. Buses 4, 5, and 6 display appreciable changes. The most significant effect occurs at bus 5, whose inertia collapses from 13.91 s

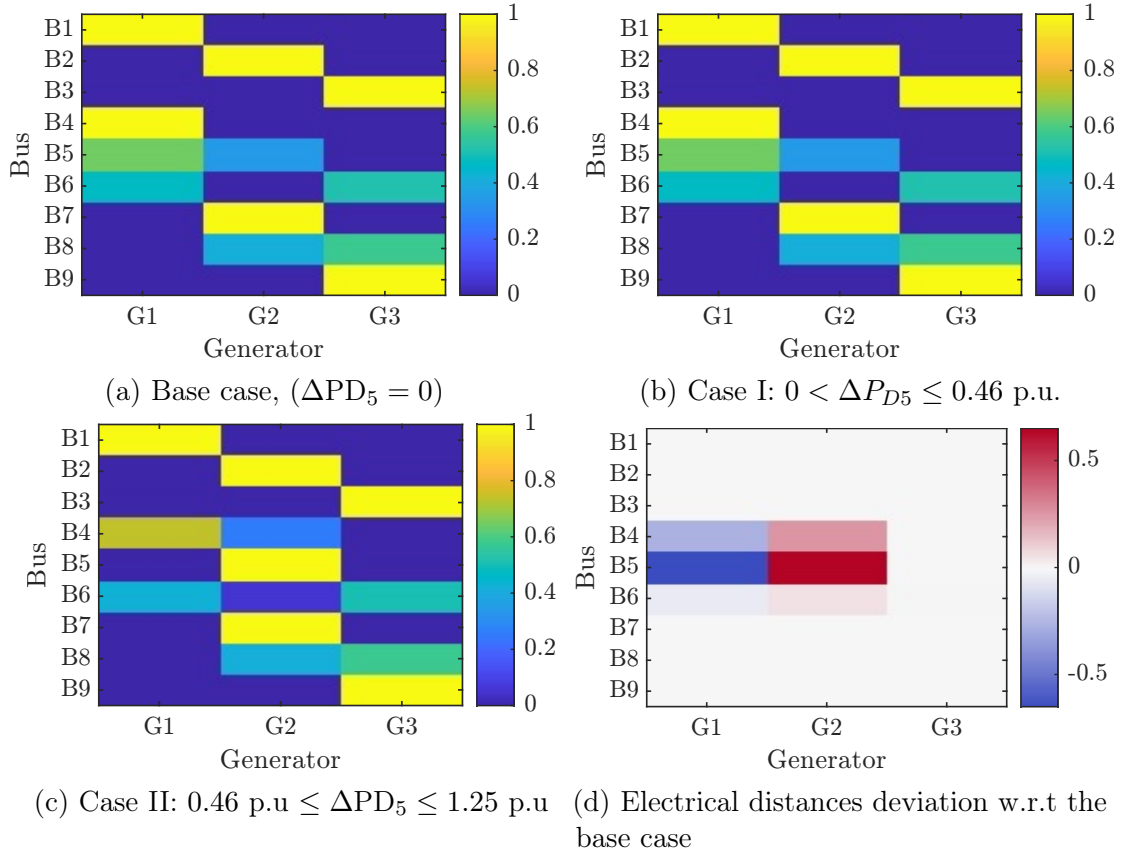


Figure 4.4: Electrical distances computed under varying load at bus 5.

to 3.398 s once SG_1 loses reachability and the bus becomes electrically supported solely by SG_2 . Bus 4, conversely, experiences an increase in inertia from 11.818 s to 15.11 s, reflecting its new status as the electrically closest point to the remaining contributing generator. Bus 6 shows a smaller increase, from 5.05 s to 5.27 s, consistent with SG_2 gaining an additional directed path via $2 \rightarrow 7 \rightarrow 5 \rightarrow 4 \rightarrow 6$. These changes confirm that inertia redistribution is driven by the change in generator reachability, and SG inertia magnitude. The spatial manifestation of this behavior is evident in the fact that, the center of inertia shifts from its base-case position near bus 5 toward bus 4 following the power flow reversal. This shift underscores that the spatial inertia landscape is highly sensitive to directed power-flow topology.

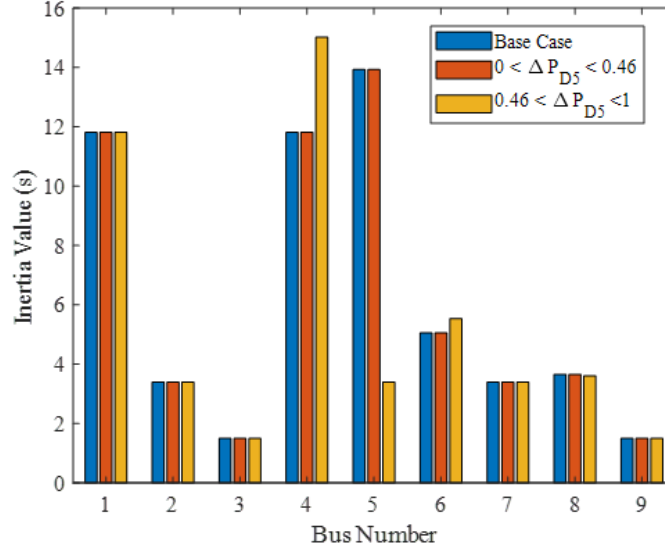


Figure 4.5: Comparison of bus inertia for the base case and two Bus-5 load-reduction scenarios.

4.1.3 Sensitivity of the Spatial Inertia Distribution to Generator Inertia Constants

This subsection examines how variations in individual generator inertia constants affect the spatial distribution of bus-level inertia. The analysis builds on the inertia distribution dependence on the electrical-distance introduced in section 3.2, and evaluates how inertia perturbations propagate across the network.

To perform this analysis, the inertia constant of each SG was independently scaled by a factor $\alpha \in \{0.50, 0.75, 1.00, 1.25, 1.50\}$, while all other generators were kept at their nominal values. For each scaling, the relative change in the estimated bus-level inertia was evaluated.

Sensitivity to SG_1 Inertia Scaling

Figure 4.6, illustrates how the bus-level inertia distribution varies when the inertia constant of SG_1 is scaled by a factor α . As shown in the figure, changes in SG_1 inertia influence only a limited subset of buses. The largest variation occurs at

buses 1 and 4, where the estimated inertia changes approximately proportionally with the scaling factor α . A smaller but still observable change is seen at bus 5, reflecting its intermediate electrical coupling to SG_1 . In contrast, the variation at Bus 6 is very small across the entire scaling range, indicating only weak sensitivity to SG_1 inertia. Buses 2–3 and 7–9 remain essentially unchanged for all scaling factors, confirming that their inertia estimates are not influenced by variations in SG_1 .

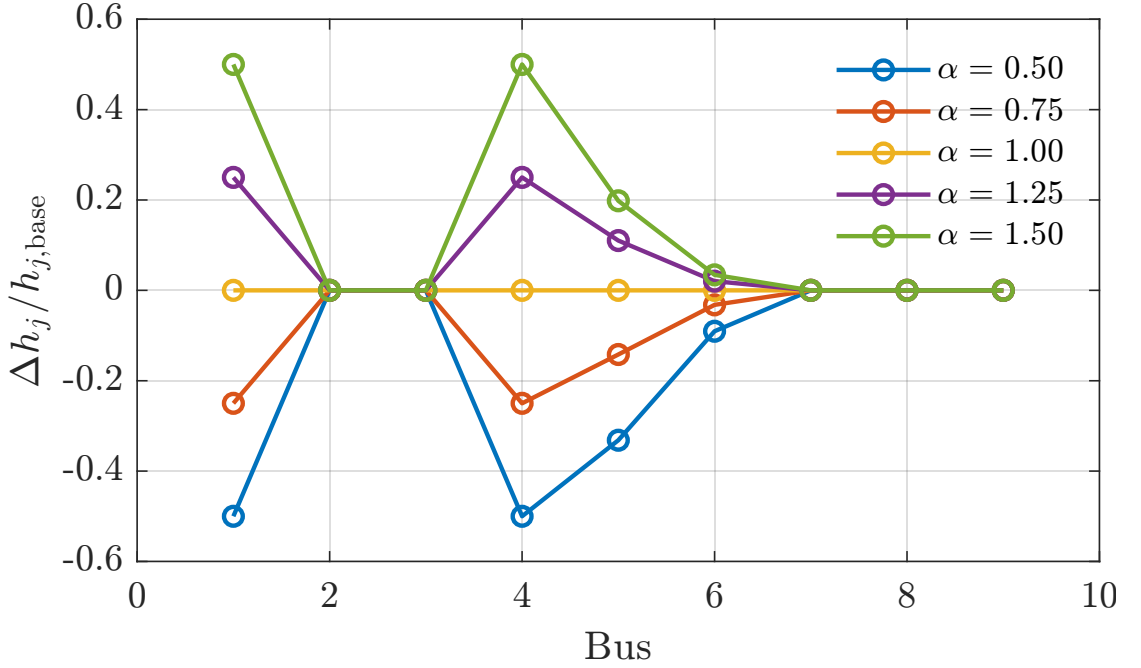


Figure 4.6: Bus-wise inertia change when only SG_1 inertia is scaled.

This behavior follows directly from the directed power-flow structure of the IEEE 9-bus system. SG_1 exports real power predominantly towards the central region (bus 5 and 6) of the network, with its two principal influence corridors given by

$$1 \rightarrow 4 \rightarrow 5 \quad \text{and} \quad 1 \rightarrow 4 \rightarrow 6.$$

Along these paths, the electrical distances d_{1j} quantify the strength of SG_1 's inertial influence at each bus. As shown in Table 4.1, buses 1 and 4 have $d_{1j} = 1$, indicating highest electrical proximity to SG_1 , whereas buses 5 and 6 have intermediate distances of 0.649 and 0.482, respectively. All other buses have $d_{1j} = 0$, reflecting

the absence of any directed real-power pathway from SG_1 .

These electrical-distance values explain the spatial pattern of sensitivity observed when SG_1 's inertia constant is scaled:

- Buses 1 and 4 exhibit identical sensitivity. Because both have $d_{1j} = 1$, the buses lie directly along SG_1 's export path and therefore inherit SG_1 's inertial influence without attenuation. Their relative inertia changes follow SG_1 's scaling exactly.
- Buses 5 and 6 exhibit attenuated—but nonzero—sensitivity. Their intermediate distances, $d_{15} = 0.649$ and $d_{16} = 0.482$, imply that SG_1 contributes inertia to these buses through downstream branches. The sensitivity is therefore weaker than at buses 1 and 4, consistent with the reduced electrical closeness.
- The sensitivity at bus 5 is greater than at bus 6. The electrical distance d_{15} is higher than d_{16} meaning bus 5 is electrically closer to SG_1 than bus 6. Consequently, changes in SG_1 inertia propagate more strongly to bus 5 than to bus 6, producing a larger relative inertia change.

These observations reveal a clear spatial ordering of SG_1 's influence:

$$d_{14} = 1 > d_{15} = 0.649 > d_{16} = 0.482 > d_{1j} = 0, \quad j \in \{2, 3, 7, 8, 9\},$$

showing that the inertia sensitivity is highly localized around SG_1 and decays rapidly along the branching power-flow paths beyond bus 6 (Figure 4.6). This one-to-one correspondence between electrical distance and observed sensitivity confirms that the electrical-distance formulation accurately captures how synchronous-generator inertia propagates through the network.

Sensitivity to SG_2 Inertia Scaling

Figure 4.7 shows the variation in bus-level inertia when the inertia constant of SG_2 is scaled over the range $\alpha \in [0.50, 1.50]$. The response is highly localized, with

observable sensitivity only at Buses 2, 5, 7, and 8, while Buses 1, 3, 4, 6, and 9 remain essentially unchanged across the full scaling range.

The strongest sensitivity is observed at Buses 2 and 7. At both buses, the normalized inertia deviation $\Delta h_j/h_{j,\text{base}}$ varies approximately from -0.50 at $\alpha = 0.50$ to $+0.50$ at $\alpha = 1.50$, with intermediate values of about -0.25 at $\alpha = 0.75$, 0 at $\alpha = 1.00$, and $+0.25$ at $\alpha = 1.25$. This indicates a proportional dependence of the equivalent bus inertia on the scaling applied to SG_2 -consistent with their electrical distances $d_{2j} = 1$, $j \in \{2, 7\}$.

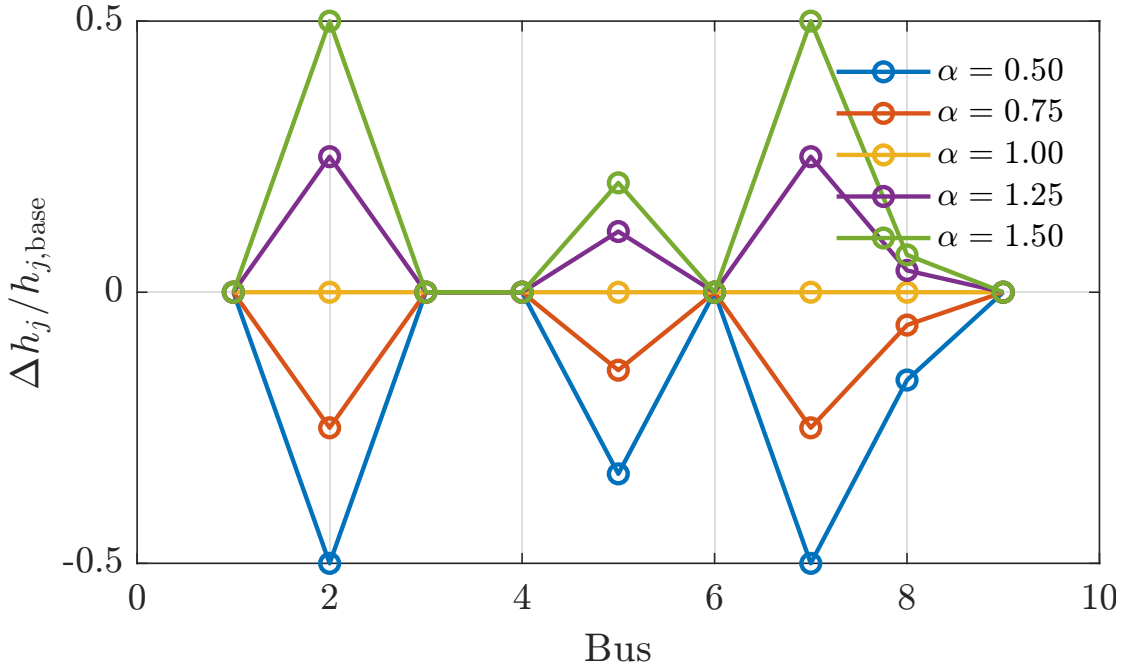


Figure 4.7: Bus-wise inertia change when only SG_2 inertia is scaled.

Bus 5 also exhibits moderate sensitivity, its normalized inertia deviation changes approximately from -0.34 at $\alpha = 0.50$ to $+0.21$ at $\alpha = 1.50$, passing through about -0.14 , 0 , and $+0.12$ at the intermediate scaling levels. This confirms that SG_2 contributes to the equivalent inertia at bus 5, although its influence is attenuated relative to Buses 2 and 7. This observation suggest that, inertia at bus 5 is jointly supported by multiple generators.

Sensitivity to SG_3 Inertia Scaling

Scaling the inertia of SG_3 produces the largest variations at Buses 3, 6, 8, and 9, as shown in Figure 4.8. These buses are electrically close to SG_3 (with $d_{33} = 1$, $d_{36} = 0.518$, $d_{38} = 0.576$, and $d_{39} = 1$) and therefore, form its principal region of influence. The strong sensitivity at these locations reflects the direct propagation of SG_3 's inertia changes along its dominant power-transfer paths.

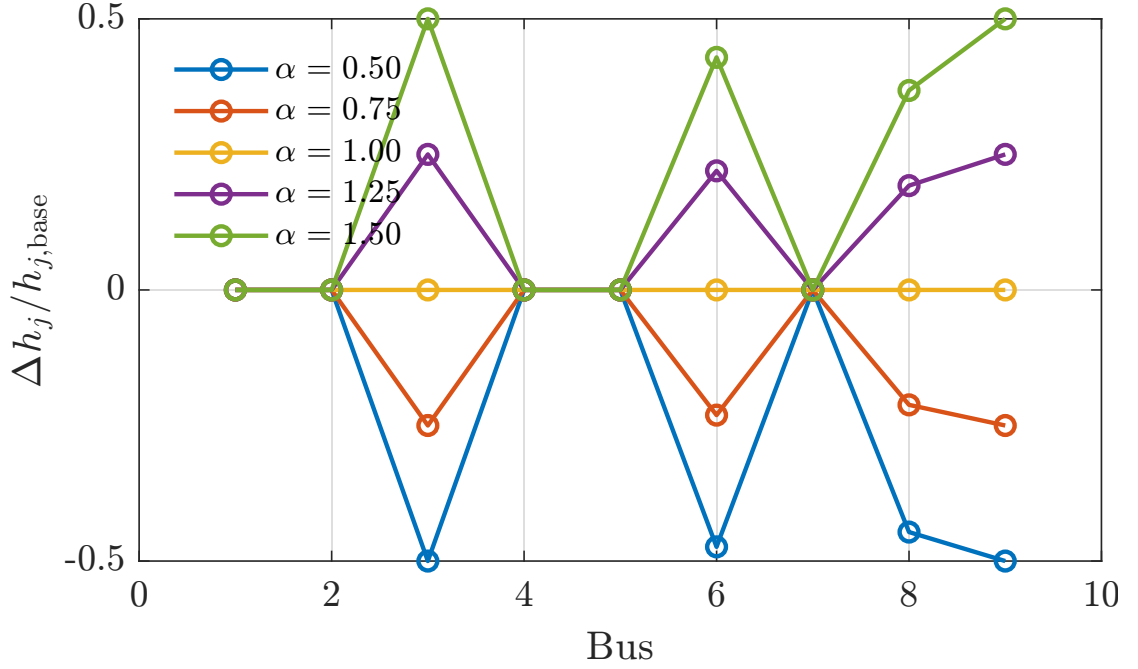


Figure 4.8: Bus-wise inertia change when only SG_3 inertia is scaled.

A secondary effect is observed at Bus 8, whose response is slightly attenuated relative to Buses 3 and 9. This is because its inertia is jointly supported by SG_2 and SG_3 . This mixed influence arises from the convergence of the SG_2 and SG_3 flow corridors at Bus 8, consistent with its intermediate electrical distances $d_{28} = 0.424$ and $d_{38} = 0.576$.

Buses 1, 2, 4, 5, and 7 exhibit negligible variation across the scaling range, indicating that they are electromechanically distant from SG_3 . Their responses confirm that generator-inertia perturbations propagate only along flow-aligned pathways permitted by the electrical distances d_{kj} , with rapid attenuation outside SG_2 's influence region.

Across the three generator-scaling experiments, inertia sensitivity remains spatially localized around the perturbed generator and decays with electrical distance, confirming that inertia scaling follows the same connectivity structure observed in the electrical-distance maps.

4.1.4 Application to a Larger Power System: IEEE 39-Bus Test System

The proposed inertia-estimation method is applied to the IEEE 39-bus test system to assess its performance in a larger, more meshed network containing ten synchronous generators.

The estimated bus-level inertia values are presented in Figure 4.9. The distribution exhibits a clear spatial structure across the network. Buses electrically close to high-inertia generators exhibit the largest inertia values, reflecting strong electromechanical coupling to dominant inertia sources. For example, Buses 9 and 1 show comparatively high inertia, consistent with their close electrical proximity to highest inertia generator SG_1 .

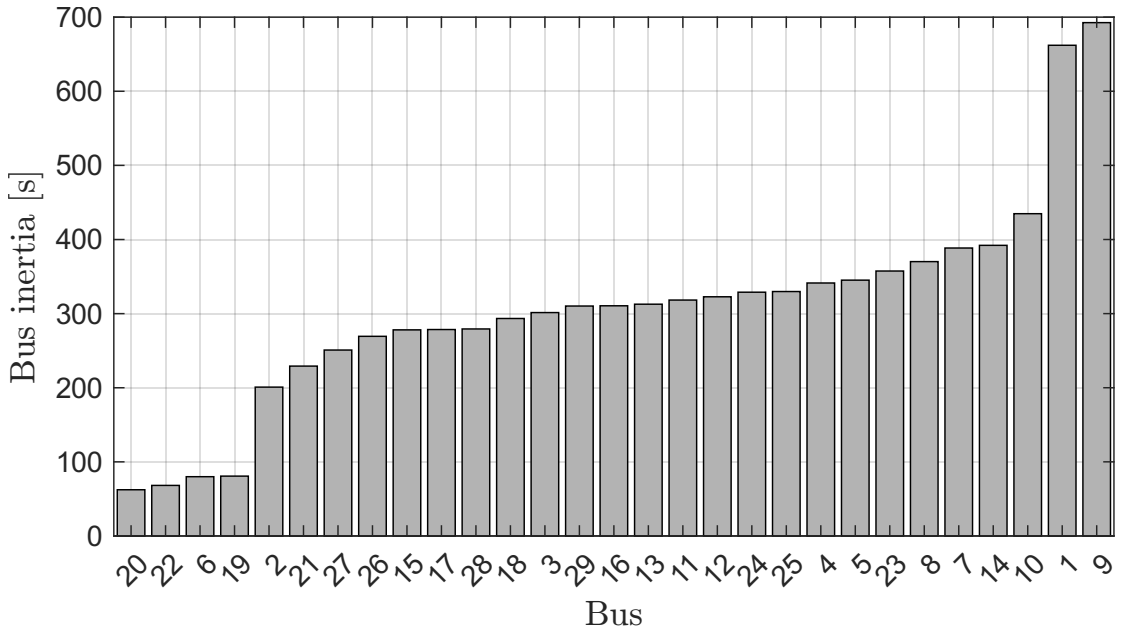


Figure 4.9: Estimated bus-level inertia for the IEEE 39-bus test system.

