OPTIMIZING THE CONJUNCTIVE USE OF SURFACE WATER AND GROUNDWATER IN WATER STRESSED RIVER BASINS: CASE OF OLIFANTS RIVER BASIN, SOUTH AFRICA

by

GISLAR EDGAR KIFANYI

Submitted in partial fulfilment of the requirements for the degree

DOCTOR TECHNOLOGIAE: CIVIL ENGINEERING

in the

Department of Civil Engineering

FACULTY OF ENGINEERING AND THE BUILT ENVIRONMENT

TSHWANE UNIVERSITY OF TECHNOLOGY

Supervisor: Prof. J.M. Ndambuki

Co-Supervisor: Prof. S.N. Odai

October 2017

DECLARATION

"I hereby declare that the thesis submitted for the degree (Doctor Technologiae: Civil Engineering), at the Tshwane University of Technology (TUT) is my own original work and has not previously been submitted to any other institution of higher education. I further declare that all sources cited or quoted are indicated and acknowledged by means of a comprehensive list of references".

G.E. Kifanyi

DEDICATION

To my parents Mr and Mrs Edgar Kifanyi for their commitment and foresight throughout my sojourn in South Africa. To my late cousin Benedict S Killagane who passed away during the study, who demonstrated long standing love and kindness in pursuit of this feat. Your love and kindness remain unforgettable. May GOD grant your gentle soul eternal rest.

ACKNOWLEDGEMENT

Firstly, I thank the Almighty GOD with whom this research work was possible. I would like to express my sincere gratitude to Prof. Julius Musyoka Ndambuki and Prof. Samuel Nii Odai for accepting to work with me as my supervisors. Indeed, your patience, tireless support, encouragement and constructive criticisms made the success of this academic achievement. I say thank you so much and GOD bless you. I also extend my sincere gratitude to TUT for the financial support granted to me towards the completion of this research work. Moreover, I would like to extend my sincere gratitude to the Faculty of Engineering and Built Environment and the Department of Civil Engineering family members. Your support and kindness will remain unforgettable to me. I say thank you so much and GOD bless you all in your respective careers. Furthermore, I extend my thanks to my employer Mbeya University of Science and Technology for granting me study leave as well as moral and financial support. My heartfelt thanks and appreciation extends to TUT Scientists Network Outcomes Water and Sanitation (SNOWS) project team for their support and encouragement. Special thanks to the Department of Water and Sanitation (DWS), Agriculture Research Council (ARC), Council of Geoscience (CG), Water Resources Commission (WRC) and the South African Weather Service (SAWS) for their assistance with data. To my relatives, brothers and sisters, and sister in-law Grace Ndelwa Killagane, I say thanks for your prayers and GOD bless you all. To my parents, moms Agnes and Mariana Mguluka and dad Edgar Kifanyi, your words of encouragement “work and pray never ever give up!” were the driving force towards this academic achievement. I say thank you so much for your prayers and GOD bless you.

ABSTRACT

One of the new techniques currently used to address water shortage problems in the developed countries is the optimum conjunctive water use. Optimum conjunctive water use demands that the surface and subsurface reservoirs are fully characterized if deterministic methods are to give reliable results. However, in real world phenomena, full characterization of surface – groundwater reservoirs is neither practically nor economically feasible. This research, therefore, aimed at developing a combined simulation-optimization quantitative conjunctive water use management model which can sustainably manage water resources taking into account input parameter uncertainty. Response matrix technique was used to combine simulation model with optimization model (procedure). The novelty of this research work is that determination of optimum conjunctive water use was determined under scanty data and uncertain condition. Surface water and groundwater conceptual models were developed, and integrated to form a conjunctive water use conceptual model which was converted into numerical simulation model for both deterministic and stochastic simulations. MODFLOW-2000 and RIVER Package (RIV) (together referred to as conjunctive water use simulation model) supported in Visual MODFLOW 2014.2 Classical Interface was used to determine aquifer system responses (drawdowns). These drawdowns were assembled as response matrices and then incorporated into an optimization management model as drawdowns constraints coefficients. The simulation-optimization problems were solved and analysed through "Active-Set" (Sequential Quadratic Programming (SQP) optimizer (algorithm)) implemented under the MATLAB 2014a environment. The Retrospective Optimization Approximation (ROA) method was used for solving the stochastic optimization problem and to

investigate the impact of uncertainty on optimal management strategies. ROA procedure solves and evaluates a sequence of optimization sub-problems in an increasing number of realizations. Results indicated that the study area aquifer has potential groundwater resource which is undeveloped. Deterministic approach underestimates the water withdrawal rates. The optimal withdrawal rates designed based on ROA approach were relatively higher than those designed based on deterministic approach. Moreover, the overall percentages of contribution of surface water and groundwater sources to the total water demand obtained through ROA approach was about 58% and 42%, respectively while the overall percentages contribution obtained through deterministic approach was about 85% and 15%, respectively. This is about $\pm$27 % variation (i.e., Differences between the approaches realized) of percentages of contribution of the two water sources to the total water demand. Furthermore, findings indicated that ROA conjunctive water use management technique has potential to ensure sustainability of limited water resources of river basins. Through ROA approach the expected total optimal objective function value converged to its maximum value within a relatively few iterations (6 to 8 iterations) in about 2.30 Hrs computational time. In conclusion, results demonstrated that the ROA approach is a promising technique for use in managing conjunctive water use under uncertainty conditions. It is recommended that guidelines for determination of the sequence of sample sizes for use in ROA method framework should be established. The use of parallel computer processors to enhance computational time efficiency for large optimization problems should be explored. Quantitative methods for determination of weights for estimating values of objective functions should be investigated. The application of the ROA approach to multi-objectives optimization problems should be explored.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

LIST OF ACRONYMS

2D	Two Dimensional
3D	Three Dimensional
AMSL	Above Mean Sea Level
ARC	Agricultural Research Council
CG	Council of Geoscience
CLT	Central Limit Theorem
Conj.use	Conjunctive Water Use
CPW	Combined Pumping Well
CRV	Conductance of Riverbed Soil Materials
CSWD	Combined Surface Water Diversion
D/S	Downstream
DC	Diversion Canal
DDA	Dam Direct Abstraction
DEM	Digital Elevation Model
DP	Deep Percolation
DWA	Department of Water Affairs
DWAF	Department of Water Affairs and Forestry

DWS	Department of Water and Sanitation
ERCSOP	Estimates Retrospective Conjunctive Water Use Sample Path Optimization Problems
ERGSOP	Estimates Retrospective Groundwater Sample Path Optimization Problems
GHB	General Head Boundary
GIS	Geographic Information System
GW	Groundwater
HRV	Head of River
$i.i.d.$	Independent Identically Distributed
IPA	Irrigation Plantations Area
IWMI	International Water Management Institute
KNP	Kruger National Park
LLN	Law of Large Numbers
NP	National Park
Opt.	Optimal
PN	Problem Number
PW	Pumping Well

QC	Quaternary Catchment
RA	Retrospective Approximation
RCDA	River Course Direct Abstraction
REFIID	Recharge From Irrigation and Industrial water Demand zones
RER-DR	Reservoir Release Direct to River
R-Flow	Return Flow to the river course from water users
RIV	River Package
ROA	Retrospective Optimization Approximation
RSA	Republic of South Africa
RVBOT	Riverbed Bottom elevation
RZ	Recharge Zone
S/A	Surface water-Aquifer
SA	South Africa
SAA	Sample Average Approximation
SAWS	South African Weather Service
SLLN	Strong Law of Large Numbers
SNOWS	Scientists Network Outcomes Water and Sanitation
SOSP	Sample path Optimization Sub-Problem

SOSPkRi	Sample Path Optimization Sub-Problem k of Realizations i, where k and i are numbers of sub-problems and realizations, respectively
SOSPs	Sample path Optimization Sub-Problems
Std	Standard Deviation
SW	Surface Water
SWD	Surface Water Diversion
SW-RDVDZ	Surface Water River Diversion Direct to Demand Zones
SW-RE-DVDZ	Surface Water Reservoir Diversion Direct to Demand Zones
TCOP	True Conjunctive water use Optimization Problem
TGOP	True Groundwater Optimization Problem
TUT	Tshwane University of Technology
U/S	Upstream
US	United States
USGS	United States Geological Survey
WMA	Water Management Area
WRC	Water Resources Commission

CHAPTER ONE: INTRODUCTION

1.1 BACKGROUND

Most parts of the world, in recent years, are experiencing acute shortages of fresh-water in terms of both quality and quantity due to various factors such as increasing population and changing land use activities. An increase in water demand for domestic, industrial and irrigation use as well as the conflicting demands for sustainability of the ecosystem, has resulted in water stressed river basins in many regions of the world (Roberts, 2010).

Southern Africa has water shortage problems due to the geographical climatic conditions and financial constraints of the regions (Randell, 1999; Hornby *et al.,* 2016), thus adding new water supply structures is virtually impossible (Randell, 1999). According to Randell (1999), allocation of water resources among river riparians within the region is becoming a great challenge to water resources management as populations dependent on water resources is rapidly expanding, consequently putting greater pressure on already overstressed river basin systems. In such situations, a question of how to effectively manage the use of this limited water resource to maximize overall benefit is inevitable.