In contrast, buses located near lower-inertia generators, or electrically distant from major inertia sources, display reduced effective inertia. This behavior is evident at Buses 20, 22, 6, and 19, where the estimated inertia values are the lowest in the system. Their reduced inertia reflects weaker coupling to high-inertia generators and greater electrical separation from high inertia SGs.

Between these two extremes, several buses form intermediate clusters whose inertia values reflect blended support from multiple generators. These regions correspond to meshed portions of the network, where generator influence overlaps, producing inertia levels that are neither dominated by a single source nor uniformly distributed. The resulting structure reveals the formation of spatial inertia zones shaped by generator proximity and network connectivity.

Compared to the IEEE 9-bus system, these results show that the qualitative behavior of the inertia distribution is preserved as system size increases. Buses aligned with a dominant directed generator paths exhibit inertia values closely reflecting that generator, whereas buses embedded within meshed regions display blended inertia contributions.

4.1.5 Dynamic Interpretation of the Estimated Spatial Inertia

The initial rate of change of frequency (RoCoF) across network buses provides a direct dynamic indicator of the spatial distribution of inertia. Figure 4.10 shows the bus-frequency deviations obtained from the time-domain simulation experiment designed in subsection 3.2.8.4. The fastest frequency drops (highest RoCoF) are observed at lowest inertia buses 3 and 9, while buses located closer to stronger or multiple inertia sources exhibit smaller deviations. This response ordering is consistent with the inertia estimates, bus 9 is electrically dominated by generator SG_3 , which has the lowest inertia constant, whereas buses 5, 6, and 8 receive composite inertia contributions from two SGs and therefore exhibit

comparatively milder frequency drop.

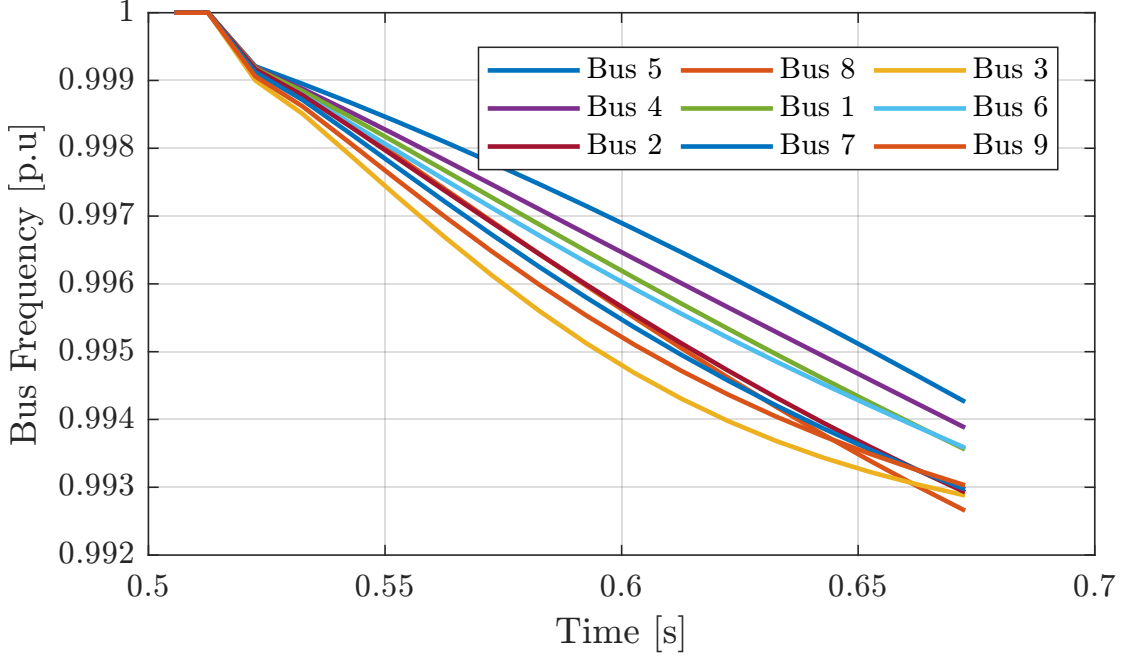
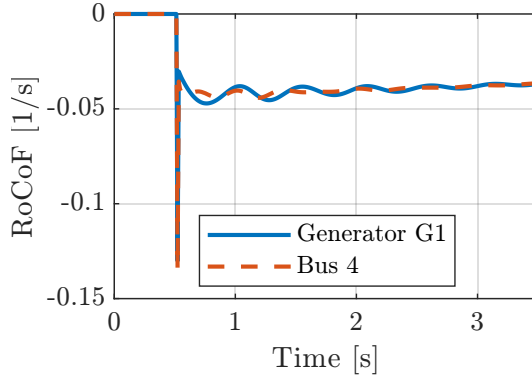


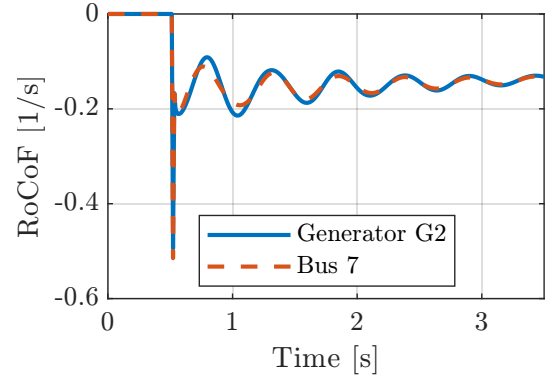
Figure 4.10: Bus frequency deviation following a 25 MW step load increase at buses 5, 6, and 8.

A clearer correspondence emerges when examining the RoCoF trajectories directly. Figure 4.11 reveals distinct clustering of RoCoF responses that aligns with the spatial inertia structure inferred from the electrical-distance matrix in Table 4.1. Specifically, bus pairs (1,4), (2,7), and (3,9) exhibit nearly identical RoCoF dynamics, indicating that each pair is predominantly supported by a single upstream SG. This behavior reflects unity electrical distance to their respective generators, buses 1 and 4 to SG_1 , buses 2 and 7 to SG_2 , and buses 3 and 9 to SG_3 .

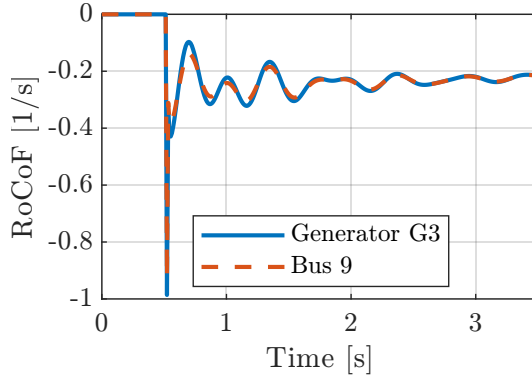
In contrast, the load buses 5, 6, and 8 exhibit intermediate RoCoF responses that cannot be attributed to a single generator. Instead, their RoCoF trajectories closely match weighted combinations of the RoCoF responses of the generator pairs supplying them. Bus 5 follows a weighted contribution from SG_1 and SG_2 , bus 6 from SG_1 and SG_3 , and bus 8 from SG_2 and SG_3 . These weighted responses are consistent with their respective electrical distances and confirm that their effective inertia arises from the combined influence of multiple generators rather than dominance by a single inertia source.



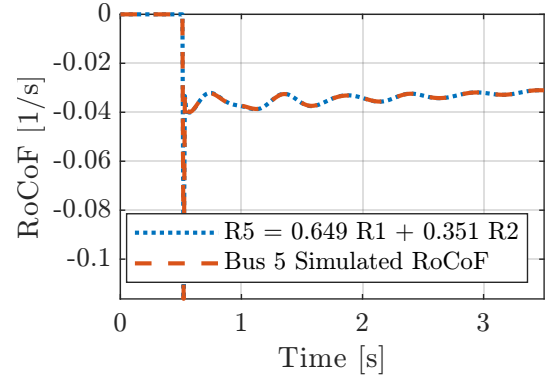
(a) RoCoF at SG_1 and bus 4



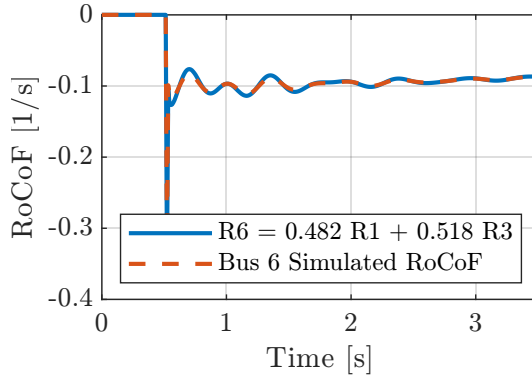
(b) RoCoF at SG_2 and bus 7



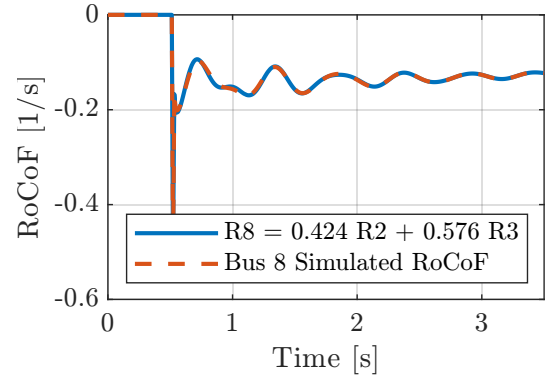
(c) RoCoF at SG_3 and bus 9



(d) RoCoF at bus 5 and its weighted contribution from SG_1 and SG_2



(e) RoCoF at bus 6 and its weighted contribution from SG_1 and SG_3



(f) RoCoF at bus 8 and its weighted contribution from SG_2 and SG_3

Figure 4.11: RoCoF-based bus clustering.

Overall, the observed RoCoF patterns provide strong dynamic validation of the offline inertia estimation method. Buses with high estimated inertia exhibit slower frequency dynamics and smaller RoCoF magnitudes, whereas buses with low estimated inertia respond more sharply. Moreover, the emergence of generator-dominated clusters and composite RoCoF behavior at meshed load buses confirms that the electrical-distance formulation accurately captures both single-generator dominance and multi-generator inertia blending in the network.

4.1.6 Bus Frequency Coherency and Generator Dominance Structure

To validate the generator dominance structure, time-domain simulation designed in subsection 3.2.8.4 was used to obtain bus frequency trajectories following a disturbance. Figure 4.12 shows the frequency differences between bus pairs (1,4), (2,7), and (3,9). In each case, the difference signal exhibits an oscillatory pattern that is almost perfectly out of phase with the corresponding generator swing, confirming that the buses within each pair behave coherently and share the same inertial support from their upstream generator.

The behavior of the meshed load buses 5, 6, and 8 provides a more detailed validation of the composite inertia representation. Figure 4.13 shows the frequency differences between: Bus 8 and generators SG_2 and SG_3 [Figures 4.13(a)–(b)]; Bus 6 and generators SG_1 and SG_3 [Figures 4.13(c)–(d)]; Bus 5 and generators SG_1 and SG_2 [Figures 4.13(e)–(f)].

The corresponding difference signals are in phase with the generator–bus pairs predicted by the electrical-distance matrix. For example, the frequency difference between bus 8 and bus 3 (Δf_{83}) is in phase with the difference between bus 3 and bus 9, while the difference between bus 8 and bus 2 (Δf_{82}) is in phase with the difference between bus 2 and bus 7. This implies that the frequency oscillation at bus 8 is effectively a superposition of the oscillations at buses 2 and 3, in line with

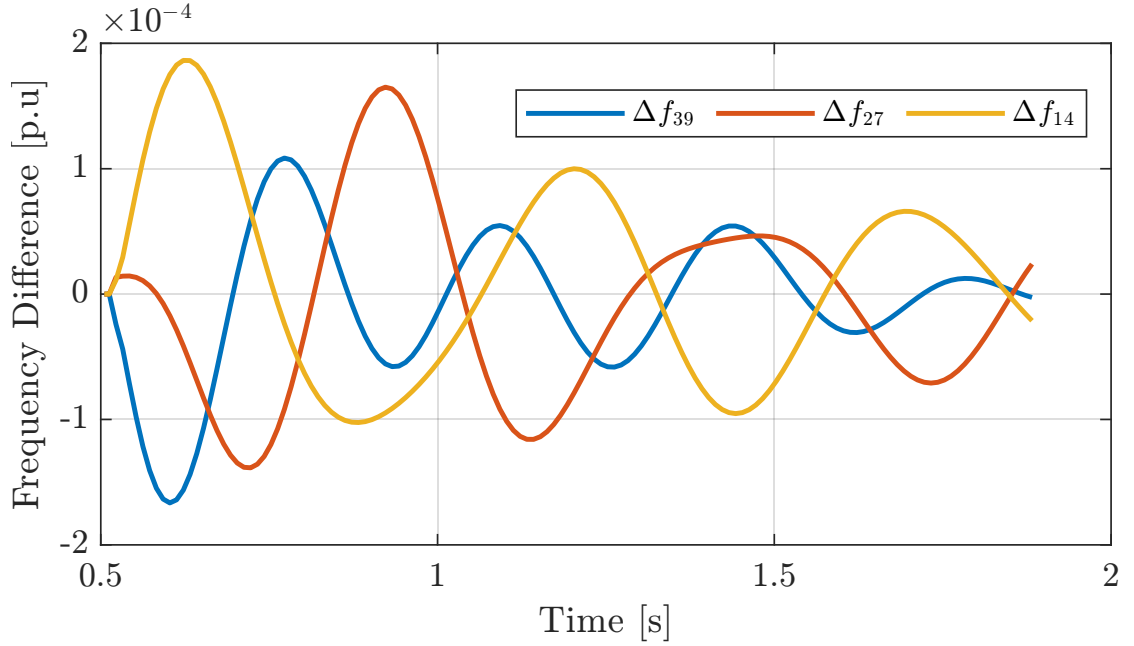


Figure 4.12: Bus frequency differences for generator-dominated bus pairs.

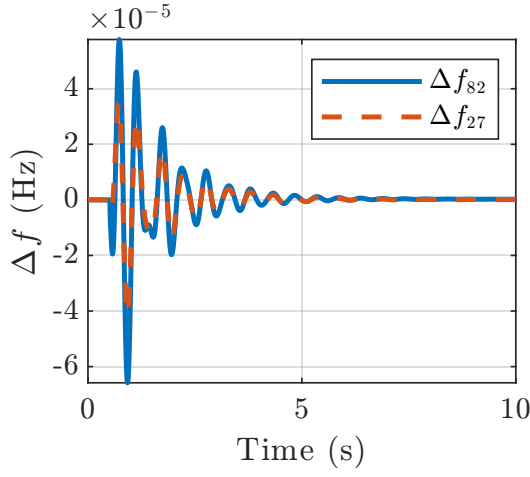
the interpretation that its inertia is the composite effect of SG_2 and SG_3 .

Similarly, (Δf_{62}) and (Δf_{61}) are in phase with (Δf_{72}) and (Δf_{41}) , respectively, confirming that bus 6 receives inertial support from both SG_1 and SG_3 . The same behavior is observed at bus 5, whose frequency differences align with those of buses 1 and 2, consistent with a composite inertia arising from SG_1 and SG_2 .

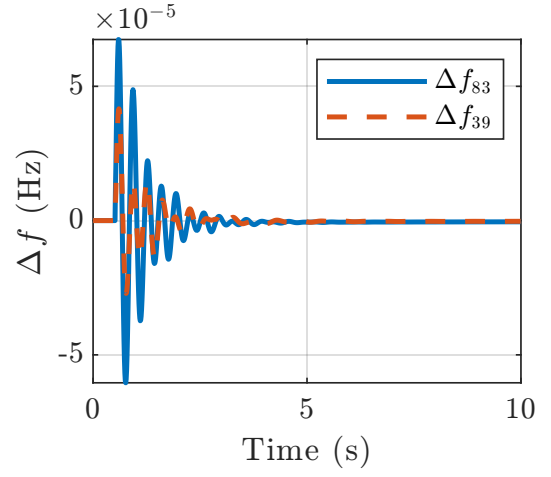
These observations provide a, time-domain corroboration of the generator dominance structure embedded in the offline electrical-distance formulation. Without using any dynamic measurements in its construction, the proposed method correctly predicts which generators dominate the inertial behavior of each bus and how composite inertial effects arise at meshed load buses.

4.1.7 Comparative Analysis with Existing Inertia Estimation Methods

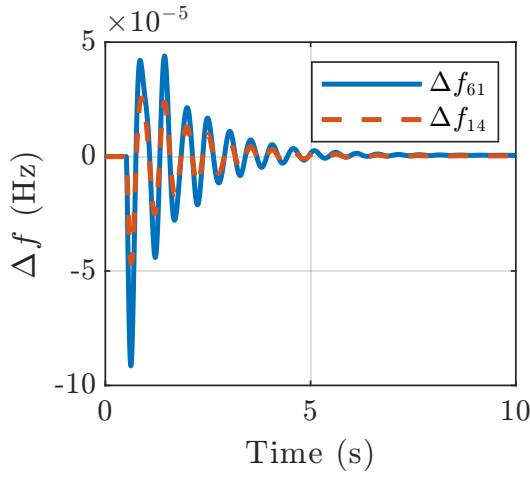
To evaluate the effectiveness of the proposed inertia estimation framework, a comparative ranking analysis was performed as described in Section 3.2.8.5. Figure 4.14



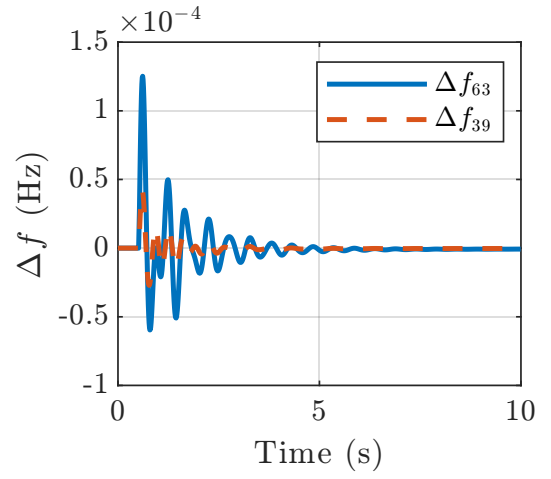
(a) Coherence of bus 8 with generator 2.



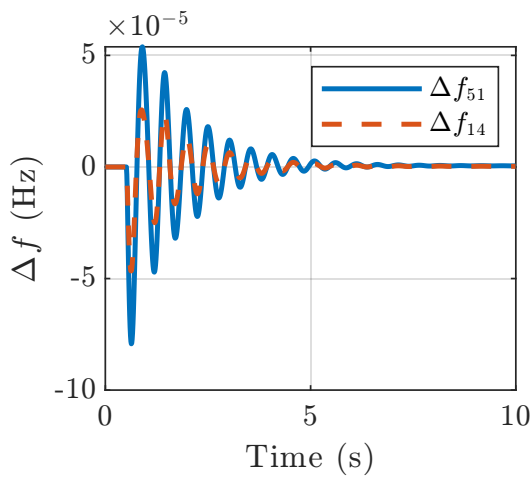
(b) Coherence of bus 8 with generator 3.



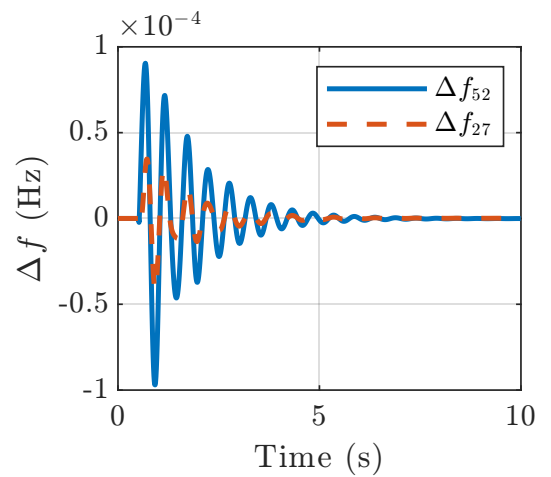
(c) Coherence of bus 6 with generator 1.



(d) Coherence of bus 6 with generator 3.



(e) Coherence of bus 5 with generator 1.



(f) Coherence of bus 5 with generator 2.

Figure 4.13: Bus coherency with generator frequency oscillations following a 25 MW load step at buses 5, 6, and 8.

presents the bus inertia rankings obtained using three different approaches for the IEEE 9-bus system: the proposed spatial inertia formulation, the analytical nodal inertia method of [33], and the time-domain index-based approach of [30].