Mahjoub *et al.* (2011) argues that, by considering the fact that most of the region in Southern Africa have a problem in terms of water resources, conjunctive water use of surface water and groundwater can be considered as a proper technique for water security. This is because conjunctive water use of water resources relies on the principle that by using surface water when it is plentiful, recharging aquifers and conserving groundwater supplies in wet periods, water will then be available for future pumping in the dry periods when surface water supplies are short (Mariño, 2001). Moreover, conjunctive water use involves coordination of the use of groundwater and surface water systems.

However, in managing conjunctive water use of surface water and groundwater systems, it is important to realize that surface water and groundwater systems are usually interconnected and thus should be treated accordingly. In short, surface water system can be defined as one of the components of hydrological cycle system that comprises all water collecting on the ground or in a stream, river, lake, wetland, or ocean while groundwater is the water system that is found beneath the earth's surface.

Moreover, it should be noted that groundwater is a significant component of many river basins that is essential for sustaining stream flow during dry periods (Sahuquillo & Lluria, 2002). Depending on hydraulic head, surface water bodies can also flow to groundwater systems. Thus, for sustainability of surface water and groundwater resource systems, development of an efficient management tool for optimizing the conjunctive water use of surface water and groundwater resources exploitation is of paramount importance.

However, one barrier to the development of conjunctive water use management systems is lack of guidelines on how to operate such systems and how to evaluate the benefit of capacity expansion. As a result, groundwater has traditionally been used only as a backup supply during times of shortage (Lettenmaier & Burges, 1979). Moreover, several literatures (Peralta, 2001; Jolly *et al.*, 2010; Katambara,

2011) have added that, apart from lack of guidelines there is also inadequate data for groundwater and surface water to sufficiently model such systems. However, in spite of these data scarcity problems, water resource developments must continue to take place to satisfy the economic and social development needs of communities (Mazvimavi, 2003).

Thus, there is a significant need to develop an effective conjunctive water use management tool which can suitably manage simultaneously groundwater and surface water in river basins that have scarce data and uncertainty conditions. This is because uncertainties in input parameters to water resource problems have been recognized as impediment to designing efficient water resource management strategies (Ndambuki *et al.,* 2003). Moreover, nature and interdependence of surface water and groundwater coupled with size and complexity of river basin systems as well as legal and administrative issues, has intensified use of simulation and optimization models for conjunctive water use management (Mariño, 2001).

Application of optimization techniques is most challenging in water resources systems area when it comes to the management of water resources systems both in terms of quality and quantity. This is because of the large number of decision variables involved, stochastic nature of the inputs, and multiple objectives (Ndambuki *et al.,* 2003; Datta & Harikrishna, 2005). However, according to Mariño (2001), by applying simulation-optimization methods one can quickly and efficiently identify best options for control and planning when a range of options exists. Therefore, this research focuses on the application of simulation-optimization approach based on optimization technique which is suitable for optimizing the conjunctive water use of surface water and groundwater in a water stressed river basin with scarce data and uncertain conditions.

1.1.1 The Role of Conjunctive Water Use in South African Water Security

South Africa is a water scarce country. Allocation of water resources among river riparian is becoming a great challenge to water resources managers (planners and decision makers) as population dependent on water resources is rapidly expanding, putting more pressure on already overstressed river basin systems (Randell, 1999). The existing water resource available in South Africa comprises of 77 percent surface water, 9 percent groundwater and 14 percent re-used water of return flows (Waterwiki.net, 2006; DWAF, 2008). Moreover, a recently updated study on water resource availability in South Africa shows that 98 percent of surface water has already been allocated for use, leaving only 2 percent available for future development (Underwood, 2009; Cape Town Green Map, 2009).

It is important to note that South Africa's surface water resources have been highly developed and utilized, and in many instances over-exploited (Lévite *et al.,* 2003; Warburton, 2011). Of the 19 water management areas, 10 are currently water stressed (Warburton, 2011). Moreover, over 80 percent of South Africa is underlain by relatively low-yielding aquifer systems (Woodford *et al.,* 2007). In such situations, the question of how to utilize efficiently this limited water resource to maximize overall benefit is of paramount importance.

Conjunctive water use technique, if properly administered has the potentials to alleviate many problems associated with water shortages in South Africa. Mahjoub *et al.* (2011) pointed out that by considering the fact that most of the Southern African countries have a problem in terms of water resources, conjunctive water use of surface water and groundwater can be considered as a proper technique for enhancing water security.

Moreover, Peralta (2001) asserted that conjunctive water use management is needed in many areas to assure sustainable availability of water resources. This is because conjunctive use of water resources relies on the principle that by using surface water when it is plentiful, recharging aquifers and conserving groundwater supplies in wet years, water will then be available for future pumping in the dry years when surface water supplies are short (Mariño, 2001). However, it should be realized that, as in any other aspects of modelling, conjunctive water use simulation optimization modelling is prone to be associated with uncertainties; hence, it is important that uncertainties should be properly addressed for efficient management, planning and reliable decision making.

1.1.2 Uncertainties in Conjunctive Water Use Simulation-Optimization Modeling

Usually models that optimize water resources based on objective function and constraints have a simulation component. This is because simulation component is used to predict the physical behaviour of the systems response when subjected to external stresses (Basagaoglu and Mariño, 1999; McKinney *et al.,* 1999). Peralta (2001) argues that predicting how diversions and pumping affect the coupled water resources requires data describing the physical system. These data can include groundwater and stream stages (or heads), flow rates, and parameters such as hydraulic conductivity, transmissivity and storativity. It is important to note that for these predictions to be accurate, the input parameters should be accurate as well.

Similarly, in order to have an accurate optimum conjunctive water use (withdrawals) of surface water and groundwater, it is imperative that the simulations should be as accurate as possible. This implies that the input parameters must be adequate and accurate. Hence, the accuracy of optimum conjunctive water use management models is highly dependent on the availability and accuracy of the input parameters.

Uncertainty in estimating the optimum conjunctive water use (surface water – groundwater withdrawals) lies not only on variability of the hydrological cycle and the climate but also on lack of adequate data and perfect knowledge about the surface water – groundwater system interactions, errors in historic data and inherent variability of system parameters both in space and time. Hence, one should expect uncertainties when predicting surface water – groundwater conjunctive use system responses (i.e., when surface water – groundwater system is subjected to external stresses). Bear *et al.* (1992) pointed out that the degree of uncertainty is increased in most cases by the lack of sufficient data for parameter estimation and model validation. Errors in observed data used for parameter identification also contribute to uncertainty in the estimated values of model parameters (Bear *et al.,* 1992; Ndambuki *et al.*, 2005).

Various methods for introducing uncertainty into the models and the modelling process have been proposed. For example, one approach is to employ Monte Carlo Methods in which the various possibilities are represented in a large number of simulated realizations. Another approach is to construct stochastic models in which the various coefficients are represented as probability distributions rather than deterministic values. Thus, management decision makers must make use of such predictions in the decision-making process (Bear *et al.,* 1992).

Unfortunately, most researchers used to determine optimum conjunctive water use based on deterministic approach (i.e., an approach which assumes that all the input data are sufficiently measured and known without error). This assumption is not valid, because in reality only a few set of data are available to define current situation of river basin system with certainty. Hence, there is a need to develop a tool which can efficiently determine optimum conjunctive water use, taking into account the scarcity of data and uncertainty conditions.

1.1.3 Research Motivation

In South Africa, many river basins (including Olifants River basin, the Great Letaba River catchment (the study area)) are water stressed with scarce hydrological data. Hughes (2005) argued that development and successful application of hydrological models in Southern Africa has been seriously hampered by high degree of spatial and temporal variation in hydrometeorological variables and resulting stream flows, a lack of adequately long or continuous records, a lack of information on land use changes and both spatial and temporal variations in water utilization as well as lack of quantitative understandings of the mechanisms of critical hydrological and hydrogeological processes.

Lévite *et al.* (2003) used demand management approach to analyse water demand management scenarios in water-stressed river basin in South Africa and found that in South Africa, water resources in river basins are almost fully allocated. They also observed that within the limits of data availability, it appears that, in Olifants river basins water users are not able to meet their water requirements from the river basins. It was found that in the river basins even the ecological reserve is not being fully met during dry periods. This is because in dry years, the river has zero flow as it flows into the Kruger National Park (IWMI, 2007).

Moreover, Lévite *et al.* (2003) study revealed that demand management approach is inefficient during dry periods. Furthermore, Sami and Hughes (1996) asserted that very few contributions in South Africa have focused on integrating groundwater and surface water estimation models and it is difficult to apply both locally and internationally available surface water-groundwater models due to lack of detailed hydrogeological information in many parts of South Africa.

However, despite these challenges, there is a need to integrate and conjunctively manage surface water and groundwater resources in a country such as South Africa, where optimal management of all water resources is of paramount importance. Still *et al.* (2010) emphasise that realities of water demands and availability dictate the urgency for a new understanding that realizes the potential of conjunctive water use management in South African social-ecological and economic development. Further, they stated that: “whether we can achieve this depends on the attributes of the water storage system and our willingness to accept that it can be done within the constraints of acceptable levels, and that although environmental flows will be suboptimal, the long-term prospects will be enhanced.”

This highlights the need to develop an alternative water resources management tool which can efficiently manage conjunctively surface water and groundwater in the river basin. Thus, this need for an alternative water resources management approach is the main motivation for this research whose aim was to develop a model capable of optimizing the conjunctive water use of surface water and groundwater resources taking into account the scarcity and uncertainty of data.