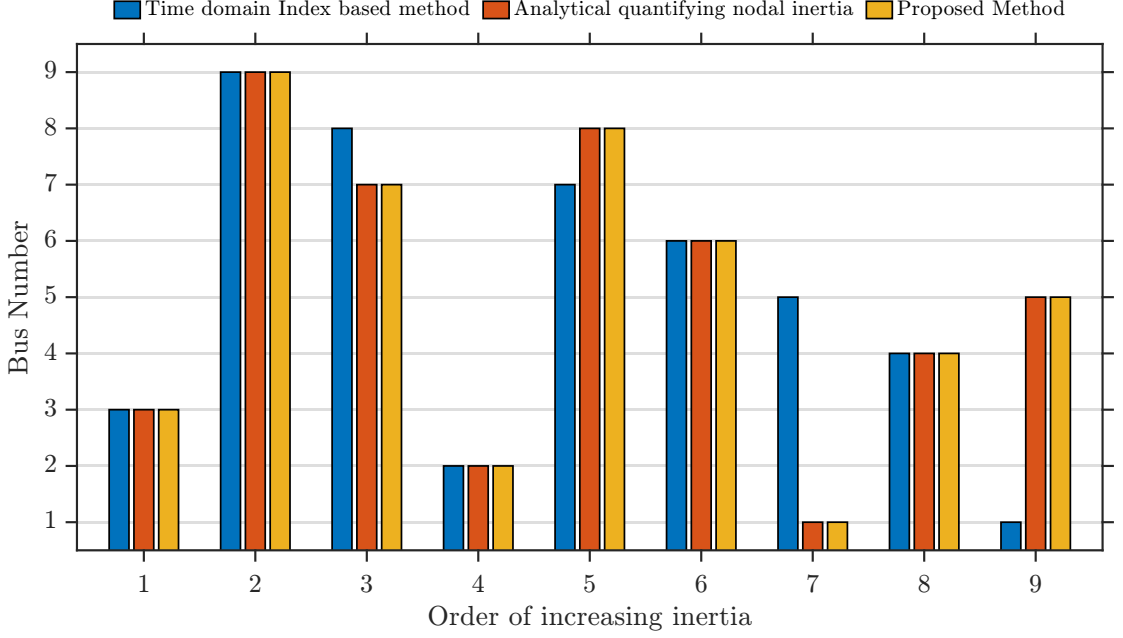


Figure 4.14: Comparison of inertia distribution estimation techniques on the IEEE 9-bus system.

The results show that the proposed method produces the same ordering as the analytical formulation in [33]. Both methods yield the following inertia ranking (from lowest to highest inertia):

$$3 < 9 < 7 < 2 < 8 < 6 < 1 < 4 < 5.$$

This ordering indicates that buses 3 and 9 possess the lowest effective inertia, while buses 4 and 5 exhibit the highest inertia levels.

In contrast, the time-domain index-based method yields a slightly different ordering,

$$3 < 9 < 8 < 2 < 7 < 6 < 5 < 4 < 1,$$

which differs mainly in the relative positions of buses 7, 8, and 5. These deviations arise from the formulation of the time-domain index d_j , which is computed as the

squared deviation between the bus frequency trajectory $f_j(t)$ and the center-of-inertia frequency $f_{\text{COI}}(t)$ over a finite time window. Consequently, the index reflects not only the spatial inertia distribution, but also the dynamic characteristics of the specific disturbance applied during the simulation.

Despite these differences, all three approaches consistently identify buses 3 and 9 as the lowest-inertia locations in the system, and buses 4 and 5 among the highest. The close agreement between the proposed method and the analytical formulation, together with the consistency of the resulting ranking with the observed time-domain frequency dynamics, provides strong evidence that the proposed inertia estimation technique accurately captures the underlying physics governing inertia distribution in the network.

4.2 Minimum Bus-Level Inertia Thresholds

The procedure outlined in Figure 3.2 is now applied to the IEEE 39-bus test system to compute spatially resolved minimum inertia thresholds under the selected large disturbance.

4.2.1 Spatial Distribution of Disturbance due to Load Change

The disturbance considered in this study is a load change event corresponding to the disconnection of the largest individual load in the IEEE 39-bus system. Under the base-case operating condition, the highest active-power demand is connected at Bus 39. Although Bus 39 also hosts a synchronous generator, only the load component is removed during the disturbance, while the generator remains in service.

The disturbance magnitude is therefore defined as $\Delta P_c = 1104.0$ MW, applied at bus $c = 39$. The selection of this disturbance enables assessment of frequency security during the initial inertial-response interval, in which system dynamics are

dominated by the release of stored kinetic energy from synchronous machines and the resulting RoCoF.

The largest load change disturbance is adopted to establish conservative minimum inertia limits that guarantee frequency security under the most severe expected generation–load imbalance caused by load variation. Since the required inertial response scales directly with disturbance magnitude, satisfying the RoCoF constraint for this disturbance implicitly ensures compliance for all smaller load changes within the assumed operating conditions. This selection therefore follows a conservative disturbance-based design philosophy commonly adopted in power-system frequency stability assessment.

Figure 4.15 illustrates the spatial distribution of the propagated disturbance magnitude $|\Delta P_i|$ across all buses of the IEEE 39-bus system.

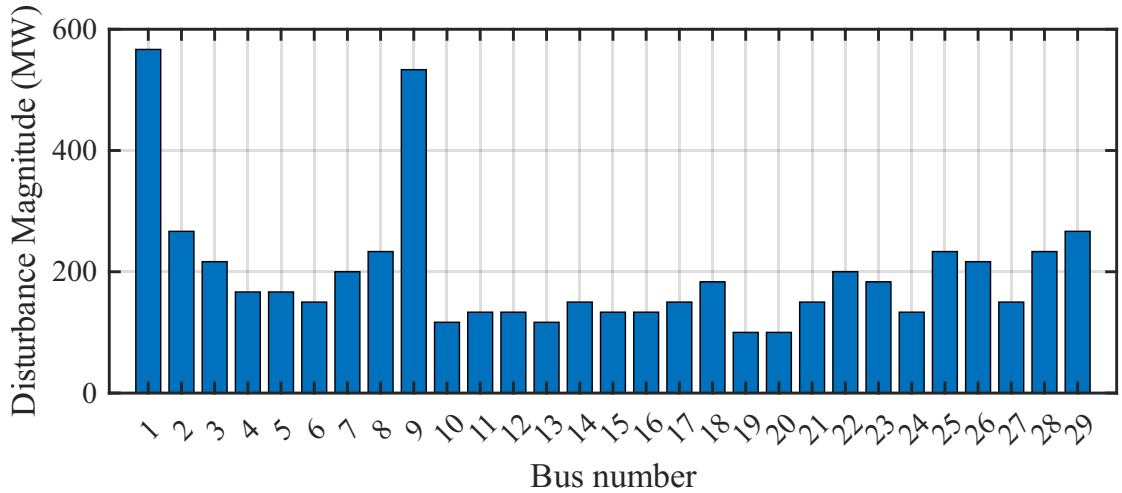


Figure 4.15: Spatial distribution of propagated disturbance magnitude resulting from a 1104.0 MW load disconnection at Bus 39.

The results reveal a strongly non-uniform disturbance distribution. Buses electrically close to Bus 39 experience larger equivalent power imbalances, whereas electrically distant buses observe attenuated disturbance effects. This behavior demonstrates that disturbance visibility is governed primarily by network electrical coupling rather than geographical proximity.

4.2.2 Computed Minimum Bus-Level Inertia Thresholds

Substituting the propagated disturbances into equation (3.43) and enforcing a uniform RoCoF limit of

$$|\dot{f}_i|_{\max} = 1 \text{ Hz/s}, \quad \forall i \in \mathcal{B},$$

the minimum bus-level inertia thresholds $h_{\min,i}$ are computed for all buses in the system.

Although the RoCoF limit is uniform, the resulting inertia requirements are spatially non-uniform due to differences in disturbance exposure and electrical coupling. Figure 4.16 compares the base-case bus inertia distribution with the derived minimum inertia thresholds and shows the corresponding security margin at each location. The security margin is defined as the difference between the available base-case inertia and the minimum inertia required to satisfy the RoCoF constraint.

The results reveal substantial spatial variation in minimum inertia requirements. Buses electrically closer to the disturbance location require higher inertia levels, reflecting stronger exposure to the propagated imbalance, whereas electrically remote buses require comparatively lower inertia support. This confirms that inertia adequacy is fundamentally a local network property rather than a uniform system-wide quantity.

Comparison with the base-case operating condition shows that all buses satisfy the inertia security constraint, as indicated by positive security margins at every location. The smallest margin is observed at Bus 20, where the available inertia exceeds the required threshold by approximately 17.3 s (about 38.5% above the minimum requirement), identifying this bus as the most critical from an inertia-security perspective. In contrast, several buses exhibit very large margins, indicating substantial tolerance to further reductions in synchronous inertia.

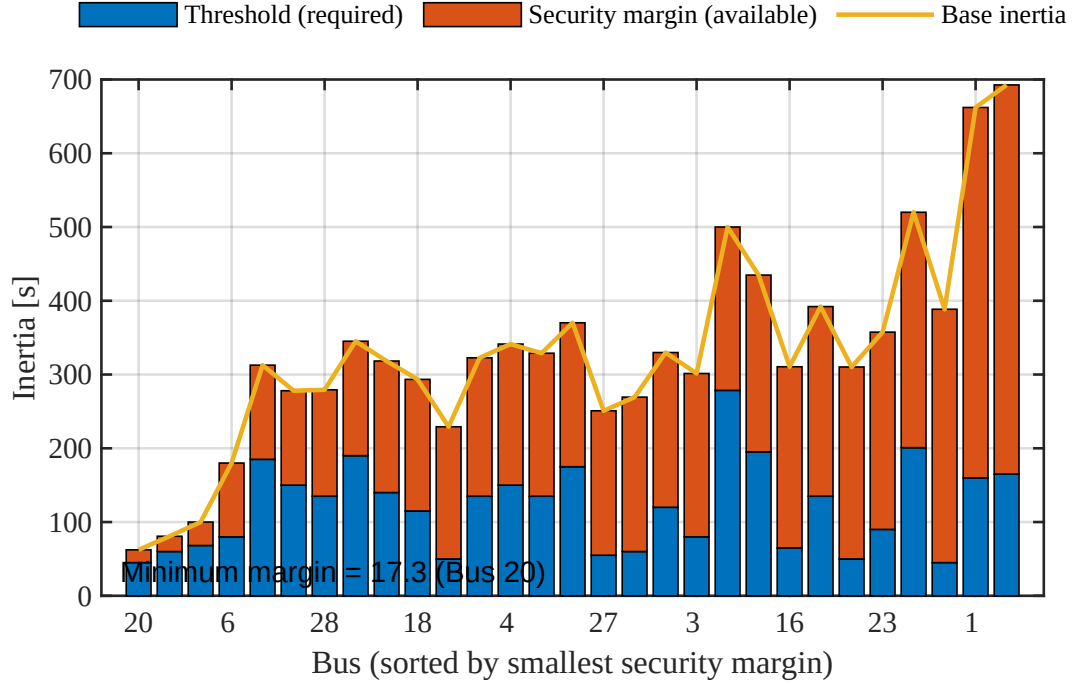


Figure 4.16: Comparison of base-case bus inertia and minimum inertia thresholds showing available security margins under a 1 Hz/s RoCoF limit.

Overall, the system possesses significant inertia headroom under the selected load change disturbance scenario. The spatial distribution of security margins establishes quantitative limits for inertia reduction and directly informs the inertia-aware optimization framework developed in Section 3.4.

4.3 NSGA-II Optimization Results

This section presents the results of the inertia-constrained multi-objective optimization developed in Chapter 3. The NSGA-II framework combines the spatial inertia estimation method developed under the first objective with the minimum bus-level inertia thresholds derived under the second objective to screen candidate operating points for spatial frequency-security compliance. As a result, the optimization produces feasible Pareto-optimal dispatch solutions that balance VRES penetration, transmission losses, and synchronous-generator commitment, while

satisfying bus-level inertia requirements. For each candidate solution, bus-level inertia is re-estimated using the proposed analytical formulation, and compliance with RoCoF-based thresholds is verified before Pareto ranking. The aim is to maximize inertia-less VRES dispatch while maintaining spatial frequency security.

4.3.1 PV Plant Allocation in the Modified IEEE 39-Bus System

Prior to performing the inertia-constrained optimization studies, the IEEE 39-bus test system was modified to include utility-scale PV generation. The siting and sizing of the PV plants were determined using the loss-minimization methodology described in Section 3.6.2, which follows the approach proposed in [108].

Applying this methodology resulted in PV installations at buses 3, 4, 8, 15, 16, 20, and 24. The corresponding installed capacities are 612.7 MW, 988.9 MW, 1267.2 MW, 885.7 MW, 854.6 MW, 854.6 MW, and 457.4 MW, respectively. The total installed PV capacity represents approximately 96% of the system peak load. This PV-integrated configuration defines the inertia-less VRES available scenario used throughout the subsequent optimization and validation studies.

4.3.2 Pareto Front Characteristics and Operating Point Interpretation

For the base optimization case of the modified IEEE 39-bus system under minimum inertia thresholds constraint obtained in section 4.2, the inertia-constrained multi-objective problem described in sections 3.4, and 3.5 is solved using NSGA-II. The resulting set of feasible non-dominated solutions forms the Pareto front shown in Figure 4.17. The Pareto front shows a trade-off between inertia-less VRES penetration and transmission losses for different SG commitment patterns. Each point labeled A–G corresponds to a feasible operating state with specific penetration

levels, loss values, and committed SG sets.

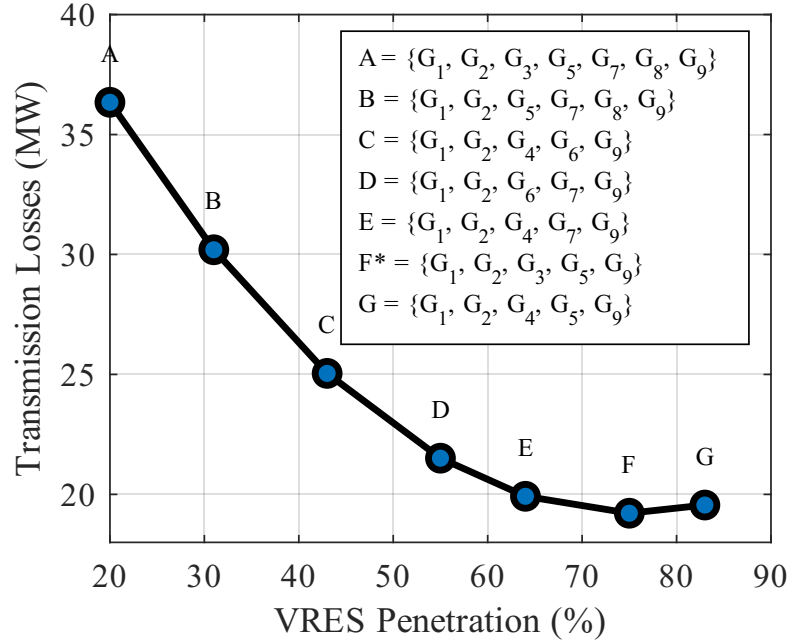


Figure 4.17: Pareto fronts and operation point shift

A numerical summary of the Pareto-optimal operating points is provided in Table 4.2. Moving from A to F increases inertia-less VRES penetration from 20% at point A to 75% at point F, while transmission losses decrease markedly from 36.34 MW to 19.22 MW, respectively. This reduction coincides with progressive changes in generator commitment, including the decommitment of units electrically distant from major load centers and the increased utilization of nearer generation sources.

Table 4.2: Key operational parameters of Pareto-optimal solutions

Point	Penetration (%)	Loss (MW)	No. SGs	Committed SGs
A	20	36.34	7	$\{G_1, G_2, G_3, G_5, G_7, G_8, G_9\}$
B	31	30.20	6	$\{G_1, G_2, G_5, G_7, G_8, G_9\}$
C	43	25.04	5	$\{G_1, G_2, G_4, G_6, G_9\}$
D	55	21.51	5	$\{G_1, G_2, G_6, G_7, G_9\}$
E	64	19.92	5	$\{G_1, G_2, G_4, G_7, G_9\}$
F (Knee)	75	19.22	5	$\{G_1, G_2, G_3, G_5, G_9\}$
G (Max)	83	19.86	5	$\{G_1, G_2, G_4, G_5, G_9\}$

Between points A and F, power loss decline monotonically despite rising VRES penetration. This behavior reflects two combined effects:

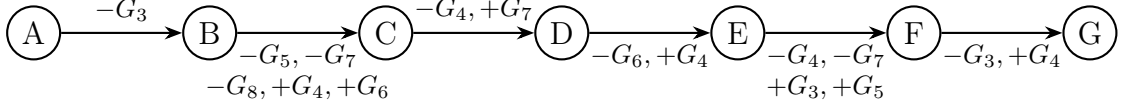
- (i) Spatial realignment of injections: inertia-less VRES plants located nearer to load centers displace SGs situated electrically farther away, reducing transmission losses.
- (ii) Selective SG decommitment: SGs that contribute little to the inertia distribution are preferentially removed.

Point F emerges as the clear knee point, it achieves a high penetration of approximately 75% while yielding the lowest total losses (19.22 MW). Beyond this point, losses begin to rise (19.86 MW at G) even as penetration increases further to 83%. This behavior marks the onset of diminishing returns, where any additional decommitment of SGs reduces the network's ability to route power through efficient paths. To satisfy the spatial inertia thresholds, the system is forced to reroute power through longer paths, which increases transmission losses despite the higher VRES penetration.

Point G represents the final feasible operating state because there exists no other combination of five (or fewer) SGs that can support additional inertia-less VRES penetration while simultaneously maintaining all bus-level inertia requirements. Any further attempt to decommit an SG or to alter the committed set in favor of higher VRES penetration would cause one or more buses in the identified bottleneck region to fall below their minimum inertia thresholds, rendering the solution infeasible. Thus, point G marks an operational limit, beyond which the spatial inertia constraint becomes the binding factor preventing further inertia-less VRES penetration.

The SG commitment sets in Table 4.2 and state transition diagram in figure 4.18 reveal the following structural trends:

- Seven and six SGs online (A–B): High inertia, low VRES utilization, and high losses. Units G_7 and G_8 remain online, but their spatial effectiveness



Legend: $+G_k = \text{SG } G_k \text{ committed}$, $-G_k = \text{SG } G_k \text{ decommitted}$

Figure 4.18: Transition of synchronous generator commitment patterns across Pareto-optimal operating points A–G.

is low. In this case, the system can absorb up to approximately 31% of the inertia-less VRES penetration without violating spatial inertia distribution threshold.

- Transition to five SGs (C–F): Five SGs are decommitted, especially those providing weak spatial inertia support. This transition drives most of the reductions in losses.
- Knee point (F): The committed set $\{G_1, G_2, G_3, G_5, G_9\}$ uniquely satisfies the spatial inertia distribution threshold and the requirement for minimal losses.
- Maximum penetration point (G): Only G_3 is replaced by G_4 ; however, this substitution increases losses because G_4 is electrically less effective at supporting the low-inertia region.

These discrete changes in commitment structure explain the piecewise-smooth shape of the Pareto front. Each segment corresponds to a commitment regime with its own dynamic and flow characteristics. Notably, five SGs appear sufficient across points C–G, but which five SGs are online critically determines loss performance and inertia satisfaction. This observation highlights the necessity of spatially aware inertia constraints, that conventional OPF would not distinguish between SGs based on their ability to support local inertia requirements.

4.3.3 Optimal solution at Pareto knee point

4.3.3.1 Generation Dispatch

The generation dispatch resulting from the NSGA-II optimization reflects the combined influence of three competing objectives—maximizing inertia-less VRES penetration, minimizing transmission losses, and reducing SG commitment while strictly satisfying all operational and spatial inertia distribution constraints. At the selected operating point (solution F in Figure 4.17), the algorithm commits a minimal, but strategically positioned subset of synchronous generators, ensuring that the spatial inertia thresholds are met throughout the network.

At this operating point, five of the ten SGs remain online: G_1 , G_2 , G_3 , G_5 , and G_9 . These units together supply 1596.7 MW, meeting approximately one quarter of the total system demand of 6254.23 MW. The remaining demand is met entirely by inertia-less VRES. Table 4.3 shows that all online SGs operate within their generation limits, with loading levels ranging from 20% to 62%. This indicates that the SGs are not dispatched to the edge of their capabilities; rather, they are retained primarily for their inertial contribution and their spatial influence on the inertia adequacy support.

The remaining five SGs— G_4 , G_6 , G_7 , G_8 , and G_{10} —are decommitted. Their absence reflects two simultaneous optimization effects: (i) VRES plants offset the need for their active-power contribution, and (ii) their electrical position offers limited marginal benefit to the spatial inertia distribution, once the committed SG set is already sufficient to meet bus level inertia requirement. Importantly, the algorithm does not favor committing the SGs geographically closest to high-demand buses, but rather those whose electrical proximity effectively lifts the bus inertia above their thresholds.

After optimization of the VRES penetration and minimization of network losses, the inertia-less plants supply 4683.5 MW, corresponding to almost 75% of the total generation. Table 4.4 shows that five of the seven VRES achieve loading levels

Table 4.3: Optimal dispatch of synchronous generators at the Pareto knee point

Bus No.	Generator name	Min. Gen. (MW)	Max. Gen. (MW)	Status	Dispatch (MW)	Loading (%)
30	G_{10}	208.0	1040	OFF	–	–
31	G_2	129.2	646	ON	130.9	20.3
32	G_3	145.0	725	ON	189.4	26.1
33	G_4	130.4	652	OFF	–	–
34	G_5	101.6	508	ON	136.1	26.8
35	G_6	137.4	687	OFF	–	–
36	G_7	116.0	580	OFF	–	–
37	G_8	112.8	564	OFF	–	–
38	G_9	173.0	865	ON	458.0	52.9
39	G_1	220.0	1100	ON	682.3	62.0
Total SGs generation					1596.7	–

above 70%, with the plants at buses 3 and 4 operating near full capacity (92% and 90%, respectively). This pattern demonstrates that the optimization algorithm preferentially dispatches VRES units that are electrically well-positioned to reduce losses, while simultaneously ensuring that their injections do not violate inertia requirement at any bus.

As shown in Table 4.4, the plants at buses 15, 20 and 24 operate at comparatively lower loading levels of 67.8%, 67.9% and 70.4%, respectively. These plants are located to the buses with minimum security margin (see figure 4.16), therefore, experience slightly reduced dispatch levels, reflecting the influence of spatial inertia constraints in shaping renewable power injections.