1.2 PROBLEM STATEMENT

One of the new techniques currently used to address water shortage problems in the developed countries is the optimum conjunctive water use of surface water and groundwater resources. It is important to note that optimal conjunctive water use demands that the surface-subsurface reservoirs are fully characterized if deterministic methods are to give reliable results. However, full characterization of surface-subsurface reservoirs is neither practical nor economically feasible. Usually, in real world phenomena, few field measurement points are taken to represent physical behaviour of the surface-subsurface reservoir system. Therefore, this research aimed at developing a tool that is capable of optimizing the conjunctive water use in water stressed river basins taking into account the scarcity of data and their uncertain condition. The novelty of this research work is that the determination of optimum conjunctive water use is sought under limited data as well as uncertain conditions.

1.3 RESEARCH OBJECTIVES AND SCOPE OF THE STUDY

1.3.1 Main Objective

The main objective of this research was to develop a model capable of optimizing the conjunctive water use of surface water and groundwater resources under uncertainty conditions through the application of simulation-optimization approach.

1.3.2 Specific Objectives

(1) To develop a surface water Conceptual model;

(2) To develop a groundwater conceptual model;

(3) To integrate the models developed in (1) and (2) to build up a conjunctive water use conceptual model;

(4) To test the model developed in point (3) above by applying it to the Great Letaba River catchment.

1.3.3 Scope of the Study

The study was designed towards developing a methodology, which addresses the area of conjunctive water use management (optimization) under parameter uncertainty conditions. The proposed methodologies combine mainly the two components of simulation and optimization, through which the effect of parameter uncertainty on the optimal conjunctive water use strategies was analysed.

1.4 THESIS OUTLINE

In this thesis, six chapters are presented to cover the subject of the study. In Chapter one, an introduction which covers firstly a background overview on conjunctive water use management, role of conjunctive water use in South African water security, and a brief on uncertainties in conjunctive water use simulation-optimization modelling are presented. Moreover, the problem statement, research objectives and a broad overview of the scope of the study are also given in this chapter.

In Chapter two, literature relevant to quantitative conjunctive water use management is reviewed. This chapter is divided into two main parts; the first part covers the general overview of surface water, groundwater, and conjunctive water use systems conceptual models and modelling aspects in deterministic as well as stochastic conditions. The second part deals with simulation models, optimization models and combined simulation-optimization models. Different techniques used to combine simulation models with optimization models (procedure) are also discussed in this part. This Chapter ends with some concluding remarks.

Chapter three mainly deals with methodology, theoretical considerations, conceptual and management models development. In this Chapter, the two methodologies proposed are introduced and developed in a systematic manner. The Chapter begins by presenting conceptual models development, followed by conjunctive water use numerical simulation and management models.

In Chapter four, application of methodology to a hypothetical example is presented. These results are obtained based on the application of the two solution methodologies proposed in Chapter three. The results obtained from the two solution methodologies are verified through an hypothetical example results determined by Tyagi *et al.* (1995). For analysis purpose, results from the solution methodologies are finally compared and presented in different graphical (pictorial) and tabular formats. This Chapter ended with a brief concluding remark.

Chapter five deals with a real world case application of the solution methodologies developed in Chapter three. Results obtained are analyzed and compared. This Chapter ends up with some concluding remarks. Chapter six gives overall conclusions based on the results presented in Chapters four and five. Moreover, this Chapter also highlights recommendations for further research. These recommendations are drawn from the discussions of the results of Chapters four and five.

CHAPTER TWO: LITERATURE REVIEW

2.1 INTRODUCTION

Development of combined simulation and optimization models for conjunctive water resources management has expanded rapidly in recent years (Barlow & Granato, 2007). This has been attributed because of the advancement in computer technology. Mok (2007) pointed out that due to the significant advancement in computer hardware and computational algorithms (such as parallel computing and evolutionary global optimizers) in the past decade, simulation-optimization models have evolved into useful conjunctive water resources management tools. Simulation-optimization models provide scientists and water resource decision makers with an improved understanding of hydrologic and hydrogeologic controls on conjunctive management options within each basin (Barlow & Granato, 2007). Usually in conjunctive water use simulation-optimization modelling, conceptual models are developed to represent the hydrologic and hydrogeologic features of hypothetical or real world systems.

Moreover, it should be realised that conjunctive simulation optimization modelling of surface water and groundwater becomes necessary when there is a strong coupling between the two systems. Coupling may be due to interaction of management objectives, or both systems. Safavi and Bahreini (2009) emphasize that water managers and decision makers must understand the surface water-groundwater systems interactions, especially under uncertainty. This is due to the fact that as in any system management, conjunctive water use is also characterized by uncertainties.

While simulation-optimization approach has provided water resources managers and decision makers with symptomatic plans which determines how best to manage the limited water resource, in real world phenomena quite often the available data is inadequate to give deterministic approach reliable results, hence this gap has prompted researchers to develop tools which are capable of addressing issues of uncertainty.

In the subsequent sub-sections, a review of the most relevant issues related to this study is done. In general, the literature review can be grouped into two perspective classes namely deterministic and stochastic management approaches. In this study, conceptual models were developed for conjunctive water use numerical model simulations to determine aquifer system responses (aquifer system hydraulic heads and drawdowns) when subjected to unit pumping rate in both deterministic and stochastic approaches. These responses were assembled in the form of coefficients matrices known as response matrix and were incorporated in optimization models to represent simulation model in the simulation-optimization modelling process.

2.2 CONCEPTUAL MODELS

Conceptual models are developed to represent hypothetical or real world systems. They are simplified representation of all potential parameters, and interpretation of the characteristics and dynamic responses of hypothetical or real world systems. In modelling conjunctive water use of surface water and groundwater systems, both water storage systems are characterized. This involves identifying water inflows and outflows, land use patterns, hydrological and hydrogeological processes. In recent years, discretization process has been automated through the use of geographic information system (GIS) technology. Model features of interest (including geometry, attributes and boundary conditions) are described using arcs, points, polylines, and polygons in a GIS database, and the process of discretizing these data sets to a grid is automated. This process of defining model data using GIS objects is usually referred to as a conceptual model approach (Boisson & Cabal, 1998; Jones *et al.,* 2000).

2.2.1 Surface Water Conceptual Models

Surface water conceptual models are the representation and interpretation of the characteristics and dynamics of a surface water system which is based on an examination of all available hydrological data for a modeled area (Mandle, 2002). This includes the external configuration of the system, location and rates of inflow and outflow. Moreover, conceptual models do avoid parameter identifiability problems by estimating averaged parameters via calibration procedures (Le ROY, 2005).

According to Bredenkamp *et al.* (1995) a conceptual model is usually one in which the rainfall is routed through one or more "storage reservoirs" to manipulate the following hydrological components:

- The interception loss
- Soil moisture absorption
- Infiltration rate
- Evapotranspiration loss

Moreover, conceptual models may include the geometry, material properties, boundary conditions, general flow patterns as well as system stresses (such as surface water withdrawals) (Salis *et al.,* 2006).

In general, conceptual models refer to models with a structure of interconnected storages and often referred to as “soil moisture accounting” models. However, in developing conceptual models, field characterization is necessary. Mandle (2002) emphasized that field characterization is a necessary prerequisite to the development of a conceptual model.

According to Johnson (2010) to develop appropriate river basin models, information is required on:

- Precipitation and drainage area (to determine input volume of water);
- Land use, soil types and/or permafrost (to determine how much water infiltrates to the water table);
- Slope (to determine the rate water reaches the drainage outlet);
- Land cover of vegetation and lake/river abundance and size (to determine rates of evapotranspiration);
- Climate data (to determine seasonality, runoff and/or snow-water equivalent);
- Stream flow data and water-level data (to understand discharge and storage diversion data).

Moreover, the effects of climate and hydrologic fluctuation on water supply can be included in water management models through prescription of climatic and hydrologic scenarios (Mckinney *et al.,* 1999; Mayer & Muñoz-Hernandez, 2009). It should be realized that often surface water modeling is undertaken to establish sustainable limits on the amount of surface water that can be withdrawn from a river system. However, because of the close and complex relationships between precipitation and the hydrological processes, stochastic output series can be generated based on rainfall variations, and then used as input to a deterministic or regression model (Bredenkamp *et al.,* 1995). Moreover, according to Draper (2001) a monthly rather than annual time step is preferable so as to capture both inter-season and inter-annual uncertainty, and the effects of monthly varying surface storage constraints.

Ajami *et al.* (2008) pointed out that hydrologic models are simple mathematical conceptualization of complex and spatially distributed watershed processes that can be used to provide estimates of current and future hydrologic events. However, the reliability of these models depends on proper parameter and state estimation. Hence, it is important that uncertainty be addressed because uncertainties not only affect ability of the model to accurately estimate real world system response but also impact on reliability of water resources decision making.

2.2.2 Groundwater Conceptual Models

A groundwater conceptual model is a simplified representation of the hydrogeologic setting and the response of the flow system to stress (ASTM International, 2002). Moreover, a conceptual model of groundwater system provides a generalized description and interpretation of the hydrogeologic framework of the groundwater system behavior. The basic components of a groundwater conceptual model are the sources of water to the region and sinks of water from the region, the physical boundaries of the region, and the distribution of hydraulic properties within the region (Leon & Ty Ferré, 2003). Bredenkamp *et al.* (1995) pointed out that a conceptual model is usually one in which the rainfall is routed through one or more "storage reservoirs" to manipulate the following hydrological components: (i) the interception loss; (ii) soil moisture absorption; (iii) infiltration rate; and (iv) evapotranspiration loss.

Moreover, according to Palmer *et al.* (2007) an underlying assumption of groundwater flow simulation models is that the aquifer material behaves as a porous permeable media. Normally, in modeling, simplification is introduced as a set of assumptions which expresses the nature of the system and those features of its behavior that are relevant to the problem under investigation (Bear *et al.,* 1992). These assumptions will relate, among other factors, to the geometry of the investigated domain, the way various heterogeneities will be smoothed out, the nature of the porous medium (e.g., its homogeneity, isotropy), the properties of the fluid (or fluids) involved, and the type of flow regime under investigation.

The formation of a conceptual model is critical to the development of a more quantitative representation of the subsurface hydrology, such as a numerical groundwater flow model. A calibrated, numerical groundwater flow model allows for prediction of the impacts of changes to the hydrologic conditions on subsurface flow. However, this requires precise definitions of the physical boundaries of the region, the water fluxes into and out of the system, and the hydraulic properties distributed throughout the region (Leon & Ty Ferré, 2003).

Mandle (2002) suggested that, questions to be asked in developing a conceptual model, should include, but are not limited to:

(i) Are there adequate data to describe the hydrogeological conditions at the site?

(ii) In how many directions is groundwater moving?

(iii) Can the groundwater flow or contaminant transport be characterized as one, two-or three-dimensional?

(iv) Is the aquifer system composed of more than one aquifer, and is vertical flow between aquifers important?

(v) Is there recharge to the aquifer by precipitation or leakage from a river, drain, lake, or infiltration pond?

(vi) Is groundwater leaving the aquifer by seepage to a river or lake, flow to a drain, or extraction by a well?

(vii) Does it appear that the aquifer's hydrogeological characteristics remain relatively uniform, or do geologic data show considerable variation over the site?

(viii) Have the boundary conditions been defined around the perimeter of the model domain, and do they have a hydrogeological or geochemical basis?

(ix) Do groundwater-flow and contaminant source conditions remain constant, or do they change with time?

(x) Are there receptors located downgradient of the contaminant plume?

(xi) Are geochemical reactions taking place in onsite groundwater, and are the processes understood?

Moreover, Palmer *et al.* (2007) added that a model of a river basin can potentially help to answer the following questions:

(i) How will groundwater withdrawals associated with proposed growth scenarios affect flows in nearby streams?

(ii) How will significant new groundwater withdrawals in one of the basin's tributary watersheds affect wells in that watershed, both upstream and downstream of the withdrawals?

(iii) How will new groundwater withdrawals affect aquifer levels in adjacent watersheds?

It is important to know that mathematical models describing groundwater flows are well established. Most of these mathematical models are based on the balances of water volumes which result in partial differential equations or difference equations (Haimes, 1973). Moreover, Haimes (1973) emphasized that the aquifer system parameters such as transmissivity and storage coefficient are an integral part of the equations and must be known in order to predict the response of the aquifer to various demands placed on it.

However, it should be affirmed that in reality, many real-world aquifers are characterized by a few measurement points, which are used to derive the aquifer characteristics (Ndambuki *et al.,* 2003). Thus, since conceptual models are a simplified representation of a real world system behavior response, this may imply that conceptual models significantly contribute to uncertainty in predictions of groundwater flow. Krom and Lane (2009) pointed out that hydrogeologic conceptual models are a key source of uncertainty in predictions of groundwater flow. Frequently there is insufficient information to fully characterize the aquifer systems (Ndambuki *et al.,* 2003; Krom & Lane, 2009).

Moreover, Krom and Lane (2009) asserts that since, uncertainty is guaranteed in a groundwater model, it is imperative to develop groundwater conceptual models in a manner that clearly exposes the uncertainty by using methods that allow this uncertainty to be explored. Hence, this research sought to develop a water resources management tool that can account for uncertainty brought about by scarcity of data to fully characterize the hydrogeology characteristics. In this study, surface water and groundwater conceptual models were developed and integrated (combined) to form a conjunctive water use conceptual model.

2.2.3 Conjunctive Water Use Conceptual Models

Conjunctive water use conceptual models are a simplified representation and interpretation of complex dynamics of hydrologic processes and hydrogeologic settings, and the response of surface water-groundwater flow systems to stress. Though are simplified, conceptual models have the ability to mimic the physical processes of hydrological and hydrogeological systems in a more realistic manner by incorporating mechanisms of surface and subsurface flows. Most developments in conceptual model approach have focused on source/sink and boundary condition data; however, a major part of the conceptual model is an idealized representation of hydrostratigraphy (Jones *et al.,* 2002). Model features (including geology, geometry, attributes and boundary conditions (i.e., source/sink objects such as river networks and recharge zones)) are generated using GIS as shape files data, arcs, points, polylines, and polygons. In this study, the process of discretizing data to grid cells was done using Visual MODFLOW (software which supports GIS) a three dimensional (3D) finite-difference numerical simulation model.

2.3 SIMULATION MODELS

Simulation models predict physical system response to natural and managed stimuli (Fayad *et al.,* 2012). These models use mathematical expressions, for surface water system simulation model. For example, Meigh *et al.* (1999) used a Grid-Based Approach to estimate water scarcity for Eastern and Southern Africa. A monthly time step was used to compute total runoff in a month. Having generated the local runoff from each grid cell, the flows were routed between cells and summed up to give the total runoff in each cell. The equation for total runoff developed was as follows:

$$Q_{Si} = Q_{Li} + \sum(1 - L)(Q_{Ui} - Q_{Ci} + Q_{Ri}) \pm Q_{Ti} \qquad \ldots\ldots\ldots\ldots\ldots (2.1)$$

where Q_{Si} is the total runoff for the current cell in month i; Q_{Li} is the locally generated runoff for the current cell in month i; Q_{Ui} is the total runoff routed from adjoining upstream cells in month i; Q_{Ci} is the water consumed in adjoining upstream cells in month i; Q_{Ri} is the routed return flows arising from adjoining upstream cells in month i; Q_{Ti} is the long distance transfer of flows out of or back into the current cell; and L is the proportional transmission loss term for flows out of the upstream cells.

The transmission loss L is included to represent reductions in river flows due to evaporation and infiltration, which can be high in semi-arid regions. Lakes, reservoirs and wetlands can cause considerable modifications to the flow pattern and are therefore treated separately in the model. Each such component may be represented by a simple water balance equation of the form:

$$S_i = S_{i-1} + Q_{in} + A(P - E) - Q_{out} \qquad \ldots\ldots\ldots\ldots\ldots (2.2)$$

where S_i is the storage at the end of month i; S_{i-1} is the storage at the end of month $i-1$; Q_{in} is the inflow; Q_{out} is the outflow; P is the rainfall; E is the evaporation; and A is the surface area, all in month i.

Moreover, for stream flow, relationships with other hydrological variables can be derived for example by using Manning's equation (Basagaoglu *et al.,* 1998):

$Q = \left(1/n\right) A R^{2/3} S^{1/2}$ .... (2.3)

where Q is normal flow (discharge); n is Manning's roughness coefficient; A is cross sectional area; R is hydraulic radius; and S is channel slope. It is essential to note that from equation (2.3) the relationship of stream flow with stream stage is non-linear. However, for big rivers the relationship can be assumed linear. This is because basically for big rivers, the width of the river is assumed to be very large compared to its depth.

In conjunctive water use system, simulation models account for physical behaviour of surface water-groundwater systems (Safavi *et al.,* 2009). Simulation of hydraulically connected surface water-groundwater system can be done to determine responses of the systems when aquifer is subjected to the stresses (such as groundwater withdrawals). The simulation modelling of conjunctive water use system enables one to understand the interactions behaviour of the surface water-groundwater systems.

Based on the Darcy's law and the law of conservation of mass, the following equation can be derived for the flow through a nonhomogeneous anisotropic porous media (Ndambuki, 2001):

$$\frac{\partial}{\partial x}\left(T_x \frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(T_y \frac{\partial h}{\partial y}\right) \pm W = S\frac{\partial h}{\partial t} \quad \text{.. (2.4)}$$

where h is the hydraulic potential (L), $T_{x,y}$ are the components of the transmissivity tensor (L^2T^{-1}), x, y are Cartesian coordinates (L), W is a source or sink term (LT^{-1}), S is the storage coefficient (-), and t is time (T). For hydraulically connected surface water-groundwater system, equation (2.4) is the basis of surface water-groundwater simulation models. The component '$+W$' may represent the net recharge which often depends on rainfall and/or surface water systems while the component '$-W$' may represent discharge of the aquifer system to surface water bodies (such as stream, rivers, or lakes). From equation (2.4), it can be noted that simulation models attempts to predict changes in groundwater levels (hydraulic head), and stream or river stages in response to changes in stresses such as groundwater withdrawal (pumping), surface water flow losses, and recharge.

Simulation model enables fast, inexpensive and non-disruptive examination and testing of a large number of scenarios prior to actually implementing a particular decision in the "real" world environment (Better *et al.,* 2008). However, it should be realized that the input parameters for the simulation equations if precisely known leads to deterministic results, otherwise, stochastic (Ndambuki, 2001). Moreover, since simulation approximates reality it also permits the inclusion of various sources of uncertainty and variability into forecasts that impact performance (Better *et al.,* 2008).