4.3.3.2 Transmission Losses

The behavior of transmission losses across the Pareto front provides important insight into how the network efficiency evolves under combined VRES penetration

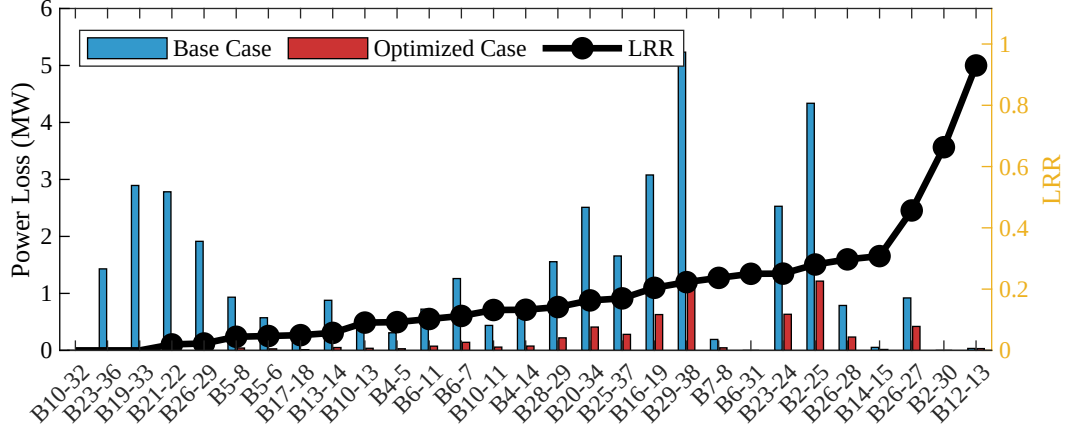
Table 4.4: Optimal dispatch of solar PV power plants at the Pareto knee point

Plant location	Capacity (MW)	Dispatched (MW)	Loading (%)
3	612.3	563.43	92.0
4	854.6	770.68	90.2
8	1267.2	1017.31	80.3
15	885.7	600.87	67.8
16	854.6	737.60	86.3
20	988.8	671.78	67.9
24	457.4	321.87	70.4
Total VRES generation		4683.5	

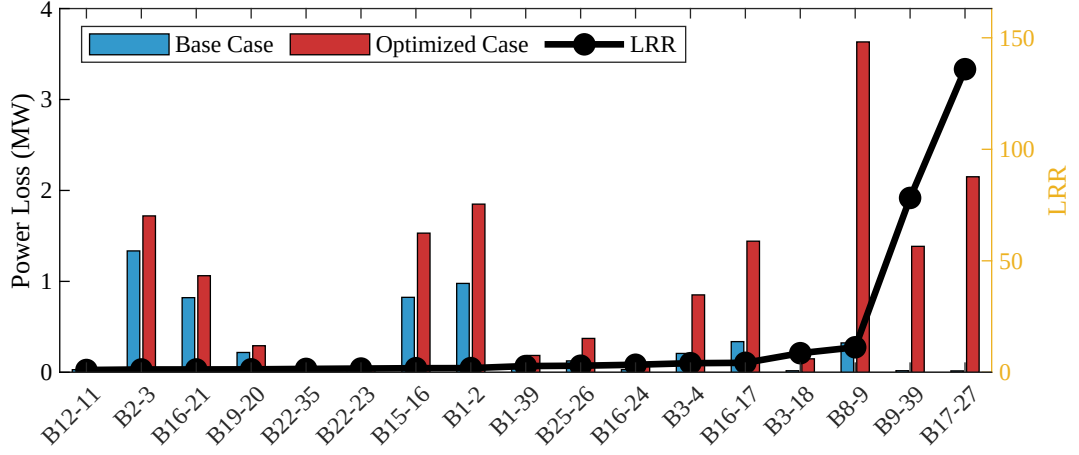
and spatial inertia constraint. As shown in Figure 4.17, total losses decline steadily from point A to the knee point F, falling from approximately 36 MW to 19.22 MW. Compared with the about 43 MW power loss from running power flow study of the base IEEE 39 bus test system, the reduction achieved at point F corresponds to an overall improvement of about 56%. This confirms that high VRES penetration, when guided by spatial inertia requirements, can simultaneously enhance both frequency security and steady-state efficiency.

To understand where these improvements originate, Figure 4.19 presents a branch-level analysis of losses using LRR values, defined in equation (3.75). This metric provides a relative comparison between the optimized loss on branch $(i-j)$ and its loss in the base case. Values of $LRR < 1$ indicate reduced losses, whereas $LRR > 1$ indicate increases.

Subplot 4.19a shows that a majority of the branches exhibit $LRR < 1$, indicating that high VRES penetration leads to widespread loss reductions. Notable examples include branches $B19-33$, $B16-19$, $B23-24$, $B2-25$, $B21-22$, and $B29-38$, which in the base case accounted for 6 to 12% of total system losses. After optimization, these branches experience losses below 3 %. This improvement arises because the optimized dispatch places significant VRES injections closer to major load centers,



(a) Branches with improved/equal losses ($LRR \leq 1$).



(b) Branches with increased losses ($LRR > 1$).

Figure 4.19: Comparison of transmission line power loss for base and optimized case.

shortening average transfer distances and reducing line loading. The effect is especially pronounced in long, high-impedance lines that, in the base case, carried substantial SG-generated power originating far from load buses.

Subplot 4.19b highlights branches where $LRR > 1$. While these branches exhibit increased losses under the optimized schedule, their base-case losses were initially extremely small. Consequently, even modest increases in absolute loss result in LRR values exceeding unity, despite the actual magnitude of additional loss being insignificant. These localized increases reflect the system's need to reroute small

amounts of power through alternative paths to satisfy bus level spatial inertia constraints. They do not signal deterioration of system efficiency; instead, they represent a controlled redistribution of flows that support global optimization goals.

This behavior becomes particularly relevant beyond the knee point. The transition from operating point F to point G requires the replacement of G_3 with G_4 to maintain spatial inertia adequacy. Although this change facilitates higher VRES penetration, it alters the geometry of power injections, forcing certain flows through longer paths and thereby increasing losses marginally (from 19.22 MW to 19.86 MW).

4.3.3.3 Inertia Distribution

The spatial inertia distribution in the optimized operating point provides essential insight into how the NSGA-II framework meets the bus-level frequency stability constraint imposed by the minimum inertia thresholds. Figure 4.20 compares the minimum required inertia at each bus—derived from the RoCoF-based criterion developed in section 3.3 with the optimized inertia values obtained from the scheduled set of SGs. The minimum inertia profile serves as a strict lower bound required for local frequency stability, while the optimized profile reflects the actual inertia support available under the co-optimized SG–VRES dispatch.

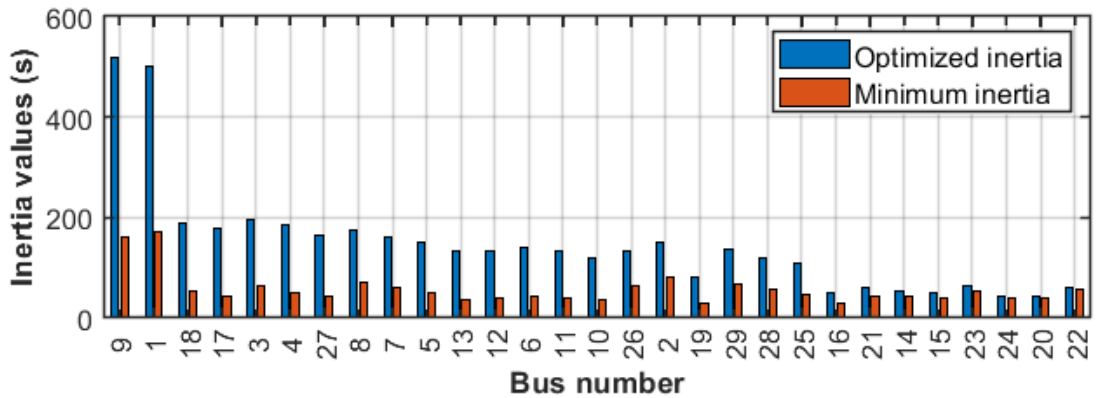


Figure 4.20: Spatial inertia distribution across all buses, computed on a 100 MVA system base.

The resulting distribution is shaped by the SG commitment pattern that emerged at

the Pareto knee point. Out of the ten available SGs, the optimization decommitted five of them $-G_4, G_6, G_7, G_8$, and G_{10} . This reflects the system’s ability to maintain both inertia adequacy and low transmission losses, while relying on only five SGs. The remaining units, G_1, G_2, G_3, G_5 , and G_9 , form the minimal set capable of supporting the spatial inertia requirements.

A key observation from Figure 4.20 is that the optimized inertia at buses 14–16 and 20–24 lies only slightly above their corresponding minimum inertia thresholds. Based on the values shown in the figure, the inertia security margins in this region range approximately from 7% to 50%, obtained by using criteria established in 3.5.7.3 expressed in percentage of their respective minimum inertia.

The smallest margin occurs at bus 22 (approximately 7%), indicating that this location operates closest to the inertia feasibility boundary, while the largest margin occurs at bus 16 (approximately 50%). These relatively small margins confirm that this cluster of buses forms the inertia-constrained region that ultimately limits further VRES penetration.

The role of these SGs is not symmetric. Their contribution to the inertia landscape depends strongly on their electrical proximity to the vulnerability zone in the network. As shown in Figure 4.21, the red-bounded region (R_2)—comprising buses 14–16 and 21–24, exhibits the lowest inertia margins. These buses are electrically closely coupled, forming a dynamic subregion whose inertial response is dominated by contributions from neighboring SGs. Therefore, the optimization retains SGs that efficiently support this region: G_2, G_3 , and G_5 , which are electrically adjacent to the bottleneck, whose placement also enhances local inertia. The inclusion of G_1 , despite its distance from the region, is explained by its high inertia.

This spatial configuration reveals a frequency-stability bottleneck; the optimization converges to a solution where the inertia of the buses in the red region lies close to their minimum allowed values. Any further reduction in SG commitment, or a significant reallocation of VRES injections, would violate the local inertia constraints. Thus, the bottleneck region effectively determines the maximum

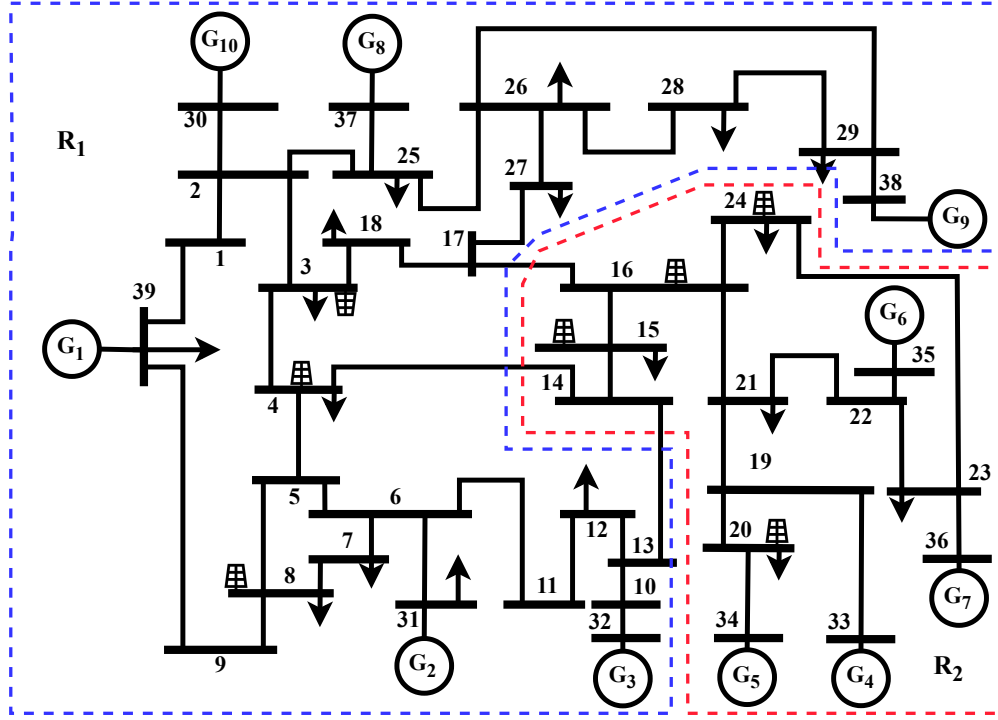


Figure 4.21: Inertia-constrained region (red bounded area) where bus-level inertia approaches the minimum permissible threshold.

achievable VRES penetration and the minimum online SGs required to secure the system.

4.3.4 Sensitivity Analysis

4.3.4.1 Impact of RoCoF limits on the feasible operating region

Sensitivity of the Pareto optimal solutions to RoCoF limits is evaluated using the bus-level inertia thresholds derived in section 3.3. Figure 4.22 presents the Pareto fronts obtained for RoCoF limits of 1, 2, and 3 Hz/s, illustrating the trade-off between VRES penetration and transmission losses under progressively relaxed frequency security requirements.

Under the strictest limit of 1 Hz/s, the feasible operating region is most constrained. The inertia thresholds necessitate the commitment of a larger number

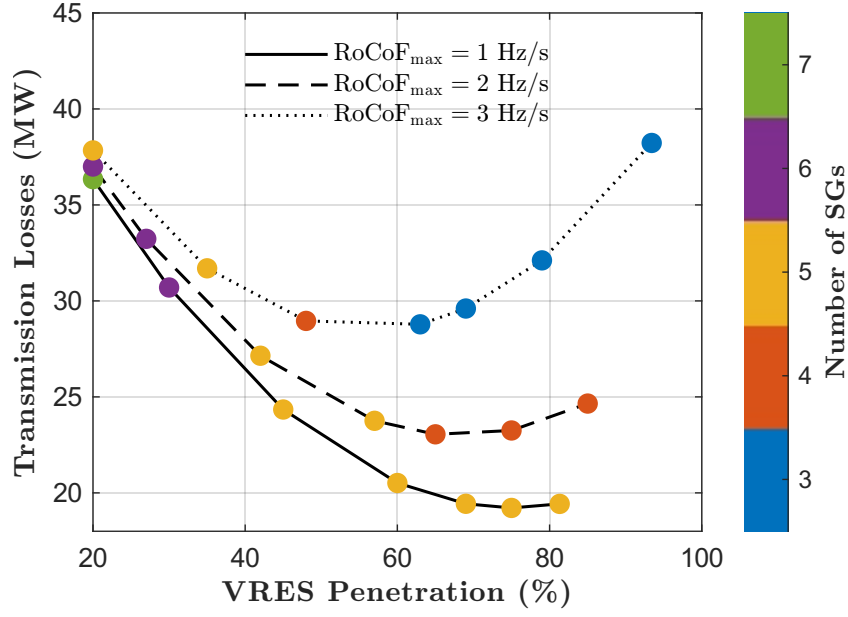


Figure 4.22: Pareto fronts for different RoCoF settings.

of synchronous generators, which remain spatially distributed across the network. This distributed commitment pattern produces balanced real-power injections and reduces electrical transfer distances. Consequently, branch loading is moderated, resulting in the lowest transmission losses among the three cases. Despite the tight stability constraint, the system accommodates VRES penetration levels of up to approximately 82

When the RoCoF limit is relaxed to 2 Hz/s, the inertia requirement at each bus decreases, enabling greater displacement of synchronous generation. This expands the feasible solution space and increases the achievable VRES penetration to roughly 86%. However, the reduced number of committed SGs leads to a more concentrated spatial pattern of power injections. Active power must, therefore, traverse longer electrical paths to reach load centers, increasing branch currents and associated losses relative to the 1 Hz/s scenario.

The most relaxed setting of 3 Hz/s yields the largest feasible region and allows VRES penetration levels approaching 95%. While this reflects significant operational flexibility from a VRES integration perspective, the sparse distribution of SGs produces the least efficient power-flow configuration. Power transfers rely more

heavily on longer, higher-impedance corridors, resulting in the highest transmission losses among all scenarios.

Overall, the RoCoF sensitivity study highlights a clear operational trade-off. Stricter RoCoF limits enforce geographically distributed SG support, improving both frequency stability margins and network efficiency. Conversely, relaxed limits enable higher renewable sources utilization but at the expense of increased transmission losses. These findings demonstrate that the selected dynamic security standard fundamentally shapes the feasible operating envelope and should, therefore, be explicitly embedded within operational planning frameworks.

4.3.4.2 Load and VRES Variation at a RoCoF Limit of 1 Hz/s

To assess temporal robustness, the optimization framework is evaluated across time-varying operating conditions derived from the scenario construction described in section 3.5.7.6. Figure 4.23 shows the 24-hour load and PV availability profiles mapped onto the IEEE 39-bus system. Three representative operating points are selected for detailed analysis: Hour 8 — peak demand, low PV output, Hour 13 — solar generation peak, Hour 17 — moderate load and PV conditions

Figure 4.24 presents the corresponding Pareto fronts under the fixed RoCoF limit of 1 Hz/s. Because the leftmost point of each Pareto front represents 0% VRES penetration (full SG commitment), the baseline loss level directly reflects the network loading associated with each hour. Higher demand conditions, therefore, begin at higher loss values.

At hour 8, the system experiences peak loading while PV availability is minimal. A large number of synchronous generators remain online to meet demand, and inertia requirements are inherently satisfied. As VRES penetration increases, localized renewable injections displace synchronous output, reducing electrical transfer distances and producing a gradual decline in transmission losses along the Pareto front.

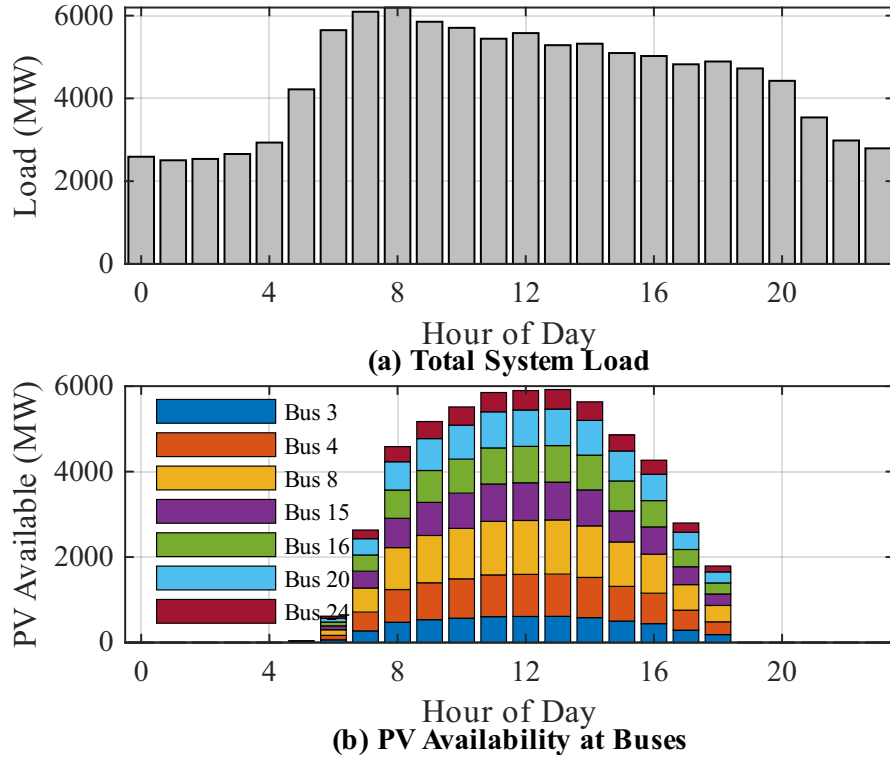


Figure 4.23: Mapped 24-hour load and PV availability profile for the IEEE 39-bus system.

At hour 13, PV availability exceeds system demand, creating the potential for very high renewable penetration. However, the 1 Hz/s RoCoF limit imposes stringent inertia thresholds that prevent excessive displacement of synchronous machines. A minimum subset of SGs must remain committed to maintain spatial inertia adequacy.

At hour 18, both load and PV output are moderate. Lower demand reduces branch currents and baseline losses, while available PV generation offsets a significant share of SGs output. The SG commitment required to satisfy inertia thresholds is already sufficient to supply demand, allowing full absorption of available PV without violating stability constraints. Consequently, the Pareto front for hour 17 lies lowest on the loss axis, reflecting the combined benefits of reduced network utilization and renewable displacement of conventional generation.

Taken together, the hourly sensitivity analysis yields three key observations. First, system loading establishes the baseline loss regime through its influence on corri-

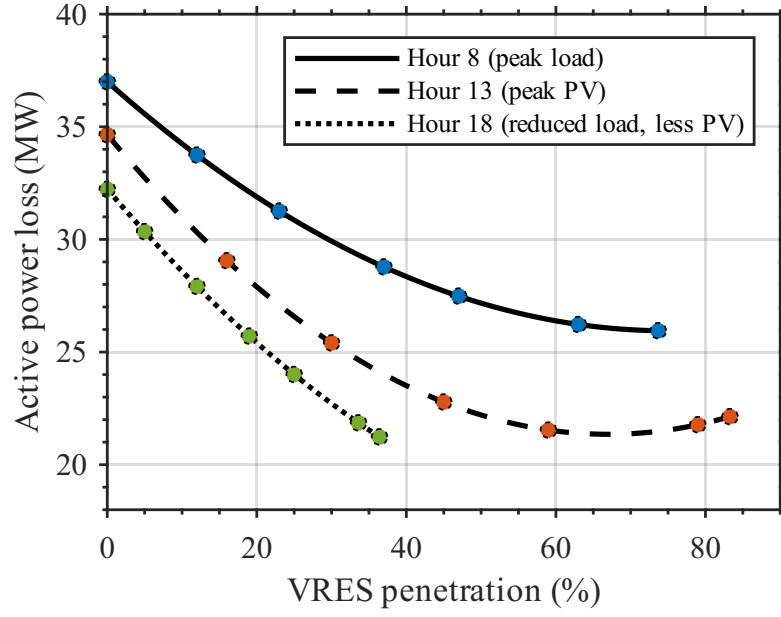


Figure 4.24: Pareto fronts under hourly load and PV conditions at a RoCoF limit of 1 Hz/s.

dor utilization. Second, renewable availability alone does not determine feasible penetration; it must be evaluated jointly with spatial inertia requirements. Third, the co-optimization framework adapts effectively to temporal variability, while maintaining both inertia adequacy and efficient power-flow patterns. These results confirm the operational robustness of the proposed approach under realistic time-varying conditions.