As it has been realized that simulation models mainly describe the stress-response relationship of an aquifer or surface water system, its use to seek out optimal management schemes requires trial and error approach. This poses a great challenge in terms of time, labour, as well as reliability of the optimal management scheme that is obtained from such an approach (Ndambuki *et al.,* 2000). Moreover, Better *et al.* (2008) pointed out that the range of parameter values and the number of parameter combinations are too large for decision makers to enumerate and test all possible scenarios; hence, they need a way to guide the search for good solutions.

However, it is important to realize that without simulation, many real-world problems are too complex to be modelled by tractable mathematical formulations that are at the core of pure optimization methods such as scenario optimization and robust optimization (Better *et al.,* 2008), hence, one must resort to simulation, which cannot easily find the best solutions (Better *et al.,* 2008; Ndambuki *et al.,* 2003). This approach despite its pitfalls is widely used with many applications (Safavi & Bahrein, 2009; Young & Bredehoeft, 1972).

2.4 OPTIMIZATION MODELS

One of the new techniques in the present century in water resources management is optimum conjunctive water use of surface water and groundwater (Mahjoub *et al.,* 2011). Conjunctive water use optimization modelling is a technique that can be used to determine maximum or minimum objective values (such as withdrawal rates) from both surface water and groundwater while meeting constraints with respect to water levels and stream flow (Czarnecki *et al.,* 2003).

Generally, optimization models are mathematical formulations characterized by an objective function and a set of constraints. Objective function measures the level of achievement of the objective while the constraints define the feasible region from where the optimal solution is to be sought. Thus, optimization models are designed to find solutions that are best according to the specified objective(s), usually economic, while satisfying a set of stated physical, technological, environmental, and other constraints (Labadie, 1993; Ndambuki *et al.,* 2003; Yevjevich, 1979).

Traditionally, conjunctive water use models have been formulated as optimization models. However, owing to the fact that water resources allocation is spatially dependent and operating rules are temporal in nature, many researchers prefer to construct conjunctive water use models by combining optimization and simulation models (Ho *et al.,* 2007). Moreover, Mckinney *et al.* (1999) insists that models that optimize water resources based on an objective function and constraints must have a simulation component. This is important because simulation component tends to account for physical behaviour aspects of the systems (Basagaoglu & Mariño, 1999).

Moreover, simulation-optimization approach can resolve the conundrum facing the simulation method and optimization method by combining both methods. Optimizers designed for simulation embody the principle of separating the method from the model. In so doing, the optimization problem is defined outside the complex system, thus, the evaluator (i.e., the simulation model) can change and evolve to incorporate additional elements of the complex system, while the optimization procedures remain the same (Better *et al.,* 2008).

Furthermore, of particular importance to water resources river basin scale analyses are models of two fundamental types (Hantush & Mariño, 1989): (1) models that simulate water resources behaviour in accordance with a predefined set of rules (actual or hypothetical) governing water allocations and infrastructure operations; and (2) models that optimize and select allocations and infrastructure based on an objective function (economic or other) and accompanying constraints. In simulation-optimization perspectives, the optimization procedure is usually based on metaheuristic search algorithms that uses the outputs from the system simulation model (evaluator), thus measures the merit of the inputs that were fed into the simulation model (Better *et al.,* 2008).

2.5 SIMULATION-OPTIMIZATION MODELS

In an effort to design more reliable optimal conjunctive water use schemes, the two management models (simulation and optimization) have to be combined so as to capture the advantages of the subtle features characterizing each of them. Simulation optimization approach can efficiently handle a larger number of scenarios than traditional optimization approaches, as well as multiple sources and types of risk and uncertainties (Better *et al.,* 2008). Simulation and optimization methods have been widely used to determine operating strategies for regional water supplies (Basagaoglu *et al.,* 1998).

Simulation-optimization approach is practical because it is unlikely that optimal surface water and groundwater conjunctive use management alternatives which comply with groundwater hydraulics and surface water flow or stream stages could be determined by either simulation or optimization alone. Ndambuki *et al.* (2003)

pointed out that aquifer management models that combine simulation with optimization reveal a lot of details regarding the interactions between social and economic forces on one hand and the water resources (surface and subsurface) on the other hand.

Moreover, Better *et al.* (2008) asserted that recent advances in simulation-optimization techniques has led to new opportunities to solve problems more efficiently, specifically, in applications involving risk and uncertainty. Simulation-optimization techniques surpass the capabilities of other optimization methods not only in the quality of solutions but also in their interpretability and practicality.

Generally, simulation-optimization tools are designed to solve optimization problems of the form (Better *et al.,* 2008):

Minimize $\mathbf{F}(\mathbf{x})$ (Objective function),

Subject to $\mathrm{A}\mathbf{x} \leq \mathbf{b}$ (Constraints on input variables),

$\mathrm{g_l} \leq \mathbf{G}(\mathbf{x}) \leq \mathrm{g_u}$ (Constraints on output measures),

$\mathrm{l} \leq \mathbf{x} \leq \mathrm{u}$ (Bounds),

where the vector $\mathbf{x}$ of decision variables includes variables that range over continuous values and/or variables that only take on discrete values (both integer values and values with arbitrary step sizes). The objective function $\mathbf{F}(\mathbf{x})$ is, typically, highly complex. In the context of simulation-optimization, generally, $\mathbf{F}(\mathbf{x})$ represents an output performance measure obtained from the simulation and it is a mapping from a set of values $\mathbf{x}$ to a real value. The constraints represented by inequality $\mathrm{A}\mathbf{x} \leq \mathbf{b}$ are usually linear (given that no linearity in the model is embedded within the simulation itself), and both the coefficient matrix A and the right hand side values corresponding to vector $\mathbf{b}$ are known. The constraints represented by inequalities of the form $\mathrm{g_l} \leq \mathbf{G}(\mathbf{x}) \leq \mathrm{g_u}$ impose simple upper and/or lower bound requirements on an output function $\mathbf{G}(\mathbf{x})$ that can be linear or nonlinear. The values of the bounds $\mathrm{g_l}$ and $\mathrm{g_u}$ are known constants. All decision variables $\mathbf{x}$ are bounded and some may be restricted to be discrete. Moreover, each evaluation of $\mathbf{F}(\mathbf{x})$ and $\mathbf{G}(\mathbf{x})$ requires an execution of a simulation of the system.

One of the primary advantages of the simulation-optimization model is that it provides a structured means to evaluate trade-offs between sustained rate of groundwater withdrawals and surface water depletion (Barlow *et al.,* 2003). However, incorporation of simulation model within an optimization-based management model is complex and difficult and takes considerably large computational time to achieve any optimal solution (Safavi *et al.,* 2009). To overcome this problem, embedding technique and response matrix approach are generally used to incorporate simulation model within management optimization models (Gorelick, 1983).

2.5.1 Embedding Approach

The embedding technique for solving groundwater management problems was first invented by Aguado and Remson (1974). In this approach, numerical approximations of the flow equations are included directly as constraints in the optimization model (Aguado & Remson, 1974; Ndambuki *et al.,* 2000). Moreover, the unknown surface water and groundwater variables (stream stages, groundwater heads and source/sink) become decision variables in the optimization method. This method, not only solves the problem once (as opposed to the response matrix approach), but also produces a substantial amount of information regarding the behaviour of the aquifer under consideration.

However, since each element within the modelled domain is represented by an equation in the optimization problem, the resulting problem is usually huge, hence the main disadvantage to its application (Ndambuki *et al.,* 2000; Ndambuki *et al.,* 2003). Though, Peralta *et al.* (1991) asserted that for small problems solved in a steady state, embedding models require less processing time than a comparable response matrix model. Moreover, they further noted that sometimes, embedding model requires less memory than a response matrix model. The required computer memory is frequently a function of the number of nonzero values in constraint equations. For embedding models, the number of non-zeroes is fixed for a particular study area while for response matrix models, the number of non-zeroes can increase dramatically in proportion to the number of pumping cells and cells at which heads must be constrained.

2.5.2 Response Matrix Approach

The response matrix approach uses superposition and linear systems theory to simulate groundwater flow (Peralta *et al.,* 1991). In the response matrix approach also known as technological matrix approach, the influence of a unit change in an independent decision variable such as pumping at a pre-selected well location upon a variety of dependent variables like drawdown at specified potential control observation points is determined. Then superposition process is performed to calculate their total response at specified potential control points resulting from all decision variables (Ndambuki, 2001).

In other words, a response matrix consists of linear influence coefficients that describe the response of the potentiometric surface to a unit volume of extraction or injection of groundwater (Peralta *et al.,* 1991). The main shortcoming of this approach is that the number of simulations to be performed to generate the response (the size of the response matrix is a function of stress locations and observation points considered) as well as re-calculating the response matrix whenever the boundary conditions and stress locations are changed. However, the final optimization problem to be solved is smaller than that obtained from the embedding method (Ndambuki *et al.,* 2000; Ndambuki *et al.,* 2003).

2.6 QUANTITATIVE CONJUNCTIVE WATER USE MANAGEMENT UNDER UNCERTAINTY

In an effort to find out alternative solutions to the acute water shortage problems, conjunctive water use of surface water and groundwater resources has become increasingly a common practice in most parts of the world. Conjunctive water use involves withdrawal of both groundwater and surface water resources (Czarnecki *et al.,* 2003). Conjunctive water use is often incidental as water users intuitively shift between surface water and groundwater sources to cope with changes and water shortages (Dudley & Fulton, 2006).

It is important to understand that conjunctive water use management is the management of hydraulically connected surface water and groundwater resources in a coordinated manner (Czarnecki *et al.,* 2003; Foster *et al.,* 2010). It engages the principles of conjunctive water use, where surface water and groundwater are used in combination to improve water availability and reliability. But, it also includes important components of groundwater management such as monitoring, evaluation of monitoring data to develop local management objectives, and use of monitoring data to establish and enforce local management policies (Dudley & Fulton, 2006).

Conjunctive water use management has two principal operational components, irrespective of whether it is practiced by individuals or implemented by a group of individuals as a coordinated program. These components are commonly referred to as recharge methods and recovery of groundwater (Dudley & Fulton, 2006). Moreover, for optimal (sustainable) conjunctive water use recovery and recharge of groundwater it needs to be balanced (Dudley & Fulton, 2006; Foster *et al.*, 2010). However, it is of great importance to realize that natural recharge is a natural random variable (i.e., a stochastic process), hence, for a reliable optimal conjunctive water use management decision, it is imperative that variables such as recharge should be treated in a stochastic manner.

Moreover, it should be realized that when dealing with water resources management, the management water system is subject to many uncertainties and risky environment (Vucetic & Simonovic, 2011). Simonovic (2009) pointed out that uncertainty in water resource management exists in two basic forms: uncertainty caused by inherent hydrologic variability and uncertainty caused by a fundamental lack of knowledge/data. Hence, it is very important that decision makers should be provided with tools that incorporate uncertainty in decisions making process (Vucetic & Simonovic, 2011).

2.6.1 Uncertainties in Conjunctive Water Use System

Conjunctive simulation of surface water and groundwater becomes necessary when there is a strong coupling between the two systems. Coupling may be due to interaction of conjunctive water use systems or management objectives, or both. Moreover, Safavi and Bahreini (2009) emphasize that water managers and decision makers must understand the surface water-groundwater systems interactions, especially under uncertainty. This is due to the fact that like in any system management, conjunctive water use system is characterized by uncertainties. In conjunctive water use system, uncertainty is inherent in both groundwater and surface water systems.

In groundwater system, uncertainty arises from boundary conditions as well as aquifer parameters such as hydraulic conductivity/transmissivity and storage coefficient, while in surface water system, uncertainty stems from the natural variability of the climatic inputs (such as river flow, precipitation, air temperature) and from model parameters (Safavi & Bahreini, 2009; Ndambuki, 2001).

Moreover, in conjunctive water use management, there are two basic types of uncertainty, namely, model and parameter (Safavi & Bahreini, 2009). Model uncertainty is caused by the limitations in the mathematical models used to simulate the physical system imposed by the simplifying assumption. Parameter, or data uncertainty, is caused by inadequate data and errors in the data measurements, incomplete knowledge of spatial or temporal variations, and heterogeneities that have not been identified during data collection (Ndambuki *et al.,* 2005; Safavi & Bahreini, 2009). Hence, it is important that uncertainty should be precisely dealt with for efficient water resources management planning and reliable decision making.

2.6.2 Managing Conjunctive Water Use Systems under Uncertainty

Usually, management models (such as simulation-optimization models) are developed to assist water resource managers and decision makers to solve water resources management problems. Simulation-optimization mathematical expressions are developed to describe a system and its response to the system inputs for various system excitations. Like many other management systems, management of conjunctive water use is characterized by uncertainty. Unfortunately, most of the times, in the world of deterministic management approaches often we choose to “ignore” uncertainty in order to come up with a unique and objective solution to a problem (Better *et al.,* 2008).

However, it should be realized that the results derived from such deterministic approach are questionable as to whether they really represent a practical solution of the problem. Gorelick (1990) questioned whether deterministic optimization makes sense given the limited understanding of the real world system of interest. Moreover, Safavi and Bahreini (2009) emphasized that water managers and decision makers must understand the interactions between surface water and groundwater systems, especially under uncertainty. This is because the existence of uncertainties limits the ability of the model to predict system behaviour with definiteness under various management decisions (Ndambuki *et al.,* 2005).

Moreover, it is important to realize that the natural recharge that largely replenishes groundwater stocks is stochastic as it depends on weather. According to Hafi (2006), the stochastic nature of both recharge (supply) and demand for groundwater in the current time period results in uncertainties of future groundwater stocks. This is why it has been recognized that principally, conjunctive water use is an approach that recognizes this connection and tries to utilize overall water supply more efficiently (Dudley & Fulton, 2006). However, due to this realization of the mutual connection of these two distinct water resource systems the modelling task of conjunctive water use system becomes more complicated, as the best allocation rules for surface water and groundwater resources need to be found when surface water allocations, groundwater recharge, aquifer soil material, rainfall and crop evaporative demands are unknown and uncertain (Hafi, 2006).

Moreover, Ajami *et al.* (2008) pointed out that uncertainty projection is very important for effective decision making and water resources management, because incomplete accounting of uncertainties may lead to unreliable and inefficient management of water resources. Therefore, it is imperative that uncertainty be precisely addressed. This is because uncertainty does not only affect the ability to predict the system response, but also determines the type of management policies appropriate for use (Safavi & Bahreini, 2009; Weihua Li, 2011).

In the field of optimization, there are different methods designed to deal with uncertainties. Thus, the exact values of the parameters (e.g., the data) of the optimization problem are not known with absolute certainty, but may vary to a greater or lesser degree depending on the nature of the problems they represent. In other words, there may be many possible “realizations” of the parameters, each of which is a possible scheme (Better *et al.,* 2008).

One of the oldest traditional approaches of incorporating uncertainty into optimization models is through post-optimality sensitivity analysis. With this method, the system is modelled as if the values of uncertain quantities are known precisely (deterministic), and then the effects of changes in particular parameters is assessed (i.e., the parameters are changed by a certain percentage value and examination of optimal results carried out). This approach simply provides a means of identifying those parameters to which system performance is most sensitive (Ndambuki *et al.,* 2005).

Other traditional methods of tackling uncertainty are scenario-based approaches to optimization, such as scenario optimization (Better *et al.,* 2008; Indriyani *et al.,* 2010). However, these approaches only consider a very small subset of possible scenarios, and the size and complexity of models they can handle are very limited (Better *et al.,* 2008). Hence, these are the shortcomings of these approaches.

Another approach of tackling uncertainty is to replace the uncertain quantities with their expected values or some critical (worse-case) value and then proceed to solve the management problem with a deterministic approach. However, it should be realized that if uncertainty in the parameters is reasonably small and does not critically affect the operation of the system, expected values can still give optimal solutions that are reasonable (Willis & Yeh, 1987).

Probabilistic (or chance constrained) technique is another method which has been applied for resolving the issue of uncertainty (Gorelick, 1990; Hantush & Marino, 1995; Poojari & Varghese, 2008; Sahinidis, 2004; Vucetic & Simonovic, 2011; Willis & Yeh, 1987). In the probabilistic or chance-constraint approach, the focus is on the reliability of the system (that is the system's ability to meet feasibility in an uncertain environment). This reliability is expressed as a minimum requirement on the probability of satisfying constraints (Sahinidis, 2004).

However, Ahmed and Shapiro (2008) found that there are two main difficulties with chance-constrained problems, first, is on checking feasibility of a given candidate solution exactly. In general, it is impossible since this requires evaluating quantiles of random functions. Second, the feasible region induced by chance constraints is, generally, non-convex leading to severe optimization challenges. Moreover, Wagner *et al.* (1994) pointed out its pitfall that it can only control the probability of violating a given constraint without explicitly mitigating the extent and consequences of the effects arising from such violations.

Probability theory has a long history of application in the field of water resources management. Hydrologic processes are random and thus the uncertainty as a result of variability may be appropriately quantified using the probabilistic approach (Vucetic & Simonovic, 2011). This approach treats the stochastic constraints in a probabilistic framework. This means that the fulfilment of the stochastic inequalities is based on pre-determined probability levels, hence optimal solutions are sought from the feasible set inscribed by these inequalities (Ndambuki, 2001). A prerequisite for using the probabilistic approach is the requirement of a prior knowledge of the probability density functions, and their joint probability distribution function. However, in real-world systems, data is usually lacking to provide such information and where available, approximations still need to be made to estimate appropriate distributions (Vucetic & Simonovic, 2011).

Another technique to handle uncertainty is by using fuzzy set approach. This method can be used for the representation of perceived qualitative ambiguity sources of uncertainty that may not be measurable, giving results with some precision (Katambara, 2011; Vucetic & Simonovic, 2011). Moreover, fuzzy programming considers random parameters as fuzzy numbers and constraints are treated as fuzzy sets. Some constraint violation is allowed and degree of satisfaction of a constraint is defined as the membership (i.e., “class”) function of the constraint (Katambara, 2011; Sahinidis, 2004). Thus, fuzziness measures the degree to which an event occurs, not whether it occurs, a contrast to probability theory (Vucetic & Simonovic, 2011). This approach suffers from generalization. It represents solution in terms of “a class” by means of appropriate fuzzy measures such as linguistic rules, logic inferences. Therefore, an intended best optimal policy is likely not to be achieved.