4.3.5 Computational Performance and Practical Viability

Beyond solution quality, the practical usefulness of an inertia-aware optimization framework depends critically on its computational performance. This subsection evaluates the execution time and convergence characteristics of the proposed NSGA-II-based scheduling framework, when applied to the modified IEEE 39-bus system. The objective is to assess whether the framework is computationally tractable for operational planning studies, particularly day-ahead scheduling under high VRES penetration.

4.3.5.1 Execution Time and Convergence behavior

The optimization framework was implemented in MATLAB R2023a and executed on a workstation equipped with an AMD Ryzen[®] 7 4700U processor (2.0 GHz), 16 GB RAM, and a 477 GB SanDisk SSD. The system operates on a 64-bit architecture with integrated AMD Radeon[™] graphics.

The average runtime for a complete optimization cycle was approximately 32 minutes. During each run, the algorithm evaluated roughly 15,000 candidate solutions (50 individuals across 300 generations), each requiring a full AC power flow solution and inertia-distribution assessment. These power flow evaluations accounted for approximately 65% of the total computation time, reflecting their dominant role in enforcing network feasibility and operational constraints. The remaining 35% of the runtime was associated with genetic operations (selection, crossover, and mutation) and post-processing related to the evaluation of bus-level inertia metrics.

Figure 4.25 illustrates the convergence behavior of the optimization process using the hypervolume indicator as a performance metric. The hypervolume reaches approximately 95% of its final value within the first 150 generations, indicating rapid convergence toward a high-quality Pareto front. Subsequent generations mainly refine the front locally rather than producing substantial improvements. This behavior confirms that the search process efficiently balances exploration and exploitation.

4.3.5.2 Assessment of Practical Implementability

The observed execution time of approximately 32 minutes remains compatible with the operational workflow of day-ahead generation scheduling. In practical power-system operation, generation scheduling is typically formulated as a unit commitment problem that determines the commitment and dispatch of generating units over a daily horizon based on forecasted demand and operational constraints [111]. These scheduling tasks are performed offline prior to system

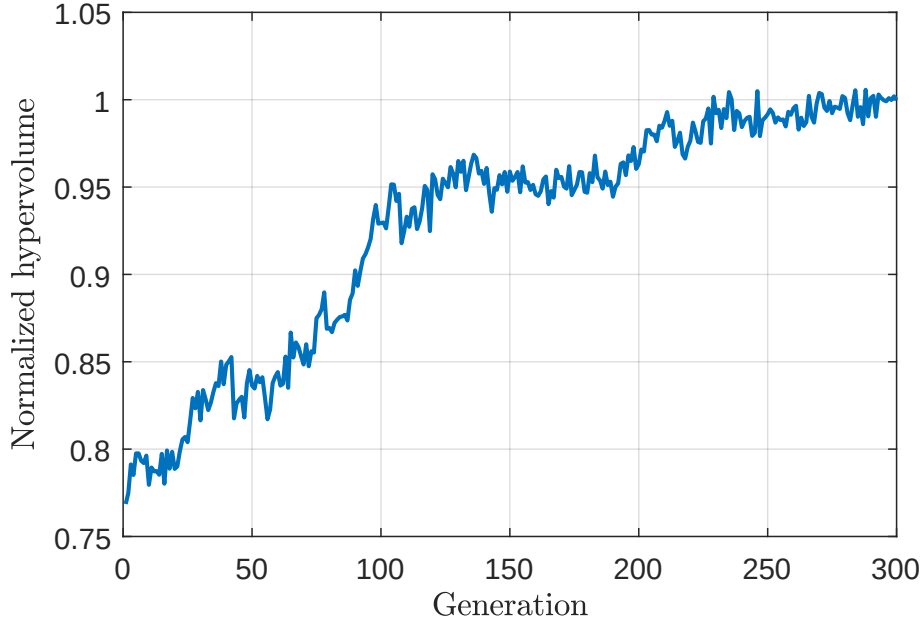


Figure 4.25: Hypervolume convergence characteristics of the NSGA-II optimization for the IEEE 39-bus system.

operation, allowing the use of computationally intensive optimization procedures.

Within this context, the proposed framework operates within a practical computation window while simultaneously performing dispatch optimization and frequency-stability enforcement through spatial inertia constraints. This integrated formulation eliminates the need for sequential “optimize–then–validate” procedures that can otherwise produce infeasible or dynamically insecure operating points.

The generation of a full Pareto-optimal solution set further enhances operational flexibility. System operators can select operating points that best match prevailing conditions or respond to contingencies by transitioning to neighboring feasible solutions without re-running the optimization. In this context, the computational cost is justified by the framework’s ability to deliver both optimality and guaranteed inertia adequacy in a single unified process.

4.4 Validation of the Proposed Inertia-Aware Optimization Framework

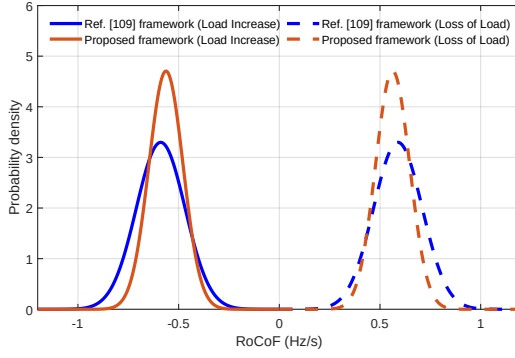
This section addresses the fourth specific objective by benchmarking the proposed inertia-aware scheduling framework against established frequency-security assessments and by testing its robustness under realistic operating variability. The focus is on two complementary validation layers introduced in section 3.7: (i) probabilistic robustness under time-varying load and VRES uncertainty, and (ii) analytical consistency with a kinetic-energy (KE) based frequency-security benchmark.

4.4.1 Stochastic Frequency Stability Results Under Fixed PV Penetration

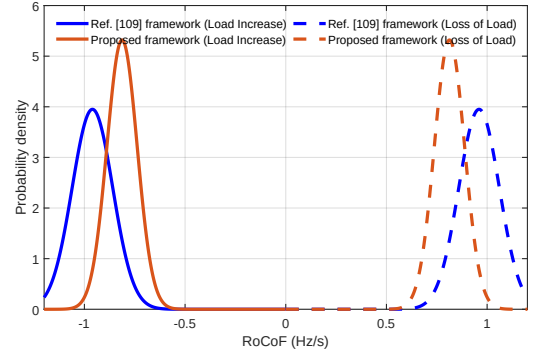
The stochastic validation procedure described in Section 3.7.1 was applied to evaluate whether the operating points produced by the proposed spatial inertia-aware optimization remain more strongly aligned with desirable frequency stability outcomes than those obtained from the benchmark methodology. Following the procedure outlined in Section 3.7.1, both frameworks were evaluated under identical Monte Carlo operating scenarios generated from the stochastic load and PV availability realizations defined by $\alpha_{\text{load}}^{(s)}(t)$ and $\gamma_{\text{PV}}^{(s)}(t)$. For each stochastic scenario, dispatch solutions obtained from the proposed NSGA-II inertia-aware scheduling framework and the benchmark approach were subjected to the two disturbance events: (i) outage of the largest load and (ii) an equivalent sudden load increase of identical magnitude. The resulting dynamic responses were evaluated using the frequency stability indices, namely the RoCoF and the frequency nadir.

Figure 4.26 summarizes the probability density functions of the RoCoF and frequency nadir across all stochastic operating scenarios for the two representative network areas considered in the study. Area R1 corresponds to a relatively high-inertia region, whereas Area R2 represents a lower-inertia region that is more

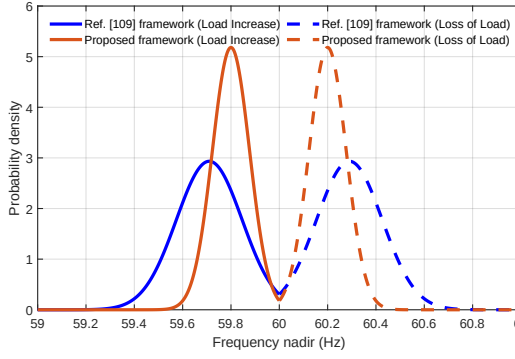
sensitive to frequency disturbances.



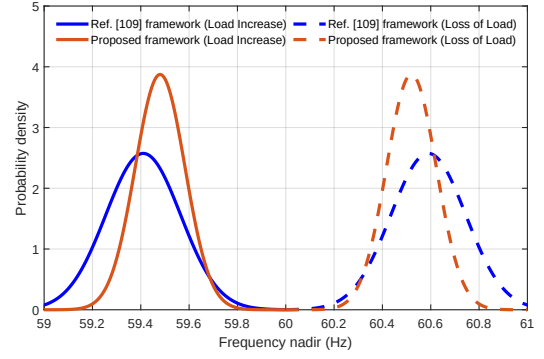
(a) RoCoF probability density across 24-hour variability for area R1.



(b) RoCoF probability density across 24-hour variability for area R2.



(c) Nadir probability density across 24-hour variability for area R1.



(d) Nadir probability density across 24-hour variability for area R2.

Figure 4.26: Probabilistic frequency-security assessment under time-varying load and VRES conditions.

The quantitative stochastic benchmarking results are summarized in Table 4.5. The metrics are computed using the definitions provided in Section 3.7.1.

For Area R1, the RoCoF distributions indicate that the proposed framework consistently reduces both the mean magnitude and the variability of RoCoF following disturbances. Specifically, under the load increase disturbance the benchmark approach produces an average RoCoF of approximately -0.589 Hz/s with a standard deviation of 0.110, whereas the proposed framework yields a mean value of -0.563 Hz/s and a standard deviation of 0.0771. This corresponds to a 4.41% reduction in the mean RoCoF magnitude and a 29.91% reduction in RoCoF variability. Symmetric behavior is observed for the load outage disturbance, where the

Table 4.5: Quantitative stochastic benchmarking of the proposed framework against the benchmark method

Area	Metric	Benchmark	Proposed	Improvement
R1	Mean RoCoF magnitude (Hz/s)	0.589	0.563	4.41% reduction
R1	RoCoF standard deviation	0.110	0.0771	29.91% reduction
R1	Mean frequency nadir (Hz)	59.70	59.80	+0.10 Hz
R1	Nadir standard deviation	0.136	0.077	43.38% reduction
R2	Mean RoCoF magnitude (Hz/s)	0.920	0.813	11.63% reduction
R2	RoCoF standard deviation	0.101	0.075	25.74% reduction
R2	Mean frequency nadir (Hz)	59.41	59.48	+0.07 Hz
R2	Nadir standard deviation	0.155	0.103	33.55% reduction

mean RoCoF magnitude decreases from 0.589 Hz/s to 0.563 Hz/s.

The frequency nadir results for Area R1 exhibit a comparable trend. During load increase events the average nadir improves from approximately 59.70 Hz under the benchmark method to 59.80 Hz with the proposed framework. This represents an absolute improvement of 0.10 Hz and a 33.33% reduction in the frequency drop from the nominal value of 60 Hz. In addition, the standard deviation decreases from 0.136 to 0.077, corresponding to a 43.38% reduction in nadir variability. Although the improvement in the mean nadir is relatively modest in this high-inertia region, the significant reduction in dispersion indicates that the proposed scheduling framework produces operating points whose frequency trajectories remain more consistently bounded across stochastic operating conditions.

More pronounced improvements are observed in Area R2, which represents the lower-inertia region of the system. In this area, the benchmark methodology

produces average RoCoF values approaching the operational security limit of 1 Hz/s. Under load increase disturbances, the benchmark RoCoF reaches an average magnitude of 0.920 Hz/s, whereas the proposed framework reduces this value to 0.813 Hz/s. This corresponds to an 11.63% reduction in mean RoCoF magnitude. The proposed framework also decreases the standard deviation from 0.101 to 0.075, equivalent to a 25.74% reduction in RoCoF variability. Similar improvements are observed for load outage disturbances.

The frequency nadir results further confirm this behavior. During generation deficit disturbances in Area R2, the average nadir increases from approximately 59.41 Hz under the benchmark approach to 59.48 Hz with the proposed framework. This represents an absolute improvement of 0.07 Hz and an 11.86% reduction in the frequency drop from the nominal value. The dispersion of the nadir distribution also decreases from 0.155 to 0.103, corresponding to a 33.55% reduction in nadir variability. This reduction in variability suggests that the spatial inertia constraints embedded in the proposed optimization lead to dispatch solutions that provide more reliable frequency support in regions where inertia scarcity would otherwise amplify frequency deviations.

The stochastic violation count provides an additional security indicator. Based on the Monte Carlo sample results, the number of RoCoF violations above the 1 Hz/s security limit is reported in Table 4.5 where applicable. If the sample-level violation count is not reported, the probability density functions in Figure 4.26 still show that the proposed framework shifts the RoCoF distributions farther from the 1 Hz/s security limit, particularly in Area R2.

These results directly address the validation question posed in Section 3.7.1. Under identical stochastic operating conditions and equal PV penetration levels, the proposed spatial inertia-aware optimization consistently produces dispatch solutions that yield lower RoCoF magnitudes, higher frequency nadirs during deficit disturbances, and reduced variability of frequency stability indices compared with the benchmark methodology. The improvement is particularly evident in the lower-

inertia area, where the proposed framework reduces the mean RoCoF magnitude by 11.63% and reduces RoCoF variability by 25.74%. These findings highlight the importance of explicitly accounting for the spatial distribution of inertia when scheduling synchronous generation in systems with high penetration of inertia-less VRES.

4.4.2 Consistency with Kinetic-Energy–Based Frequency Stability Metrics

The analytical consistency assessment described in Section 3.7.2 is now applied to evaluate whether the inertia allocation obtained from the proposed optimization framework reproduces the well-established relationship between system kinetic energy and frequency-stability indices. In particular, the RoCoF and frequency nadir predicted using the kinetic-energy-based analytical benchmark of [110] are compared with the corresponding indices obtained from time-domain simulations.

For the selected Pareto-optimal operating point (knee-point), the spatial inertia values $\{h_i\}$ produced by the NSGA-II optimization define the equivalent bus kinetic energy KE_i according to equation (3.90). A disturbance magnitude is then specified and mapped to each bus to obtain the reflected disturbance ΔP_i , following the procedure outlined in Section 3.3. The pair $(KE_i, \Delta P_i)$ therefore characterizes the effective inertial support and disturbance magnitude experienced at each bus.

Using these quantities, RoCoF and frequency nadir are first estimated analytically from the KE-based lookup relationships in [110]. The same disturbance conditions are subsequently reproduced in time-domain simulations by applying ΔP_i at each bus while maintaining the optimized inertia distribution. For every bus in the system, the resulting frequency trajectory is recorded and the RoCoF and nadir indices are extracted using equations (3.91) and (3.92).

The resulting dataset, therefore, contains; for each bus the, reflected disturbance magnitude, optimized bus inertia, analytically predicted RoCoF, nadir from the

KE mapping, and the corresponding indices obtained from dynamic simulation. These values are summarized in Table 4.6. Together, the tables provide a bus-by-bus comparison between the analytical KE-based predictions and the dynamic responses produced by the proposed spatial inertia allocation.

Figure 4.27 further evaluates the consistency of the two approaches using regression-based correlation analysis. Figure 4.27a compares the RoCoF obtained from dynamic simulation with the corresponding estimates from the KE-based mapping. The results show a clear monotonic relationship across all buses, with most points closely following the least-squares regression line and remaining within the 95% confidence interval band. This indicates that the RoCoF predicted by the analytical KE relationship remains consistent with the dynamic response produced by the optimized inertia distribution.

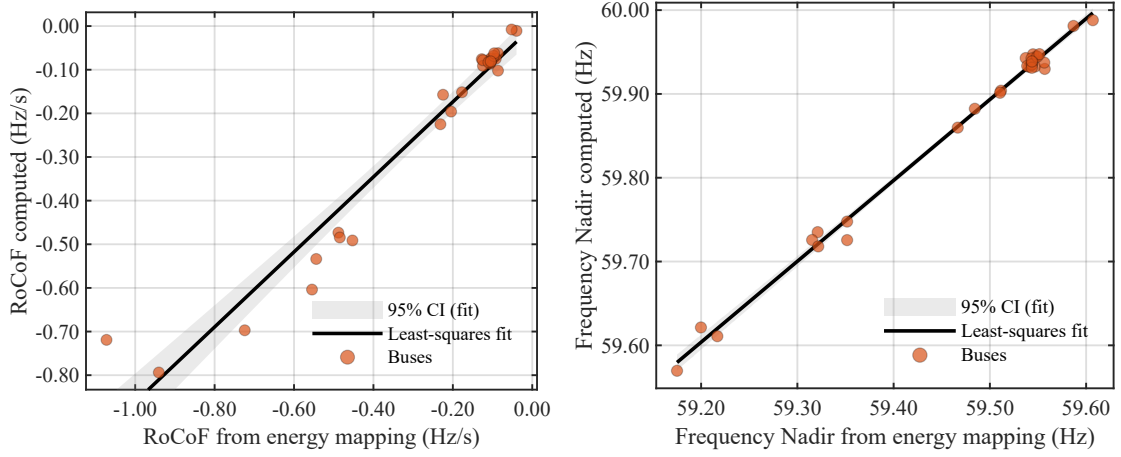
A similar comparison is shown for the frequency nadir in Figure 4.27b. The scatter distribution again follows an approximately-linear relationship between the analytical estimates and the simulated responses. As shown in the figure, the majority of bus-level results lie close to the regression line, and the confidence interval band remains narrow across the range of observed nadir values. These results indicate that the nadir predicted from the KE-based method captures the dominant relationship between available kinetic energy and frequency recovery following the disturbance.

To quantify the agreement observed in Figure 4.27, regression-based statistical metrics were computed between the KE-based estimates and the time-domain simulation results. For RoCoF, the coefficient of determination is $R^2 = 0.941$, with an RMSE of 0.064 Hz/s and an MAE of 0.048 Hz/s. This indicates strong agreement between the analytical KE-based RoCoF estimates and the simulated RoCoF values. The largest RoCoF deviation occurs at Bus 22, where the KE-based estimate is -1.118 Hz/s and the simulated value is -0.900 Hz/s, giving an absolute error of 0.218 Hz/s.

For the frequency nadir, the coefficient of determination is $R^2 = 0.955$, with an

Table 4.6: Comparison of frequency stability indicators obtained from time-domain simulation (TDS) and kinetic-energy (K.E.) mapping.

Bus No.	Reflected ΔP_i (MW)	Bus Inertia	RoCoF (Hz/s)		Nadir (Hz)	
			K.E. Mapping	TDS	K.E. Mapping	TDS
1	564	500	-0.339	-0.342	59.661	59.650
2	264	150	-0.554	-0.533	59.401	59.481
3	214	195	-0.292	-0.332	59.553	59.529
4	164	185	-0.272	-0.269	59.520	59.550
5	164	150	-0.300	-0.331	59.481	59.425
6	147	140	-0.338	-0.318	59.468	59.500
7	198	160	-0.337	-0.375	59.494	59.400
8	231	175	-0.425	-0.400	59.509	59.510
9	531	520	-0.206	-0.309	59.653	59.626
10	115	120	-0.338	-0.290	59.414	59.435
11	132	135	-0.234	-0.296	59.460	59.421
12	132	135	-0.197	-0.296	59.460	59.381
13	115	135	-0.177	-0.258	59.460	59.370
14	147	55	-0.774	-0.810	59.230	59.211
15	132	50	-0.826	-0.800	59.200	59.205
16	132	50	-0.733	-0.800	59.200	59.185
17	147	180	-0.263	-0.247	59.472	59.515
18	181	190	-0.210	-0.289	59.550	59.525
19	98	80	-0.378	-0.371	59.312	59.338
20	98	45	-0.590	-0.660	59.165	59.086
21	147	60	-0.646	-0.742	59.257	59.245
22	198	60	-1.118	-0.900	59.193	59.257
23	181	65	-0.838	-0.844	59.280	59.278
24	132	45	-0.924	-0.889	59.165	59.198
25	231	108	-0.690	-0.648	59.340	59.412
26	214	135	-0.455	-0.480	59.485	59.460
27	147	165	-0.304	-0.270	59.490	59.500
28	231	120	-0.548	-0.583	59.435	59.339
29	264	138	-0.608	-0.580	59.390	59.465



(a) RoCoF correlation between dynamic simulation and KE lookup. (b) Frequency nadir correlation between dynamic simulation and KE lookup.

Figure 4.27: Benchmark comparison of the proposed framework against the kinetic-energy-based method.

RMSE of 0.051 Hz and an MAE of 0.041 Hz. The higher R^2 value for nadir compared with RoCoF is expected because frequency nadir depends not only on the initial inertial response but also on subsequent dynamic effects, including damping and frequency recovery characteristics.