Other current popular approaches such as Monte Carlo method, stochastic optimization methods based on sample average approximation (SAA) method and its variants which are also used to tackle uncertainty in optimization problems are discussed in more detail under the following subsections, because they form part of the methodologies proposed in this study. More details of development and applications of the approaches will be presented in Chapters three, four and five of this thesis.

Monte Carlo Approach

Monte Carlo methods (also known as simulation method) are numerical methods that rely on repeated random sampling to obtain numerical results. Monte Carlo methods are stochastic numerical methods. The random sampling is based on underlying cumulative distribution function (cdf) and associated appropriate probability density function (pdf). The basic idea of the Monte Carlo methods relies on Strong Law of Large Numbers (SLLN) and Central Limit Theorem (CLT). The estimated mean of the generated random numbers represents the expected value. The distribution of the values computed for the model outcome therefore reflects the probability of values that could occur (Vucetic & Simonovic, 2011).

In conjunctive water resources management under uncertainty context, Monte Carlo approach is usually used to generate numerous realizations of the uncertain parameters. Then, for each realization, the flow equation is solved numerically to obtain the value of the dependent variable. Statistical analysis of the aggregate of computed solutions is then carried out to determine the expected value, variance, and distribution function for each location. Thereafter optimization methods are deterministically used to design optimal solutions (Woltdt *et al.,* 1990).

The Monte Carlo method is probably the most powerful and widely used method (Peck *et al.,* 1988; Safavi & Bahreini, 2009). The popularity of this method may be credited to its ability to simultaneously work with many classes of uncertain parameters (Ndambuki, 2001). Moreover, Peck *et al.* (1988) indicated that the Monte Carlo Method is possibly the most powerful method available for uncertainty analysis because it requires fewer assumptions than other methods. Furthermore, Banta *et al.* (2006) pointed out that Monte Carlo approaches have parallel processing capabilities advantage in which different runs are independent, due to this, individual simulations can be easily distributed to different computer processors (i.e., parallel computing processors).

However, Ndambuki (2001) asserted that the main drawback of the Monte Carlo approaches is the number of realizations required to give reliable statistical measures. Usually, number of realizations ranges between a few tens to thousands meaning more computational time, hence expensive. For instance, van Leeuwen *et al.* (1998) used 1000 realizations; Wagner *et al.* (1994) and Safavi and Bahreini (2009) used 100 realizations; while Wagner and Gorelick (1989), Wagner *et al.* (1992), Ndambuki *et al.* (2000), Ndambuki (2001), Ndambuki *et al.* (2003), Ndambuki *et al.* (2005), and Kifanyi *et al.* (2017) used a total of 30 realizations.

Sample Average Approximation Method and Its Variant

Sample average approximation (SAA) method is an approach for solving stochastic optimization problems by using Monte Carlo simulation. It is referred as an external Monte Carlo simulation sampling based methods. Sample Average Approximation (SAA) method is a widely used technique, often for solving large scale stochastic optimization problems based on Monte Carlo simulation (Bardossy & Raghavan, 2017).

SAA is a two phase method which uses sampling techniques and deterministic optimization to solve stochastic optimization problems (Shapiro & Wardi, 1996; Shapiro *et al.,* 2002). In literature SAA method have appeared in different names, such as sample path optimization method (Homem-de-Mello, 2003; Pasupathy, 2010), retrospective approximation (RA) method (Pasupathy, 2010; Pasupathy & Schmeiser, 2009), and retrospective optimization approximation (ROA) method (Wang *et al.,* 2012).

In classical sample average approximation (SAA) method, representative samples are sampled and then the optimization problem is solved repeatedly. In this approach, expected function of a stochastic optimization problem is approximated by a sample average estimate function derived from a random sample distribution. The resulting sample average estimated optimization problem becomes deterministic which is then solved by any appropriate deterministic optimization algorithm. The process is repeated to find out aspirant solutions and statistical estimates of their optimality gaps until convergence condition is reached.

It is important to note that SAA is not an algorithm, one needs to choose a particular standard deterministic numerical procedure (algorithm) in order to solve the SAA problem (Shapiro, 2001). This is the main motivation to SAA methods. Interested readers are referred to (Shapiro, 2001; Wang & Ahmed, 2008; Homem-de-Mello & Bayraksan, 2014) for more detail on SAA method.

However, it should be noted that classical SAA methods inherit Monte Carlo method convergence properties and hence suffer from slow rate of convergence. Usually it requires very large sample size to converge which may lead to be computationally expensive. To circumvent this drawback, researchers in recent times have adapted to a more refinement of the SAA class of methods called Retrospective Optimization Approximation (ROA) method (Pasupathy & Schmeiser, 2009; Wang *et al.,* 2012). The following subsection is intended to discuss the concepts behind the ROA method.

Retrospective Optimization Approximation Approach

Retrospective optimization approximation (ROA) method is a variant of classical SAA sampling based methods for solving stochastic optimization problems. In literature ROA method appears in various names such as variable sample size method (Homem-de-Mello, 2003). Unlike the classical SAA method which traditionally generates a single optimization problem (i.e., SAA problem) with very large sample size and solves it repeatedly to a specified error of tolerance, the ROA approach generates a sequence of sample-path optimization sub-problems with progressively increasing number of sample sizes (realizations) and then solves these sample path optimization sub-problems to a progressively decreasing error of tolerance. Hence, the ROA method increases the overall classical SAA efficiency (Pasupathy, 2010). It should be noted that ROA is not an algorithm, one needs to choose any appropriate core deterministic or stochastic numerical procedure (algorithm (e.g., Active-Set (i.e., deterministic gradient search), Genetic Algorithm (i.e., random search)) in order to solve the ROA problem (Wang *et al.,* 2012). This is one of the main advantages of the ROA approach.

The basic idea in ROA method is that the early iterations are efficient due to the fact that small sample sizes are used in early iterations (stages) to ensure that not much of computation effort is required in generating sample-path optimization sub-problems. Whereas the later iterations are also efficient because the ROA method take into account the advantage of “warm start” that is, a solution of current sample-path sub-problem is used as initial guess (solution) for the subsequent sample-path sub-problem which is probably closer to the true solution and thus not much of computation effort is required in solving subsequent sample-path sub-problems. It should be realized that the resulting ROA method estimators inherit the same properties of classical SAA estimators and that the individual ROA sample-path optimization sub-problems are solved in similar way as classical SAA approach by choosing any appropriate numerical deterministic algorithm.

The main feature of ROA method is that, it does not treat all realizations at all iterations of the optimization algorithm. Rather, ROA method framework defines a sequence of approximate optimization sub-problems which sequentially account for increasing numbers of sample size/realizations. The sequence of sample sizes (realizations) is selected heuristically (Wang *et al.,* 2012).

It should be realised that typically in optimization problems uncertainty elements can be associated within the objective function and/or constraints. Suppose uncertainty elements in optimization problems are stemmed within the constraints coefficients and we want to optimize through ROA approach (for example, groundwater quantity pumping rates of a number of spatially distributed pumping wells under geological uncertainty condition). In this case, uncertainty is expressed in terms of the assemblage of aquifer system responses (i.e., drawdowns, which are determined under hydraulic conductivity fields uncertainty conditions) due to unit groundwater pumping stress at each potential candidate pumping well location in the model domain. This assemblage set of response values (also known as response matrix coefficients) are normally developed to represent a simulation model in a simulation optimization framework.

Consider that there exist a probability space (Ω, F, P), where Ω, is a set of all possible outcomes, F denotes a set of collection of all subsets of Ω possible outcomes, and P denotes a probability measure. Now, let hydraulic conductivity field uncertainty realization be denoted by ϖ, such that $\varpi \in \Omega$, where Ω is the total possible number of realizations, thus each realization ϖ will have a different random response matrix denoted by A_{ϖ} with associated random response matrix components denoted by a_{ϖ}. Hence, the random response matrices generated may result in different optimal solutions and therefore, different optimal values of the optimization problem.

In general, the optimization management problem under uncertainty can be formulated as follows:

For a given solution set $X,$ such that $X^* \in X,\ \boldsymbol{x} \in X$, find a solution X^* that

$$Maximize/Minimize\ [f(\boldsymbol{x})] \quad s.t.\ \ \mathbb{E}_{\Omega}[\boldsymbol{G}(\boldsymbol{x}, A_{\varpi})] \leq \boldsymbol{b},\ \ \forall \varpi\ \in \Omega \quad \ldots\ldots\ldots\ldots\ldots \quad (2.5)$$

where $\mathbb{E}_{\Omega}$ represents the expectation function over the set of all realizations Ω, and $\boldsymbol{G}$ is a numerical stochastic process that computes the sample observation of constraint function $\boldsymbol{G}(\boldsymbol{x}, A_{\varpi})$ for a given $\boldsymbol{x}$ and the realization ϖ. In which, $\boldsymbol{x}$ is the vector defining the groundwater pumping rates (decision variable) spatially distributed over the model domain; $f(\boldsymbol{x})$ is objective function evaluated through estimates of $\boldsymbol{G}(\boldsymbol{x}, A_{\varpi})$ function by performing a numerical flow simulation with the hydrogeological model defined by realisation ϖ; $\boldsymbol{b}$ is constraining value column vector at control points. It should be noted that the control points and pumping well locations which are considered for developing response matrices are the same in all realisations.