Taken together, the results demonstrate that the inertia allocations obtained from the proposed optimization framework preserve the fundamental kinetic-energy relationships that govern system frequency dynamics. Although small deviations are observed at a limited number of buses, these differences are expected due to network interactions and localized dynamic effects that are not explicitly represented in the simplified KE-based analytical formulation. Nevertheless, the strong agreement between analytical estimates and dynamic simulation results confirms that the proposed spatial inertia-aware framework produces operating points that remain analytically consistent with established kinetic-energy-based frequency-stability metrics.

4.5 Chapter Summary

This chapter presented and discussed the results obtained from the methodological framework developed in Chapter 3 have been presented and discussed. The results demonstrate how spatially resolved inertia modeling can support frequency-secure operation of power systems with high penetration of inertia-less VRES.

First, the offline spatial inertia estimation method was validated using the IEEE 9-bus test system. The analytical results, sensitivity analyses, RoCoF responses, and bus-frequency coherency results confirmed that the electrical-distance formulation captures generator–bus inertial coupling and reproduces the observed frequency-response behavior. Application to the IEEE 39-bus system further demonstrated the scalability of the method and revealed non-uniform spatial inertia zones shaped by network topology, generator placement, and electrical coupling.

Second, minimum bus-level inertia thresholds were derived using RoCoF-based frequency-security constraints under a credible load-change disturbance. The results showed that inertia requirements vary significantly across the network and depend on electrical proximity to the disturbance location. This confirms that inertia adequacy is a local network property rather than a purely system-wide quantity.

Third, the derived thresholds were embedded in the NSGA-II optimization framework to enable inertia-aware scheduling of synchronous generators and inertia-less VRES. The resulting Pareto fronts showed clear trade-offs between VRES penetration, transmission losses, and synchronous-generator commitment. The results also identified spatial inertia bottlenecks that limit further VRES integration, demonstrating the importance of enforcing inertia constraints at the bus level.

The validation studies further confirmed the effectiveness of the proposed framework. Dynamic simulations verified RoCoF and frequency-nadir performance, analytical benchmarking showed consistency with kinetic-energy-based frequency-security metrics, and stochastic validation demonstrated improved frequency performance

under time-varying load and PV conditions. These results also stress-tested the key modeling assumptions stated in Chapter 3, including steady-state-based inertia allocation, linearized coupling, deterministic scheduling, RoCoF-based thresholding, and inertia-less PV representation.

Overall, the findings validate the proposed analytical and optimization framework and show that spatially informed inertia management provides a viable strategy for supporting frequency stability in low-inertia power systems with high levels of converter-interfaced renewable generation.

CHAPTER 5

CONCLUSIONS AND RECOMMENDATIONS

5.1 Conclusions

This thesis developed and validated a spatial inertia-aware generation scheduling framework for power systems with high penetration of inertia-less VRES. The study was motivated by the limitation of aggregated inertia metrics, which may indicate adequate system-wide inertia while failing to reveal local frequency-security weaknesses. The results confirm that frequency security in low-inertia systems is strongly affected by the spatial distribution of synchronous generator inertia and its electrical coupling to network buses.

First, an offline analytical method was developed to estimate bus-level inertia using electrical-distance-based coupling coefficients derived from steady-state network conditions. Validation on the IEEE 9-bus system showed that the estimated inertia distribution was consistent with dynamic RoCoF behavior and bus-frequency coherency observed in time-domain simulations. The results demonstrated that buses electrically close to dominant synchronous generators exhibit stronger inertial support, while electrically remote or weakly supported buses experience higher RoCoF.

Second, minimum bus-level inertia thresholds were derived using RoCoF-based frequency-security requirements under a credible load-change disturbance. Application to the IEEE 39-bus system showed that inertia requirements are spatially

non-uniform and depend on the electrical proximity of each bus to the disturbance location. The results identified buses 14–16 and 20–24 as inertia-critical regions, confirming that local inertia adequacy cannot be fully captured using aggregate system inertia alone.

Third, the bus-level inertia thresholds were embedded into a multi-objective NSGA-II scheduling framework to maximize inertia-less VRES dispatch while considering transmission losses and synchronous generator commitment. The optimization results showed clear trade-offs between renewable penetration, network losses, and inertia adequacy. Under a strict RoCoF limit of 1 Hz/s, the system achieved approximately 82% maximum inertia-less VRES penetration, while relaxing the RoCoF limit to 3 Hz/s increased the achievable penetration to approximately 95%, although with higher transmission losses.

The validation studies further confirmed the effectiveness of the proposed framework. Deterministic time-varying load and PV studies showed that the framework adapts generator commitment to maintain spatial inertia adequacy under changing operating conditions. Probabilistic validation under Monte Carlo load and PV variations showed improved RoCoF and frequency-nadir performance compared with the benchmark method, particularly in lower-inertia regions. Analytical comparison with kinetic-energy-based frequency-security metrics also showed strong consistency between the proposed framework and established frequency-stability relationships.

Overall, the findings demonstrate that secure integration of high inertia-less VRES requires not only sufficient total system inertia, but also adequate spatial distribution of inertia across the network. By linking offline bus-level inertia estimation, RoCoF-based local inertia thresholds, and multi-objective generation scheduling, the proposed framework provides a planning-oriented approach for identifying inertia bottlenecks and determining frequency-secure renewable dispatch levels in low-inertia power systems.

5.2 Recommendations

This thesis established a spatially resolved framework for inertia estimation and inertia-aware operational scheduling in power systems with high penetration of inertia-less variable renewable energy sources. While the proposed methodology demonstrated analytical consistency and robust performance under deterministic and probabilistic validation scenarios, several extensions are required to broaden its theoretical generality and enhance its practical applicability.

First, the proposed spatial inertia-aware optimization framework can be extended to explicitly incorporate wind generation alongside solar PV resources. Modern wind turbines equipped with converter-interfaced controls are capable of providing synthetic inertia and fast frequency response through appropriate control strategies. Incorporating such capabilities into the optimization framework would allow coordinated scheduling of synchronous generators and inverter-based resources, enabling wind plants to contribute actively to frequency stability while maintaining high renewable penetration levels.

Second, the analytical spatial inertia distribution method developed in this work is formulated using steady-state network quantities and offline calculations. While this formulation is suitable for planning and day-ahead scheduling studies, future work should investigate approaches for determining the spatial distribution of inertia in real time using measurements from phasor measurement units (PMUs) or wide-area monitoring systems. Real-time estimation of bus-level inertia availability would enable system operators to dynamically assess inertia adequacy and adapt generation dispatch or reserve allocation in response to rapidly changing operating conditions, thereby supporting online frequency-security monitoring and preventive control actions. Achieving such real-time applicability will also require improvements in scalability and computational efficiency. Although the NSGA-II-based optimization demonstrated satisfactory convergence for benchmark systems, deployment in larger transmission networks or higher temporal resolution environments

will necessitate algorithmic refinements. Techniques such as parallel evaluation of candidate solutions, warm-start strategies that exploit temporal correlation between successive operating periods, and hybrid screening approaches combining DC power flow approximation with selective AC validation could significantly reduce computational burden. These developments would facilitate the transition of the proposed framework from offline analysis toward real-time or near-real-time operational decision support.

Third, validation on large-scale and real-world power system remains a critical next step. While benchmark systems provide controlled environments for methodological development, practical power systems exhibit heterogeneous load characteristics, geographically dispersed renewable resources, and complex protection schemes. Applying the spatial inertia framework to realistic network models would enable assessment of sensitivity to load-frequency dependency, regional inertia clustering, and grid-code constraints. Such studies are necessary to translate spatial inertia concepts into industry-relevant planning and decision-support tools.

Fourth, another potential extension of this work is to investigate the derivation of minimum inertia thresholds under generator outage disturbances in addition to load change events. In the present study, load variations were used to represent active-power imbalances so that the disturbance magnitude remains independent of synchronous generator dispatch, which is particularly important when evaluating high penetration levels of inertia-less VRES. However, generator outages introduce both a power deficit and the simultaneous loss of synchronous inertia, which can influence frequency dynamics differently from load-driven disturbances. Future research may therefore extend the proposed spatial inertia-aware framework to incorporate generator outage scenarios and examine how the combined effects of power imbalance and inertia loss influence the resulting minimum inertia requirements and renewable integration limits.

Fifth, a practical industrial implementation pathway should be developed for utility planning and control-centre environments. The proposed framework can be imple-

mented as an offline decision-support tool for day-ahead or intra-day scheduling. In such implementation, network topology, generator parameters, inertia constants, operating limits, load forecasts, and renewable availability forecasts can be obtained from the energy management system, planning database, SCADA system, and state estimator. PMU data can be used to validate local frequency behavior, RoCoF response, and generator-bus coherency. The expected outputs would include bus-level inertia maps, inertia-security margins, critical inertia bottleneck regions, Pareto-optimal dispatch options, and identification of synchronous generators that are most important for maintaining local frequency security. A recommended practical implementation pathway is summarized in Table 5.1.

Table 5.1: Recommended practical implementation pathway for the proposed spatial inertia-aware scheduling framework

Implementation Stage	Required Utility Data	Expected Output	Operational
Network and operating-point preparation	Network topology, line and transformer parameters, generator ratings, generator inertia constants, SCADA/state-estimator operating point	Solved AC power-flow case representing the current or forecasted operating condition	
Spatial inertia estimation	Online synchronous generator set, generator inertia constants, bus voltages, line flows, and reduced-network coupling	Bus-level inertia distribution and generator-to-bus inertial contribution map	
RoCoF-based inertia thresholding	Credible disturbance magnitude, disturbance location, and selected RoCoF security criterion	Minimum bus-level inertia thresholds and inertia-security margins	
Inertia-aware scheduling	Generator limits, VRES availability, load forecast, bus-level inertia thresholds, and voltage limits	Pareto-optimal dispatch solutions balancing VRES penetration, losses, SG commitment, and inertia adequacy	
Operational interpretation	Optimized dispatch solutions and inertia-security margins	Identification of inertia bottleneck regions, critical SGs, and feasible renewable dispatch levels	

Collectively, these research directions aim to extend the spatial inertia paradigm from an analytically grounded planning framework toward a comprehensive operational methodology for converter-dominated power systems. By incorporating nonlinear dynamics, programmable inertia resources, scalable optimization architectures, and real-system validation, future work can further enhance the robustness, adaptability, and practical relevance of inertia-aware renewable integration.

5.3 Final remarks

The key contribution of this thesis is to demonstrate that frequency stability in low-inertia grids is governed not by total system inertia but by its *spatial distribution*. By bringing together accurate offline inertia estimation, analytically rigorous minimum inertia thresholds, and a powerful multi-objective optimization framework, this work provides a unified approach for operational planning in renewable-dominated grids. The tools and insights developed herein form a strong foundation for both academic research and real-world implementation in the transition toward sustainable, renewable dominated power systems.

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APPENDIX A

TEST SYSTEM DATA

IEEE 9 bus test system

Line	from bus	to bus	r[p.u]	x [p.u]
1	4	5	0.0100	0.0850
2	4	6	0.0170	0.0920
3	5	7	0.0320	0.1610
4	6	9	0.0390	0.1700
5	7	8	0.0085	0.0720
6	8	9	0.0119	0.0619
Transformer	1	4	0.0020	0.0576
Transformer	2	7	0.0023	0.0625
Transformer	3	9	0.0021	0.0586

IEEE 9 bus branch data on 100 MVA system base

Load	Bus Number	P [MW]	Q [MVar]
Load A	5	125	50
Load B	6	90	30
Load C	8	100	35

Load demand on IEEE 9 Bus test system

Generator	Bus Number	P [MW]	V [p.u]
G1	1	—	1.040
G2	2	163	1.025
G3	3	85	1.025

IEEE 9 bus SGs dispatch and voltage setpoints

IEEE 39 bus test system

IEEE 39-Bus System Load Data

Load	Bus	P (MW)	Q (MVar)	Load	Bus	P (MW)	Q (MVar)
Load 03	03	322.0	2.4	Load 23	23	247.5	84.6
Load 04	04	500.0	184.0	Load 24	24	308.6	-92.2
Load 07	07	233.8	84.0	Load 25	25	224.0	47.2
Load 08	08	522.0	176.0	Load 26	26	139.0	17.0
Load 12	12	7.5	88.0	Load 27	27	281.0	75.5
Load 15	15	320.0	153.0	Load 28	28	206.0	27.6
Load 16	16	329.0	32.3	Load 29	29	283.5	26.9
Load 18	18	158.0	30.0	Load 31	31	9.2	4.6
Load 20	20	628.0	103.0	Load 39	39	1104.0	250.0
Load 21	21	274.0	115.0				

IEEE 39-Bus Generator Dispatch Data

Generator	Bus	Bus Type	P (MW)	V (p.u.)
G 01	39	PV	1000.0	1.0300
G 02	31	Slack	N.A.	0.9820
G 03	32	PV	650.0	0.9831
G 04	33	PV	632.0	0.9972
G 05	34	PV	508.0	1.0123
G 06	35	PV	650.0	1.0493
G 07	36	PV	560.0	1.0635
G 08	37	PV	540.0	1.0278
G 09	38	PV	830.0	1.0265
G 10	30	PV	250.0	1.0475

Transmission Line Data of IEEE 39-Bus System (Base: 100 MVA, 60 Hz)

Line	From Bus	To Bus	Length	R (p.u.)	X (p.u.)	B (p.u.)
Line 01–02	Bus 01	Bus 02	163.06	0.0035	0.0411	0.6987
Line 01–39	Bus 01	Bus 39	99.19	0.0010	0.0250	0.7500
Line 02–03	Bus 02	Bus 03	59.91	0.0013	0.0151	0.2572
Line 02–25	Bus 02	Bus 25	34.12	0.0070	0.0086	0.1460
Line 03–04	Bus 03	Bus 04	84.51	0.0013	0.0213	0.2214
Line 03–18	Bus 03	Bus 18	52.77	0.0011	0.0133	0.2138
Line 04–05	Bus 04	Bus 05	50.78	0.0008	0.0128	0.1342
Line 04–14	Bus 04	Bus 14	51.18	0.0008	0.0129	0.1382
Line 05–06	Bus 05	Bus 06	10.32	0.0002	0.0026	0.0434
Line 05–08	Bus 05	Bus 08	44.44	0.0008	0.0112	0.1476
Line 06–07	Bus 06	Bus 07	36.50	0.0006	0.0092	0.1130
Line 06–11	Bus 06	Bus 11	32.53	0.0007	0.0082	0.1389
Line 07–08	Bus 07	Bus 08	18.25	0.0004	0.0046	0.0780
Line 08–09	Bus 08	Bus 09	144.02	0.0023	0.0363	0.3840
Line 09–39	Bus 09	Bus 39	99.19	0.0010	0.0250	1.2000
Line 10–11	Bus 10	Bus 11	17.06	0.0004	0.0043	0.0729
Line 10–13	Bus 10	Bus 13	17.06	0.0004	0.0043	0.0729
Line 13–14	Bus 13	Bus 14	40.07	0.0009	0.0101	0.1723
Line 14–15	Bus 14	Bus 15	86.09	0.0018	0.0217	0.3660
Line 15–16	Bus 15	Bus 16	37.29	0.0009	0.0094	0.1710
Line 16–17	Bus 16	Bus 17	35.31	0.0007	0.0089	0.1342
Line 16–19	Bus 16	Bus 19	77.37	0.0016	0.0195	0.3040
Line 16–21	Bus 16	Bus 21	53.56	0.0008	0.0135	0.2548
Line 16–24	Bus 16	Bus 24	23.41	0.0003	0.0059	0.0680
Line 17–18	Bus 17	Bus 18	32.53	0.0007	0.0082	0.1319
Line 17–27	Bus 17	Bus 27	68.74	0.0013	0.0173	0.3240
Line 21–22	Bus 21	Bus 22	55.55	0.0008	0.0140	0.2565
Line 22–23	Bus 22	Bus 23	38.09	0.0006	0.0096	0.1846
Line 23–24	Bus 23	Bus 24	138.86	0.0022	0.0350	0.3610
Line 25–26	Bus 25	Bus 26	128.15	0.0032	0.0323	0.5130
Line 26–27	Bus 26	Bus 27	58.32	0.0014	0.0147	0.2396
Line 26–28	Bus 26	Bus 28	188.06	0.0043	0.0474	0.7802
Line 26–29	Bus 26	Bus 29	247.97	0.0057	0.0625	1.0290
Line 28–29	Bus 28	Bus 29	59.91	0.0014	0.0151	0.2490

APPENDIX B

LIST OF PUBLICATIONS

This appendix lists peer-reviewed journal articles arising directly from the research presented in this thesis. The publications were published in peer reviewed journals.

1. E. G. Swetala, K. K. Kaberere, and J. L. Munda, “Optimal inertia-less variable renewable energy sources penetration and power loss minimization considering inertia distribution constraint,” *IEEE Access*, vol. 13, pp. 210307–210325, 2025, doi: 10.1109/ACCESS.2025.3643143.
2. Emmanuel Greddy Swetala, Keren K. Kaberere, Josiah Lange Munda, “Offline estimation of spatial inertia distribution in power systems with high renewable penetration,” *e-Prime – Advances in Electrical Engineering, Electronics and Energy*, Volume 11, 2025, 100923, ISSN 2772-6711, <https://doi.org/10.1016/j.prime.2025.100923>.

The core methodologies, models, and results presented in this thesis are primarily derived from the published journal articles listed above, with additional extensions, validations, and unifying analysis developed specifically for the thesis.

APPENDIX C

MULTI-AREA DYNAMIC MODEL AND FREQUENCY STABILITY INDICES

This appendix presents a multi-area frequency-response model of the power system. The model is provided for completeness and is not used directly in the offline inertia estimation or the NSGA-II optimization; rather, it supports the interpretation of frequency-stability indices (RoCoF and frequency nadir) employed during dynamic validation.

Multi-Area Frequency Dynamics

Following a disturbance at bus (or area) i , the frequency deviation is governed by the area swing equation

$$\frac{d\Delta f_i(t)}{dt} = \frac{1}{2H_i} (-D_i \Delta f_i(t) + \Delta P_{D,i}(t)), \quad (\text{C.1})$$

where $\Delta f_i(t)$ is the frequency deviation at bus/area i (Hz), H_i is the equivalent inertia associated with bus/area i (s), D_i is the load damping coefficient at bus/area i (pu/Hz), and $\Delta P_{D,i}(t)$ is the net active-power disturbance at bus/area i (pu), representing the post-contingency imbalance.

Equivalent Inertia from Electrical-Distance Contributions

Consistent with the offline inertia estimation framework in Chapter 3, the equivalent inertia observable at bus/area i can be expressed in terms of synchronous generator

contributions weighted by electrical distances as

$$H_i = \max_{\forall j \in i} \left(\sum_{k=1}^{n_g} \frac{d_{kj}^2}{H_k} \right)^{-1}, \quad (\text{C.2})$$

where n_g is the number of online synchronous generators, H_k is the inertia constant of generator k (s), and d_{kj} is the electrical distance between generator k and bus j (dimensionless, computed from the reduced network representation). The operator $\max(\cdot)$ reflects selection of the dominant electrically coupled contribution over buses j associated with area i .

Turbine–Governor Dynamics of Synchronous Generators

The turbine and governor dynamics associated with synchronous generation are represented using standard first-order blocks. The turbine dynamics are given by

$$\frac{d\Delta P_{sg,i}(t)}{dt} = -\frac{1}{\tau_t} \Delta P_{sg,i}(t) + \frac{1}{\tau_t} \Delta P_{G,i}(t), \quad (\text{C.3})$$

and the governor dynamics are modeled as

$$\frac{d\Delta P_{G,i}(t)}{dt} = -\frac{1}{\tau_g} \Delta P_{G,i}(t) + \frac{1}{\tau_g} (\Delta P_{P,i}(t) + \Delta P_{S,i}(t)), \quad (\text{C.4})$$

where $\Delta P_{sg,i}(t)$ is the turbine mechanical power deviation (pu), $\Delta P_{G,i}(t)$ is the governor output deviation (pu), τ_t is the turbine time constant (s), and τ_g is the governor time constant (s).

Primary and Secondary Frequency Control

Primary (droop) control is represented as

$$\frac{d\Delta P_{P,i}(t)}{dt} = -\frac{1}{R_i} \Delta f_i(t), \quad (\text{C.5})$$

where $\Delta P_{P,i}(t)$ is the primary control power contribution (pu) and R_i is the droop coefficient (Hz/pu).

Secondary frequency control (automatic generation control, AGC) is represented as

$$\frac{d\Delta P_{S,i}(t)}{dt} = -\frac{K_{S,i}}{T_{S,i}} \left(\sum_j \Delta P_{ij} + \beta_i \Delta f_i(t) \right), \quad (\text{C.6})$$

where $\Delta P_{S,i}(t)$ is the secondary control contribution (pu), $K_{S,i}$ is the AGC gain, $T_{S,i}$ is the AGC time constant (s), β_i is the frequency bias factor, and ΔP_{ij} is the tie-line power deviation (pu) between areas i and j .

The tie-line power deviation is given by

$$\Delta P_{ij} = T_{ij}(\Delta \delta_i - \Delta \delta_j), \quad (\text{C.7})$$

where T_{ij} is the synchronizing torque coefficient between areas i and j (pu/rad), and $\Delta \delta_i$ and $\Delta \delta_j$ are the voltage-angle deviations (rad) of areas i and j , respectively.

Voltage-angle deviations evolve according to

$$\frac{d\Delta \delta_i}{dt} = 2\pi \Delta f_i(t), \quad (\text{C.8})$$

where $\Delta \delta_i(t)$ is the voltage-angle deviation (rad).

State-Space Formulation

Collecting all states for area i , the dynamics can be written in state-space form as

$$\dot{X}_i = A_i X_i + B_i \Delta P_{D,i}, \quad (\text{C.9})$$

with the state vector

$$X_i = \begin{bmatrix} \Delta f_i & \Delta P_{sg,i} & \Delta P_{G,i} & \Delta P_{S,i} & \Delta \delta_i \end{bmatrix}^T, \quad (\text{C.10})$$

where A_i and B_i are the state and input matrices of the linearized multi-area model around the operating point.

The output frequency deviation is

$$\Delta f_i = C_i X_i, \quad D_i = 0, \quad (\text{C.11})$$

where C_i is the output matrix selecting the frequency state.

Laplace-Domain Representation

Applying the Laplace transform yields

$$sX_i(s) = A_i X_i(s) + B_i \Delta P_{D,i}(s), \quad (\text{C.12})$$

$$\Delta f_i(s) = C_i X_i(s). \quad (\text{C.13})$$

Solving for $X_i(s)$ gives the transfer-function relationship

$$\Delta f_i(s) = [C_i(sI - A_i)^{-1}B_i] \Delta P_{D,i}(s), \quad (\text{C.14})$$

which defines the transfer function

$$\Gamma_i(s) = C_i(sI - A_i)^{-1}B_i. \quad (\text{C.15})$$

Frequency Response in the Time Domain

In the time domain, the frequency deviation can be written as the convolution

$$\Delta f_i(t) = \int_0^t \gamma_i(\tau) \Delta P_{D,i}(t - \tau) d\tau, \quad (\text{C.16})$$

where $\gamma_i(t) = \mathcal{L}^{-1}\{\Gamma_i(s)\}$ is the impulse response corresponding to $\Gamma_i(s)$.

RoCoF Derivation

The initial rate of change of frequency (RoCoF) is obtained from (C.1) evaluated

at $t = 0^+$ as

$$\text{RoCoF}(0^+) = \left. \frac{d\Delta f_i(t)}{dt} \right|_{t=0^+} = \frac{\Delta P_{D,i}(0^+)}{2H_i}, \quad (\text{C.17})$$

where $\text{RoCoF}(0^+)$ is the initial RoCoF (Hz/s) immediately after the disturbance.

Frequency Nadir

The frequency nadir at bus/area i is defined as

$$f_{\text{nadir},i} = \min_{t \geq 0} (f_{i,0} + \Delta f_i(t)), \quad (\text{C.18})$$

where $f_{i,0}$ is the nominal (pre-disturbance) frequency (Hz).

APPENDIX D

MATLAB CODES

The simulation codes were organized into logical directories to clearly separate estimation, optimization, and validation functionalities. The high-level folder structure is summarized below:

```
Simulation_Framework/  
|  
|  
|-- inertia_estimation/  
|   |-- run_main_simulation.m  
|   |-- kron_reduction.m  
|   |-- compute_electrical_distance.m  
|   |-- estimate_bus_inertia.m  
|  
|-- optimization/  
|   |-- nsga2_main.m  
|   |-- fitness_function.m  
|   |-- constraint_handler.m  
|  
|-- dynamic_simulation/  
|   |-- multi_area_dynamic_model.m  
|   |-- frequency_indices.m
```

D.1 MATLAB Script for Inertia Estimation

```
1 %% =====
2 %   Offline Inertia Estimation
3 %   Network Topology data from MATPOWER Version 8.1
4 %% =====
5
6 clear; clc; close all;
7 %% =====
8 %% 1) INPUT SECTION
9 %% =====
10
11 caseFile = "case39.m";           % "case9.m" or "case39.m"
12 Mpc = loadcase(caseFile);
13
14 % Machine parameters provided by a function (case-dependent)
15 [baseMVA, H, Xd_prime] = get_machine_params(caseFile, Mpc);
16
17 bf = Mpc.branch(:,1);
18 bt = Mpc.branch(:,2);
19
20 nbus = size(Mpc.bus,1);
21 ng    = size(Mpc.gen,1);
22 nl    = size(Mpc.branch,1);
23
24 gen_bus = Mpc.gen(:,1);          % generator terminal bus indices
25
26 Z = Mpc.branch(:,3) + 1j*Mpc.branch(:,4);
27 y = 1 ./ Z;
28
29 %% =====
30 %% 2) PROCESSING SECTION
```

```

31 %% =====
32
33 %% -----
34 %% 2.1 Form network Y (series-only)
35 %% -----
36 Y = zeros(nbus, nbus);
37
38 for k = 1:n1
39     i = bf(k); j = bt(k);
40     Y(i,j) = Y(i,j) - y(k);
41     Y(j,i) = Y(i,j);
42 end
43
44 for i = 1:nbus
45     Y(i,i) = -sum(Y(i,:));
46 end
47
48 %% -----
49 %% 2.2 Run power flow
50 %% -----
51 sol = runpf(Mpc);
52
53 Vmag = sol.bus(:,8);
54 Vang = sol.bus(:,9) * pi/180;
55 V = Vmag .* (cos(Vang) + 1j*sin(Vang));
56
57 %% -----
58 %% 2.3 Add load admittances (equivalent Yload)
59 %% -----
60 Sload_pu = (Mpc.bus(:,3)/baseMVA) +
61             1j*(Mpc.bus(:,4)/baseMVA);
61 Y_load    = conj(Sload_pu) ./ (V .* V);

```

```

62
63 Ybus = Y + diag(Y_load);
64
65 %% -----
66 %% 2.4 Augment Ybus with generator internal admittances
67 %%     Augmented node order:
68 %%         internal gen nodes: 1..ng
69 %%         network buses:      ng+1 .. ng+nbus
70 %% -----
71 A11 = -1j * diag(1 ./ Xd_prime);
72 A22 = Ybus;
73
74 A12 = zeros(ng, nbus);
75 for g = 1:ng
76     b = gen_bus(g);
77     A12(g, b) = -A11(g,g);
78 end
79 A21 = A12.';
80
81 Y_aug = [A11, A12;
82          A21, A22];
83
84 %% -----
85 %% 2.5 Define essential buses = all PQ (load) buses
86 %% -----
87 ess_bus = find(Mpc.bus(:,2) == 1)' % MATPOWER: type 1 = PQ
88 ess_bus_aug = ng + ess_bus;
89
90 %% -----
91 %% 2.6 Kron reduction
92 %% -----
93 keep_nodes = [1:ng, ess_bus_aug];

```

```

94 all_nodes = 1:(ng + nbus);
95 elim_nodes = setdiff(all_nodes, keep_nodes);
96
97 Ykk = Y_aug(keep_nodes, keep_nodes);
98 Yke = Y_aug(keep_nodes, elim_nodes);
99 Yek = Y_aug(elim_nodes, keep_nodes);
100 Yee = Y_aug(elim_nodes, elim_nodes);
101
102 Y_red = Ykk - Yke * (Yee \ Yek);
103
104 nb_keep = length(ess_bus);
105 Ynew = Y_red(ng+1:end, 1:ng);           % (nb_keep x ng)
106
107 %% -----
108 %% 2.7 Electrical coupling terms
109 %% -----
110
111 % 2.7.1 Branch currents using Y off-diagonals
112 I = zeros(nbus, nbus);
113 for k = 1:nl
114     i = bf(k); j = bt(k);
115     I(i,j) = (V(i) - V(j)) * Y(i,j);
116     I(j,i) = -I(i,j);
117 end
118
119 % 2.7.2 Generator currents at their terminal buses
120 Ig = zeros(ng,1);
121 for g = 1:ng
122     b = gen_bus(g);
123     Ig(g) = sum(I(:, b));
124 end
125

```

```

126 % 2.7.3 Internal EMF behind Xd'
127 Vg = V(gen_bus);
128 Eg = Vg + Ig .* 1j .* Xd_prime;
129
130 % 2.7.4 Normalized current sensitivity CI (ng x nb_keep)
131 CI = abs(Eg) .* abs(Ynew. '); % (ng x nb_keep)
132 CI = CI ./ (sum(CI,1) + eps); % normalize per bus
133 % 2.7.5 Normalized synchronizing coefficient Kij
134 del_i = angle(Eg);
135 del_j = angle(V(ess_bus));
136
137 cosd = zeros(ng, nb_keep);
138 for i = 1:ng
139     for j = 1:nb_keep
140         cosd(i,j) = abs(Eg(i)) * cos(del_j(j) - del_i(i));
141     end
142 end
143
144 Kij = cosd .* abs(Ynew. '); % (ng x nb_keep)
145 Kij = Kij ./ (sum(Kij,1) + eps); % normalize per bus
146
147 %% -----
148 %% 2.8 Inertia estimation at essential buses (PQ buses)
149 %% -----
150 h_ess = 1 ./ sum((Kij .* CI) ./ (H + eps), 1); % 1 x nb_keep
151
152 %% =====
153 %% 3) OUTPUT SECTION
154 %% =====
155
156 figure;
157 bar(h_ess); grid on;

```

```

158 xlabel('PQ bus index (position in ess_bus list)');
159 ylabel('Estimated inertia (s)');
160 title(sprintf('Estimated Inertia at PQ Buses (%s)',
    caseFile));
161
162 disp('Essential buses = PQ buses (MATPOWER numbering):');
163 disp(ess_bus);
164
165 disp('Estimated inertia values (s) in the same order:');
166 disp(h_ess);
167
168 % Optional: table mapping bus number -> inertia
169 T = table(ess_bus(:), h_ess(:), 'VariableNames', {'Bus',
    'EstimatedInertia_s'});
170 disp(T);

```

MATLAB Function for Machine Parameters

```

1 function [baseMVA, H, Xd_prime] =
    get_machine_params(caseFile, Mpc)
2 %Returns baseMVA, SGs inertia (H) on system base, and Xd'.
3
4 % baseMVA : system base MVA (from MATPOWER case)
5 % Hg      : (ng x 1) inertia [s] on machine's base (baseMVA)
6 % Xd_prime : (ng x 1) transient reactance in p.u.
7
8 baseMVA = Mpc.baseMVA;
9 ng = size(Mpc.gen,1);
10
11 caseKey = lower(string(caseFile));
12 switch caseKey
13     case {"case9", "case9.m"}

```

```

14      % IEEE 9-bus
15      Hg = [4.775; 1.770125; 1.175125];      % s
16      Sg = [247.5; 192; 128];                % MVA
17      H = (Sg .* Hg) / baseMVA;              % convert to
                                         system base
18
19      Xd_prime = [0.23; 0.23; 0.15];
20
21      case {"case39", "case39.m"}
22          % IEEE 39-bus
23          Hg = [5.000; 4.329; 4.475; 3.575; 4.333; 4.350;
24                3.771; 3.471; 3.450; 4.200];
25          Sg = [10000; 700; 800; 800; 300; 800; 700; 700;
26                1000; 1000];
27          H = (Sg .* Hg) / baseMVA;
28
29          Xd_prime = [0.006; 0.070; 0.053; 0.044; 0.132;
30                      0.050; 0.049; 0.057; 0.057; 0.031]
31
32      otherwise
33          warning('No specific machine data for "%s". Using
34                  defaults.', caseFile);
35
36          Hg = 4.0*ones(ng,1);
37          Sg = baseMVA*ones(ng,1);
38          H = (Sg .* Hg) / baseMVA;
39          Xd_prime = 0.20*ones(ng,1);
40
41      end
42
43      % Sanity check
44      if numel(H) ~= ng || numel(Xd_prime) ~= ng

```

```

40         error('Machine parameter vectors must match ng = %d
           generators.', ng);
41     end
42
43 end

```

D.2 optimization

Main Driver

```

1 function run_nsga2()
2 %NSGA-II for maximizing inertia-less VRES.
3 % Produces Pareto set over: [#SGs, Loss, -Renew]
4 % Saves results into results.mat
5
6 %% -----
7 %% INPUT / CONFIGURATION
8 %% -----
9 define_constants; %#ok<*DEFNU>
10
11 caseName = 'case39';
12 mpc = loadcase(caseName);
13 [mpc, renew_buses, renew_max] = add_renewables(mpc);
14
15 % Slack selection (edit as needed)
16 slack_bus = mpc.bus(mpc.bus(:, BUS_TYPE) == REF, BUS_I);
17 slack_gen_idx = find(mpc.gen(:, GEN_BUS) == slack_bus,
18                     1);
19
20 % Limits

```

```

20 [Pmin_non_slack, Pmax_non_slack, non_slack_idx] =
    get_non_slack_limits(mpc, slack_idx);
21 Pmin_slack = 0.2 * mpc.gen(slack_idx, 9);
22 Pmax_slack =          mpc.gen(slack_idx, 9);
23
24 % Encoding sizes
25 nSGs      = numel(non_slack_idx);
26 nRenew    = numel(renew_buses);
27 nVars     = 2*nSGs + nRenew;
28
29 % Decision bounds
30 lb = [zeros(1,nSGs), Pmin_non_slack, zeros(1,nRenew)];
31 ub = [ones(1,nSGs),  Pmax_non_slack, renew_max];
32 intCon = 1:nSGs; % commitment bits are integers (0/1)
33
34 %% -----
35 %% NSGA-II OPTIONS
36 %% -----
37 opts = optimoptions('gamultiobj', ...
38     'PopulationSize',    50, ...
39     'MaxGenerations',    300, ...
40     'CrossoverFraction', 0.9, ...
41     'MutationProbability', 0.05, ...
42     'CrossoverDistributionIndex' 20, ...
43     'MutationDistributionIndex', 20, ...
44     'FunctionTolerance', 1e-4, ...
45     'PlotFcn',           {@gaplotpareto}, ...
46     'Display',           'iter');
47
48 %% -----
49 %% RUN NSGA-II
50 %% -----

```

```

51 objHandle = @(chrom) fitness_function( ...
52     chrom, mpc, renew_buses, renew_max, ...
53     slack_idx, Pmin_slack, Pmax_slack, non_slack_idx);
54
55 [x, Fval, exitflag, Output, Population, Score] =
    gamultiobj( ...
56     objHandle, nVars, [], [], [], [], lb, ub, [],
        intCon, opts);
57
58 %#ok<NASGU>
59 pareto_solutions = x;
60 objectives       = Fval;
61
62 %% -----
63 %% POST-PROCESS PARETO SOLUTIONS
64 %% -----
65 nSolutions = size(pareto_solutions, 1);
66
67 power_loss  = nan(nSolutions, 1);
68 penetration = nan(nSolutions, 1);
69 num_SGs     = nan(nSolutions, 1);
70
71 % Store inertia-related signals as cell arrays to avoid
dimension mismatches.
72 hmin_cell    = cell(nSolutions,1);
73 hcomputed_cell = cell(nSolutions,1);
74
75 for i = 1:nSolutions
76     chrom = pareto_solutions(i,:);
77
78     % Apply chromosome and get metrics from the
constraint handler

```

```

79         out = constraint_handler( ...
80             chrom, mpc, renew_buses, renew_max, slack_idx,
81             ...
82             Pmin_slack, Pmax_slack, non_slack_idx);
83
84         if ~out.success
85             continue;
86         end
87
88         power_loss(i)    = out.total_loss;
89         penetration(i)   = out.total_renew;
90         num_SGs(i)       = out.num_committed;
91
92         hmin_cell{i}     = out.h_min;
93         hcomputed_cell{i} = out.h;
94     end
95
96     %% -----
97     %% SAVE RESULTS
98     %% -----
99     save('results.mat', ...
100         'pareto_solutions', 'objectives', ...
101         'power_loss', 'penetration', 'num_SGs', ...
102         'hcomputed_cell', 'hmin_cell');
103
104     fprintf('Results saved to results.mat\n');
105 end

```

Objective Vector

```

1 function f = fitness_function(chromosome, mpc, renew_buses,
    renew_max, slack_idx, Pmin_slack, Pmax_slack,

```

```

    non_slack_idx)
2
3 %FITNESS_FUNCTION Vector objective for NSGA-II.
4
5 % Returns f = [f1, f2, f3] with penalties added.
6
7 % f1: scaled number of committed SGs (minimize)
8 % f2: scaled real power loss (minimize)
9 % f3: negative scaled renewables (minimize -> maximize
   renewables)
10
11 out = constraint_handler( ...
12     chromosome, mpc, renew_buses, renew_max, slack_idx,
13     ...
14     Pmin_slack, Pmax_slack, non_slack_idx);
15
16 if ~out.success
17     f = [1e3, 1e3, 1e3];
18     return;
19 end
20
21 f1_core = 2*out.num_committed/ .....
22     (out.num_committed + Base_Nsg);
23 f2_core = 2*out.total_loss/ .....
24     (out.total_loss + BasePloss);
25 f3_core = -2*out.total_renew /.....
26     (out.total_renew + TotalLoad);
27
28 % --- Total penalty (scalar) ---
29 penalty_total = out.penalty_slack + .....
30     out.penalty_voltage + .....

```

```

31         out.penalty_inertia;
32
33     f = [f1_core, f2_core, f3_core] + penalty_total;
34
35 end

```

Constraint and Case Handler

```

1 function out = constraint_handler(chromosome, mpc,
    renew_buses, renew_max, slack_idx, Pmin_slack,
    Pmax_slack, non_slack_idx)
2 %CONSTRAINT_HANDLER
3 % 1) Decode chromosome
4 % 2) Update MATPOWER case (commitment, SG dispatch,
    renewables)
5 % 3) Run PF
6 % 4) Compute physical metrics + penalties
7
8 % Returns struct out with:
9 %     success, results, mpc1
10 %     num_committed, total_loss, total_renew
11 %     penalty_slack, penalty_voltage, penalty_inertia
12 %     h, h_min
13
14 % -----
15 % Decode chromosome
16 % -----
17 nSGs    = numel(non_slack_idx);
18 nRenew  = numel(renew_buses);
19
20 commitment    = chromosome(1:nSGs);
21 sg_dispatch    = chromosome(nSGs+1:2*nSGs);

```

```

22     renew_dispatch = chromosome(2*nSGs+1:end);
23
24     % Enforce binary commitment
25     commitment = round(min(max(commitment, 0), 1));
26
27     % -----
28     % Apply chromosome to case
29     % -----
30     [mpc1, renew_dispatch] = update_case( ...
31         mpc, commitment, sg_dispatch, renew_dispatch, ...
32         renew_buses, renew_max, slack_idx, non_slack_idx);
33
34     % -----
35     % Run power flow
36     % -----
37     [results, success] = runpf(mpc1);
38
39     out = struct();
40     out.success = logical(success);
41     out.results = results;
42     out.mpc1    = mpc1;
43     out.h       = [];
44     out.h_min   = [];
45
46     % Default penalties
47     out.penalty_slack    = 1000;
48     out.penalty_voltage = 1000;
49     out.penalty_inertia = 1000;
50
51     if ~success
52         return;
53     end

```

```

54
55     % -----
56     % Physical metrics
57     % -----
58     out.num_committed = 1 + sum(commitment); % include slack
59     out.total_loss     = sum(results.gen(:,2)) -
        sum(results.bus(:,3));
60     out.total_renew    = sum(renew_dispatch);
61
62     P_slack = results.gen(slack_idx, 2);
63     if P_slack < Pmin_slack
64         out.penalty_slack = out.penalty_slack +
            1000*(Pmin_slack - P_slack);
65     elseif P_slack > Pmax_slack
66         out.penalty_slack = out.penalty_slack +
            1000*(P_slack - Pmax_slack);
67     end
68
69     Vmag = results.bus(:,8);
70     viol_low  = sum(max(0, 0.90 - Vmag));
71     viol_high = sum(max(0, Vmag - 1.05));
72     out.penalty_voltage = 1000 * (viol_low + viol_high);
73
74     out = compute_inertia_metrics(out, mpc1, results);
75
76 end
77
78 % =====
79 % Local helper: update_case
80 % =====
81 function [mpc1, renew_dispatch] = update_case(mpc,
        commitment, sg_dispatch, renew_dispatch, renew_buses,

```

```

renew_max, slack_idx, non_slack_idx)
82 % Update generator status and PG values for non-slack SGs
    and renewables.
83
84 mpc1 = mpc;
85
86 % Non-slack synchronous generators (commitment +
    dispatch)
87 for i = 1:numel(commitment)
88     gen_idx = non_slack_idx(i);
89
90     if commitment(i) == 0
91         mpc1.gen(gen_idx, 8) = 0; % GEN_STATUS OFF
92         mpc1.gen(gen_idx, 2) = 0; % PG
93     else
94         mpc1.gen(gen_idx, 8) = 1; % GEN_STATUS ON
95         Pmin = mpc1.gen(gen_idx, 10);
96         Pmax = mpc1.gen(gen_idx, 9);
97         mpc1.gen(gen_idx, 2) = max(Pmin,
            min(sg_dispatch(i), Pmax));
98     end
99 end
100
101 % Renewables (bounded 0..renew_max)
102 renew_dispatch = min(max(renew_dispatch, 0), renew_max);
103
104 nR = numel(renew_buses);
105 start_idx = size(mpc.gen,1) - nR + 1;
106
107 for i = 1:nR
108     gen_idx = start_idx + (i-1);
109     mpc1.gen(gen_idx, 2) = renew_dispatch(i);

```

```

110         mpc1.gen(gen_idx, 8) = 1;
111     end
112
113     % Slack generator remains ON
114     mpc1.gen(slack_idx, 8) = 1;
115 end
116
117 % =====
118 % Local helper: compute_inertia_metrics
119 % =====
120 function out = compute_inertia_metrics(out, mpc1, results)
121 % Computes inertia distribution metrics and associated
    penalty.
122
123     % Synchronous generator block
124     SG_BLOCK = 1:Nsg;
125
126     gen_status = mpc1.gen(SG_BLOCK,8);
127     comm_gen_idx = SG_BLOCK(gen_status ~= 0);
128
129     if isempty(comm_gen_idx)
130         out.penalty_inertia = 1e5;
131         return;
132     end
133
134     [~, IC_all, ~] = get_machine_params(caseFile, Mpc);
135     H_k = IC_all(comm_gen_idx).';
136
137     gen_buses = mpc1.gen(comm_gen_idx,1).';
138
139     [d, h] = compute_inertia(mpc1, gen_buses, H_k);
140

```

```

141     Pg = results.gen(comm_gen_idx, 2);
142
143     out.h = h;
144
145     [ h_min , ~ , ~ , ~ , ~ ] =
146         compute_hmin_thresholds ( mpc , C , rocof_max )
147     out.h_min = h_min;
148
149     out.penalty_inertia = 1000*(max(h_min - h)>0);
150 end
151
152 % =====
153 % Local helper: compute_inertia
154 % =====
155
156 function [d, h] = compute_inertia(mpc, gen_buses, H_k)
157 % Computes distribution factors d and estimated bus inertia h
158 % using repeated Kron reductions on Ybus.
159
160     gen_buses = gen_buses(:).';
161     H_k       = H_k(:);
162
163     [Ybus, ~, ~] = makeYbus(mpc.baseMVA, mpc.bus,
164                             mpc.branch);
165
166     bus_all      = mpc.bus(:,1).';
167     bus_ex_gen   = setdiff(bus_all, gen_buses);
168
169     ng = numel(gen_buses);
170     nb = numel(bus_ex_gen);
171
172     Y_dist = zeros(ng, nb);

```

```

172     for i = 1:nb
173         remaining = bus_ex_gen(i);
174         keep      = [gen_buses, remaining];
175         eliminate = setdiff(bus_all, keep);
176
177         Y_gg = Ybus(keep, keep);
178         Y_ge = Ybus(keep, eliminate);
179         Y_eg = Ybus(eliminate, keep);
180         Y_ee = Ybus(eliminate, eliminate);
181
182         Y_red = Y_gg - Y_ge * (Y_ee \ Y_eg);
183         Y_dist(:, i) = Y_red(1:ng, end);
184     end
185
186     Yd = imag(Y_dist);
187     col_sums = sum(Yd, 1);
188     col_sums(col_sums == 0) = eps;
189     d = Yd ./ col_sums;
190
191     denom = sum((d.^2) ./ H_k, 1);
192     denom(denom == 0) = eps;
193     h = 1 ./ denom;
194 end
195
196 % =====
197 %Local helper function: compute bus level minimum thresholds
198 % =====
199 function [h_min, dki, alpha_ki, DeltaP_bus, worstCont] =
200     compute_hmin_thresholds(mpc, C, rocof_max)
201
202 % Inputs:
203 %   mpc      : MATPOWER case struct (includes baseMVA,

```

```

203 %                                     bus, branch, gen)
204
205
206 % Outputs:
207 %   h_min           : (nbus x 1) minimum inertia threshold at
208 %                   each bus
209 %   dki             : (ng x nbus) distance factors d_{k i}
210 %   DeltaP_bus     : (nbus x 1) allocated disturbance per bus
   (p.u.)
211
212   nbus = size(mpc.bus,1);
213   ng   = size(mpc.gen,1);
214
215   if numel(rocof_max) ~= nbus
216       error('rocof_max must be nbus x 1.');
```

217 end

```

218   rocof_max = rocof_max(:);
219   rocof_max(rocof_max <= 0) = eps;
220
221   %% -----
222   % Build Ybus (MATPOWER)
223   %% -----
224   [Ybus, ~, ~] = makeYbus(mpc.baseMVA, mpc.bus,
   mpc.branch);
225
226   bus_all = mpc.bus(:,1).';      % MATPOWER bus numbering
227   gen_buses = mpc.gen(:,1).';    % generator terminal
   buses
228
229   % Kron-reduced reactances -> alpha_{k i}
230
231   alpha_ki = zeros(ng, nbus);
```

```

232
233     for i = 1:nbus
234         bus_i = bus_all(i);
235
236         % Keep all generator buses + the target bus i
237         keep = unique([gen_buses, bus_i], 'stable');
238         elim = setdiff(bus_all, keep);
239
240         Y_kk = Ybus(keep, keep);
241         Y_ke = Ybus(keep, elim);
242         Y_ek = Ybus(elim, keep);
243         Y_ee = Ybus(elim, elim);
244
245         % Kron reduction
246         Y_red = Y_kk - Y_ke * (Y_ee \ Y_ek);
247
248         % Indices within reduced matrix
249         idx_i = find(keep == bus_i, 1);
250
251         for g = 1:ng
252             b_g = gen_buses(g);
253             idx_g = find(keep == b_g, 1);
254
255             if isempty(idx_g) || isempty(idx_i)
256                 alpha_ki(g,i) = 0;
257             else
258                 % "Reactance-like" coupling proxy
259                 alpha_ki(g,i) = abs(imag(Y_red(idx_g,
260                                     idx_i))));
261             end
262         end
263     end
264 end

```

```

263
264     invAlpha = 1 ./ alpha_ki;           % (ng x nbus)
265     den = sum(invAlpha, 1);
266     den(den == 0) = eps;
267     dki = invAlpha ./ den;             % (ng x nbus)
268
269     DeltaP_pu_all = zeros(1, numel(C));
270     gen_idx_all    = zeros(1, numel(C));
271
272     for c = 1:numel(C)
273         k_idx = find(mpc.gen(:,1) == C(c).gen_bus, 1);
274         if isempty(k_idx)
275             error(' gen_bus=%d not found in mpc.gen(:,1).',
276                   C(c).gen_bus);
277         end
278         gen_idx_all(c) = k_idx;
279         DeltaP_pu_all(c) = C(c).DeltaP_MW / mpc.baseMVA;
280     end
281
282     [DeltaP_worst_pu, c_worst] = max(DeltaP_pu_all);
283     k_worst = gen_idx_all(c_worst);
284
285     worstCont = struct();
286     worstCont.k = k_worst;
287     worstCont.gen_bus = mpc.gen(k_worst,1);
288     worstCont.DeltaP_pu = DeltaP_worst_pu;
289     DeltaP_bus = (dki(k_worst, :).') * DeltaP_worst_pu;
290
291     h_min = DeltaP_bus ./ rocof_max;
292 end
293 % =====
294 % Local helper: add_renewables

```

```

294 % =====
295 function [mpc, renew_buses, renew_max] = add_renewables(mpc)
296 % Appends renewable generators to the MATPOWER case.
297
298     renew_buses = [20, 8, 4, 16, 3, 15, 24];
299     renew_max    = [854.6, 1267.2, 988.77, 854.6, 612.27,
300                    885.68, 457.38];
301     base_cost = [2 0 0 3 0 0 0];
302
303     for i = 1:numel(renew_buses)
304         mpc.gen(end+1, 1:21) = [ ...
305                                renew_buses(i), 0, 0, 0, 0, 1, 100, 1,
306                                renew_max(i), 0, ...
307                                0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0];
308         mpc.gencost(end+1, 1:7) = base_cost;
309     end
310 end
311 % =====
312 % Local helper: get_non_slack_limits
313 % =====
314 function [Pmin, Pmax, non_slack_idx] =
315     get_non_slack_limits(mpc, slack_idx)
316
317     SG_BLOCK = 1:Nsg;
318     non_slack_idx = setdiff(SG_BLOCK, slack_idx);
319
320     Pmin = (0.2 * mpc.gen(non_slack_idx, 9)).';
321     Pmax = (1.0 * mpc.gen(non_slack_idx, 9)).';
322 end

```

D.3 Multi-Area Dynamic Simulation Codes

Area Parameter Aggregation

```
1 function params = aggregate_area_params(mpc, areas, H_indiv,
2   D_indiv, R_indiv, Sb_gen, S_base)
3
4   params.N = numel(areas);
5   params.H = zeros(params.N,1);
6   params.D = zeros(params.N,1);
7   params.R = cell(params.N,1);
8   params.S_B = zeros(params.N,1);
9
10  gen_buses = mpc.gen(:,1);
11
12  for k = 1:params.N
13      area_buses = areas{k};
14      idx = ismember(gen_buses, area_buses);
15
16      % Area base and normalization to global base
17      Sb_area = sum(Sb_gen(idx));
18      params.S_B(k) = Sb_area / S_base;
19
20      % MVA-weighted averages within the area
21      H_weighted = sum(H_indiv(idx) .* Sb_gen(idx)) /
22          Sb_area;
23      D_weighted = sum(D_indiv(idx) .* Sb_gen(idx)) /
24          Sb_area;
25      R_weighted = sum(R_indiv(idx) .* Sb_gen(idx)) /
26          Sb_area;
27
28      params.H(k) = H_weighted;
```

```

25         params.D(k) = D_weighted;
26         params.R{k} = R_weighted;
27     end
28 end

```

Inter-Area Synchronizing Coefficients

```

1 function Kmat = compute_sync_coeff(res, mpc, areas, Sb_area)
2
3     define_constants;
4     V    = res.bus(:, VM);
5     ang = res.bus(:, VA) * pi/180;
6
7     nl = size(res.branch,1);
8     nAreas = numel(areas);
9     Kmat = zeros(nAreas);
10
11     for l = 1:nl
12         if res.branch(l, BR_STATUS) == 0
13             continue;
14         end
15
16         i = res.branch(l, F_BUS);
17         j = res.branch(l, T_BUS);
18         x = res.branch(l, BR_X);
19
20         Vi = V(i);  Vj = V(j);
21         deltai = ang(i);
22         deltaj = ang(j);
23
24         % Line-level synchronizing term (small-signal
           approximation)

```

```

25     C_line = (Vi*Vj/x) * cos(deltai - deltaj);
26
27     % Map buses to areas
28     ai = find(cellfun(@(b) ismember(i,b), areas), 1);
29     aj = find(cellfun(@(b) ismember(j,b), areas), 1);
30
31     if ~isempty(ai) && ~isempty(aj) && ai ~= aj
32         Kmat(ai,aj) = Kmat(ai,aj) + C_line /
33             sqrt(Sb_area(ai)*Sb_area(aj));
34         Kmat(aj,ai) = Kmat(aj,ai) + C_line /
35             sqrt(Sb_area(ai)*Sb_area(aj));
36     end
37 end
38 end
39 end

```

Multi-Area Dynamic Model Construction

```

1 function sys = build_multi_area_model(params, Cij)
2 %BUILD_MULTI_AREA_MODEL Full 6N-state multi-area frequency
   response model.
3
4 % States per area (i):
5 %   x_i = [Delta f_i, Delta Psg_i, Delta Pgov_i, Delta
   Pres_i, Delta Ps_i, Delta delta_i]^T
6
7 % Inputs:
8 %   params : struct containing per-area parameters:
9 %       .H   inertia constants per area (N x 1)
10 %       .D   damping constants per area (N x 1)
11 %       .R   droop constants per area (cell N x 1 or vector)
12 %       .TT  turbine time constants (N x 1)
13 %       .TG  governor time constants (N x 1)

```

```

14 %      .TR      RES converter time constants (N x 1)
15 %      .KS      secondary control gains (N x 1)
16 %      .TS      secondary control time constants (N x 1)
17 %      .beta frequency bias (N x 1)
18 %      .S_B      normalized area base (Sb_area/S_base) (N x 1)
19 %      .N        number of areas
20 %      Cij       : tie-line coefficient matrix (N x N)
21
22 % Output:
23 %      sys       : state-space model (6N states, 2N inputs, N
24                   outputs)
25
26 N = params.N;
27 nStates = 6*N;
28 nInputs = 2*N; % [Delta PCL_i, Delta PRES_i] for each
29                area
30 nOutputs = N; % Delta f_i for each area
31
32 A = zeros(nStates,nStates);
33 B = zeros(nStates,nInputs);
34 C = zeros(nOutputs,nStates);
35 D = zeros(nOutputs,nInputs);
36
37 f0 = 60; % nominal frequency (Hz)
38
39 for i = 1:N
40     idx = (i-1)*6 + (1:6); % states of area i
41     in_idx = (i-1)*2 + (1:2); % inputs of area i
42
43     % Parameters for area i
44     Hi = params.H(i);
45     Di = params.D(i);

```

```

44      Ri      = params.R{i};
45      TTi     = params.TT(i);
46      TGi     = params.TG(i);
47      TRi     = params.TR(i);
48      KSi     = params.KS(i);
49      TSi     = params.TS(i);
50      betai   = params.beta(i);
51      SBi     = params.S_B(i);
52
53      % Scaling factor for frequency dynamics
54      Omega = f0 / (2*Hi*SBi);
55
56      % Local A_i block (6x6)
57      Ai = zeros(6);
58
59      % (1) Frequency dynamics: Delta f_i
60      Ai(1,1) = -Omega*Di;
61      Ai(1,2) = Omega;           % from Delta Psg_i
62      Ai(1,4) = Omega;           % from Delta Pres_i
63      % Tie-line coupling via Delta delta_j is added later
64
65      % (2) Turbine: Delta Psg_i
66      Ai(2,2) = -1/TTi;
67      Ai(2,3) = 1/TTi;
68
69      % (3) Governor: Delta Pgov_i
70      Ai(3,1) = -1/(TGi*Ri);
71      Ai(3,3) = -1/TGi;
72      Ai(3,5) = 1/TGi;
73
74      % (4) RES converter: Delta Pres_i
75      Ai(4,4) = -1/TRi;

```

```

76
77      % (5) Secondary control: Delta Ps_i
78      Ai(5,1) = -KSi*betai/TSi;
79      Ai(5,5) = -1/TSi;
80
81      % (6) Angle dynamics: Delta delta_i
82      Ai(6,1) = 2*pi;
83
84      A(idx,idx) = Ai;
85
86      % Local inputs: [Delta PCL_i, Delta PRES_i]
87      Bi = zeros(6,2);
88      Bi(1,1) = -Omega;      % Delta PCL_i enters frequency
                             % equation
89      Bi(4,2) = 1/TRi;      % Delta PRES_i enters RES
                             % converter state
90
91      B(idx,in_idx) = Bi;
92
93      % Output: frequency deviation
94      C(i, idx(1)) = 1;
95  end
96
97      % -----
98      % Inter-area coupling: tie-line effects via Delta delta_j
99      % -----
100  for i = 1:N
101      for j = 1:N
102          if i ~= j && Cij(i,j) ~= 0
103              idx_i = (i-1)*6 + (1:6);
104              idx_j = (j-1)*6 + (1:6);
105

```

```

106         Hi = params.H(i);
107         SBi = params.S_B(i);
108         Omega = f0 / (2*Hi*SBi);
109
110         KSi = params.KS(i);
111         TSi = params.TS(i);
112
113         % Tie-line coupling in frequency equation
114         % through Delta delta_j
115         A(idx_i(1), idx_j(6)) = Omega * Cij(i,j);
116
117         % Coupling in secondary control through
118         % Delta delta_j
119         A(idx_i(5), idx_j(6)) = (KSi/TSi) * Cij(i,j);
120     end
121 end
122 end
123
124 sys = ss(A,B,C,D);
125 end
126
127 %=====
128 %Main
129 %=====
130
131     clc; close all;
132     define_constants;
133
134     %% -----
135     % Case + steady-state PF
136     %% -----
137
138     mpc = loadcase('case39');
139     res = runpf(mpc);

```

```

136     if ~res.success
137         error('Power flow did not converge.');
```

138 end

139

140 %% -----

141 *% Define areas*

142 %% -----

143 areas = {

144 [14 15 16 20 21 22 23]; *% Area 1 buses*

145 setdiff([1:39],[14 15 16 20 21 22 23]); *% Area 2*

146 *buses*

147 };

148 %% -----

149 *% Generator-level parameters*

150 %% -----

151 ng = size(mpc.gen,1);

152

153 D_indiv = 0.5 * ones(ng,1);

154 R_indiv = 0.05 * ones(ng,1);

155

156 *% Generator ratings (MVA)*

157 Sb_gen = mpc.gen(:, MBASE);

158 Sb_gen(Sb_gen == 0) = mpc.baseMVA;

159

160 *% Global base for area scaling*

161 S_base = 100;

162

163 params = aggregate_area_params(mpc, areas, H_indiv,

164 D_indiv, R_indiv, Sb_gen, S_base);

165 %% -----

```

166 % Dynamic control parameters per area
167 %% -----
168 N = params.N;
169 params.TT = 0.4 * ones(N,1); % turbine time
    constant (s)
170 params.TG = 0.2 * ones(N,1); % governor time
    constant (s)
171 params.TR = 0.1 * ones(N,1); % RES converter time
    constant (s)
172 params.KS = 0.3 * ones(N,1); % secondary gain
173 params.TS = 10 * ones(N,1); % secondary time
    constant (s)
174 params.beta = 1.0 * ones(N,1); % frequency bias
175
176 %% -----
177 % Compute inter-area synchronizing coefficients
178 %% -----
179 Sb_area = zeros(N,1);
180 for k = 1:N
181     idxk = ismember(mpc.gen(:,1), areas{k});
182     Sb_area(k) = sum(Sb_gen(idxk));
183     if Sb_area(k) == 0, Sb_area(k) = 1; end
184 end
185
186 Cij = compute_sync_coeff(res, mpc, areas, Sb_area); % N
    x N
187
188 %% -----
189 % Build the 6N-state model
190 %% -----
191 sys = build_multi_area_model(params, Cij);
192

```

```

193  % Inputs per area:
194  %   u = [DeltaPCL_1 .. DeltaPCL_N,   DeltaPRES_1 ..
          DeltaPRES_N]
195  %% -----
196  tEnd = 60;                % seconds
197  dt   = 0.01;             % simulation time step
198  t     = (0:dt:tEnd).';
199
200  area_fault = 2;
201  DeltaPCL_pu = 0.20;
202  tFault = 0;
203
204  % No renewable disturbance in this example (DeltaPRES =
          0)
205  DeltaPRES_pu = zeros(N,1);
206
207  u = build_step_inputs(t, N, area_fault, DeltaPCL_pu,
          tFault, DeltaPRES_pu);
208
209  % Simulate (x0 = 0)
210  y = lsim(sys, u, t);      % y is N x length(t) in ss ->
          returns length(t) x N
211  Delta_f = y;             % frequency deviations (Hz if
          model uses Hz-state; otherwise pu)
212
213  %% -----
214  % Compute dynamic performance metrics
215  %% -----
216  resultsDyn = compute_dyn_metrics(t, Delta_f);
217
218  disp(resultsDyn);
219  end

```

Input Builder

```
1 function u = build_step_inputs(t, N, area_fault,
   DeltaPCL_pu, tFault, DeltaPRES_pu)
2
3 % Inputs:
4 %   t           : time vector (nt x 1)
5 %   N           : number of areas
6 %   area_fault   : area index where load change occurs
7 %   DeltaPCL_pu  : step magnitude in p.u. for DeltaPCL in
   the faulted area
8 %   tFault       : fault application time (s)
9 %   DeltaPRES_pu : (N x 1) constant RES disturbance
   magnitudes in p.u.
10
11 % Output:
12 %   u : (nt x 2N) input matrix for lsim:
13 %       [DeltaPCL_1..DeltaPCL_N, DeltaPRES_1..DeltaPRES_N]
14
15   nt = numel(t);
16   u = zeros(nt, 2*N);
17
18   % Load disturbance steps
19   stepMask = (t >= tFault);
20   if area_fault >= 1 && area_fault <= N
21       u(:, area_fault) = DeltaPCL_pu * stepMask;
22   end
23
24   % RES disturbances (treated as constant setpoints here)
25   DeltaPRES_pu = DeltaPRES_pu(:);
26   if numel(DeltaPRES_pu) ~= N
27       error('DeltaPRES_pu must be N x 1.');
```

```

28     end
29     for k = 1:N
30         u(:, N + k) = DeltaPRES_pu(k) * ones(nt,1);
31     end
32 end

```

Metric Extraction

```

1 function metrics = compute_dyn_metrics(t, Delta_f)
2 %COMPUTE_DYN_METRICS Extracts standard frequency-response
   metrics per area.
3 %
4 % Inputs:
5 %     t           : time vector (nt x 1)
6 %     Delta_f     : frequency deviation trajectories (nt x N)
7 %
8 % Output:
9 %     metrics struct containing:
10 %     .RoCoF_max   : max |df/dt| per area
11 %     .RoCoF_time  : time index where max RoCoF occurs
12 %     .nadir       : minimum Delta_f per area
13 %     .nadir_time  : time at nadir
14 %     .steady      : steady-state Delta_f (average over last
   10% of simulation)
15
16 if size(t,2) ~= 1
17     t = t(:);
18 end
19
20 [nt, N] = size(Delta_f);
21 if nt ~= numel(t)
22     error('Delta_f must be (nt x N) consistent with t.');
```

```

23     end
24
25     dt = mean(diff(t));
26     dfdt = gradient(Delta_f, dt); % numerical derivative (nt
        x N)
27
28     RoCoF_max = zeros(1,N);
29     RoCoF_time = zeros(1,N);
30     nadir      = zeros(1,N);
31     nadir_time = zeros(1,N);
32     steady     = zeros(1,N);
33
34     for k = 1:N
35         [RoCoF_max(k), idxR] = max(abs(dfdt(:,k)));
36         RoCoF_time(k) = t(idxR);
37
38         [nadir(k), idxN] = min(Delta_f(:,k));
39         nadir_time(k) = t(idxN);
40
41         tail = round(0.9*nt):nt;
42         steady(k) = mean(Delta_f(tail,k));
43     end
44
45     metrics = struct();
46     metrics.RoCoF_max = RoCoF_max;
47     metrics.RoCoF_time = RoCoF_time;
48     metrics.nadir      = nadir;
49     metrics.nadir_time = nadir_time;
50     metrics.steady     = steady;
51 end

```