Consider random response matrix denoted by $A_{\overline{\varpi}}$ with random responses components denoted by $a_{\overline{\varpi}}$ (i.e., the responses denoted simply by $\overline{\varpi}$, that is the random hydraulic heads/water table level drawdowns). It is assumed that randomness in hydraulic heads/water table level drawdowns is only due to uncertainty arising in hydraulic conductivity of aquifer system and that $a_{\overline{\varpi}} \in \mathrm{A}_{\overline{\varpi}}$ that is $a_{\overline{\varpi}} \in A_{\overline{\varpi}}$ such that $\mathrm{A}_{\overline{\varpi}} \in \overline{\Omega}$. Moreover, P is assumed to be well defined with unknown distribution function but, what is known is expected mean values, standard deviation and/or covariance of the random responses $\overline{\varpi}$. Consider a stochastic constraint function process $\mathrm{G}(\boldsymbol{x}, \overline{\varpi}) \in \overline{\Omega}$ to be defined as $\mathrm{G}(\boldsymbol{x}, \overline{\varpi}) = \mathrm{A}_{\overline{\varpi}}\, \boldsymbol{x}$. Thus, the expected value of the function $\mathrm{G}(\boldsymbol{x}, \overline{\varpi})$ is defined as (Kifanyi *et al.*, 2017):

$$\mathbb{E}[\mathrm{G}(\boldsymbol{x}, \overline{\varpi})] = \mathbb{E}[\mathrm{A}_{\overline{\varpi}}\, \boldsymbol{x}] \quad \text{..} \ (2.6)$$

Hence, the constraint inequality in optimization problem (2.5) can be written as $\mathbb{E}[\mathrm{G}(\boldsymbol{x}, \overline{\varpi})] \leq \boldsymbol{b}$ such that the expectation $[\mathrm{G}(\boldsymbol{x}, \overline{\varpi})] = \int_{\Omega} \mathrm{G}(\boldsymbol{x}, \overline{\varpi})\mathrm{dP}(\overline{\varpi})$ is the corresponding expected value function. Therefore, inequality constraint in optimization problem (2.5) can be estimated using Monte Carlo sampling based approximation methods (in this case, ROA method) by considering a sequence of finite set of generated independent identically distributed ($i.i.d.$) samples of random response matrices of N realizations of $\varpi = \{A_{\varpi_{N_1}}, \dots, A_{\varpi_{N_k}}\}$. The expected constraint function $\mathbb{E}[\mathrm{G}(\boldsymbol{x}, \varpi_{i_k})]$ can be estimated as (Kifanyi *et al.,* 2017):

$$\mathbb{E}[\mathrm{G}(\boldsymbol{x}, \varpi_{i_k})] = [\frac{1}{N_k}\sum_{i}^{N_k} \mathrm{G}(\boldsymbol{x}, \varpi_{i_k})] \quad \dots\dots\dots\dots (2.7)$$

Thereafter, progressively evaluate the resulting sample path optimization sub-problems using ROA method framework for $k = 1, 2, \dots, N_{SP}$; where N_{SP} is the number of sample path optimization sub-problems generated. Hence, the estimates retrospective groundwater sample path optimization sub-problems can be formulated as:

$$Maximize/Minimize\ [f_{N_k}(\boldsymbol{x})]\ \ s.t.\ \ [\frac{1}{N_k}\sum_{i}^{N_k} \mathrm{G}(\boldsymbol{x}, \varpi_{i_k})] \leq \boldsymbol{b} \quad \dots\dots\dots (2.8)$$

The estimates inequality function in optimization problem (2.8) is deterministic and hence, the sample path optimization sub-problems developed becomes deterministic which can be solved by any appropriate core deterministic search optimizer algorithm. This is one of the main advantages of ROA approach. It should be noted that ROA procedure can be used in deterministic and stochastic search algorithms (Wang *et al.,* 2012). The optimization problems (formulations (2.5) and (2.8)) are referred to as the true (original) optimization problem and estimates optimization problem, respectively.

In optimization problem (2.8), the term $\frac{1}{N_k}$ defines the weight factor or probability associated with realizations ϖ_{i_k}. Different sample sizes N_k (number of realizations) are considered for each sample path optimization sub-problem generated. The performance objective function considered is the expected total optimal groundwater pumping rate. Decision variables considered are groundwater pumping rates (i.e., positive real values X, such that $\boldsymbol{x} \in X,\ X \in \mathbb{R}^{Npw}$, where Npw is the total number of pumping wells) which are spatially distributed within the model domain. In this case, it is assumed that the solution set X is closed and bounded, and hence the problem has finite number of feasible solutions.

The ROA technique is new in areas of conjunctive water use management, however, the researcher is aware that in recent years the method has predominantly been used in other fields of studies such as Petroleum engineering (Chen *et al.,* 2009; Özdogan & Horne, 2006; Wang & Reynolds, 2009; Wang *et al.,* 2012; Yeten *et al.,* 2003), and in areas of operation research (Chen & Schmeiser, 2001; Wang & Schmeiser, 2008), and the most recent application in a quantitative groundwater resource management problem (Kifanyi *et al.,* 2017). These applications however, were based upon fixed and relatively few decision variables and number of realizations. The ROA technique applied here differs from the previous approaches in the sense that it is applied to solve a sequence of optimization sub-problems (particularly designed for consideration of conjunctive water use management under uncertainty) with an increasing numbers of realizations of hydrogeological model realizations (i.e., through response matrix simulation optimization technique).

Therefore, I declare that, to the best of my knowledge this approach has never been applied before for solving conjunctive water use management problem under uncertainty. The implication of this approach to solution convergence will be discussed in Chapters four and five of this thesis. For more details on retrospective optimization approximation (ROA) method and convergence behaviour, interested readers are referred to Pasupathy (2010); Wang *et al.* (2012).

K-means Clustering Sampling Technique

Various sampling techniques can be applied to ROA framework depending on availability of data. In *k*-means clustering sampling, one can establish mapping from every realization to a finite number n of attributes. For example, in groundwater pumping perspective, attributes that can be useful is some measure of hydraulic conductivity/transmissivity (Kifanyi *et al.,* 2017). Firstly the quantity is normalized to [0 1] and then a vector of values is assigned to each realization. Using this process, one is able to identify each realization of hydraulic conductivity/transmissivity field, ϖ_{i} with a vector of attributes ρ_i, $\rho_i \in \mathbb{R}^n$. The vectors of attributes are then used in the *k*-means algorithm (Wang *et al.,* 2012).

The *k*-means algorithm provides k vectors in $\mathbb{R}^n$ (i.e., cluster centres) that minimize mean (average) distance d for a given N numbers of realizations, and can be defined in the form:

$$d = \sum_{i=1}^{n} min\left\|\rho_i - \phi_j\right\|^2, \; j = 1,2,\ldots,k$$

where ρ_i denotes a vector of attributes such that $\rho_i \in \mathbb{R}^n$; and ϕ_j denotes the coordinates of the centre of cluster j. In *k*-means approach once the cluster centres are established, a particular realization ϖ_i is assigned to one of the centers by simply computing the minimum distance as $arg_{min_{j=1,2,\ldots,k}}\left\|\rho_i - \phi_j\right\|$.

2.7 CONCLUDING REMARKS

Based on the prior overview presented in the literature pertinent to this study, this sub-section gives some concluding remarks to summarize the relevant issues drawn from the literature overview. It has emerged that conceptual models enable representation and interpretation of hypothetical or real world systems in a simplified meaningful manner and are key prerequisite tools in regional scale conjunctive water use management. It has been revealed that combined simulation-optimization models are particularly attractive for solving conjunctive water use management problems. This is because the method enables explicit expression of management objective function, while the conjunctive water use numerical model (i.e., groundwater flow numerical model) is included in the set of optimization constraints so as to guarantee hydrodynamic feasible solution.

Moreover, it has been realised that input parameters to conjunctive water use management problems are not only scarce, but also uncertain. This, therefore, implies that optimal solution strategies to such management problems also are uncertain. This implies that there is a need for new methods which are capable of tackling uncertainty in quantitative conjunctive water use optimization management problems, because the available approaches do not efficiently handle such problems. In this study, a new method (i.e., ROA method) in the context of conjunctive water use management under uncertainty is proposed and compared with a standard deterministic counterpart approach to evaluate its performance.

CHAPTER THREE: METHODOLOGY

3.1 INTRODUCTION

In chapters one and two, the main research framework followed in this study was highlighted. It was pointed out that the overall goal of this research was to develop a management tool that can be used to design a quantitative conjunctive water use of surface water and groundwater resources taking into account scarcity of data and uncertainty of the river basin parameters.

To achieve this goal, surface water and groundwater conceptual models were developed and then integrated to form conjunctive water use conceptual model. The proposed methodology recognizes the stochastic nature of the input parameters such as hydraulic conductivity of groundwater aquifer system, and/or hydraulic conductivity of river bed soil layer material. Prior to detail discussion of development of management models (optimization models) and solution methodologies, conceptual and simulation models are presented.

3.2 CONCEPTUAL MODELS

In modelling conjunctive water use of surface water and groundwater systems, both water storage systems were characterized. This involved identifying water and land use patterns, hydrological and hydrogeological processes. Figure 3.1 shows the general land use layout of the Great Letaba River catchment, the study area.

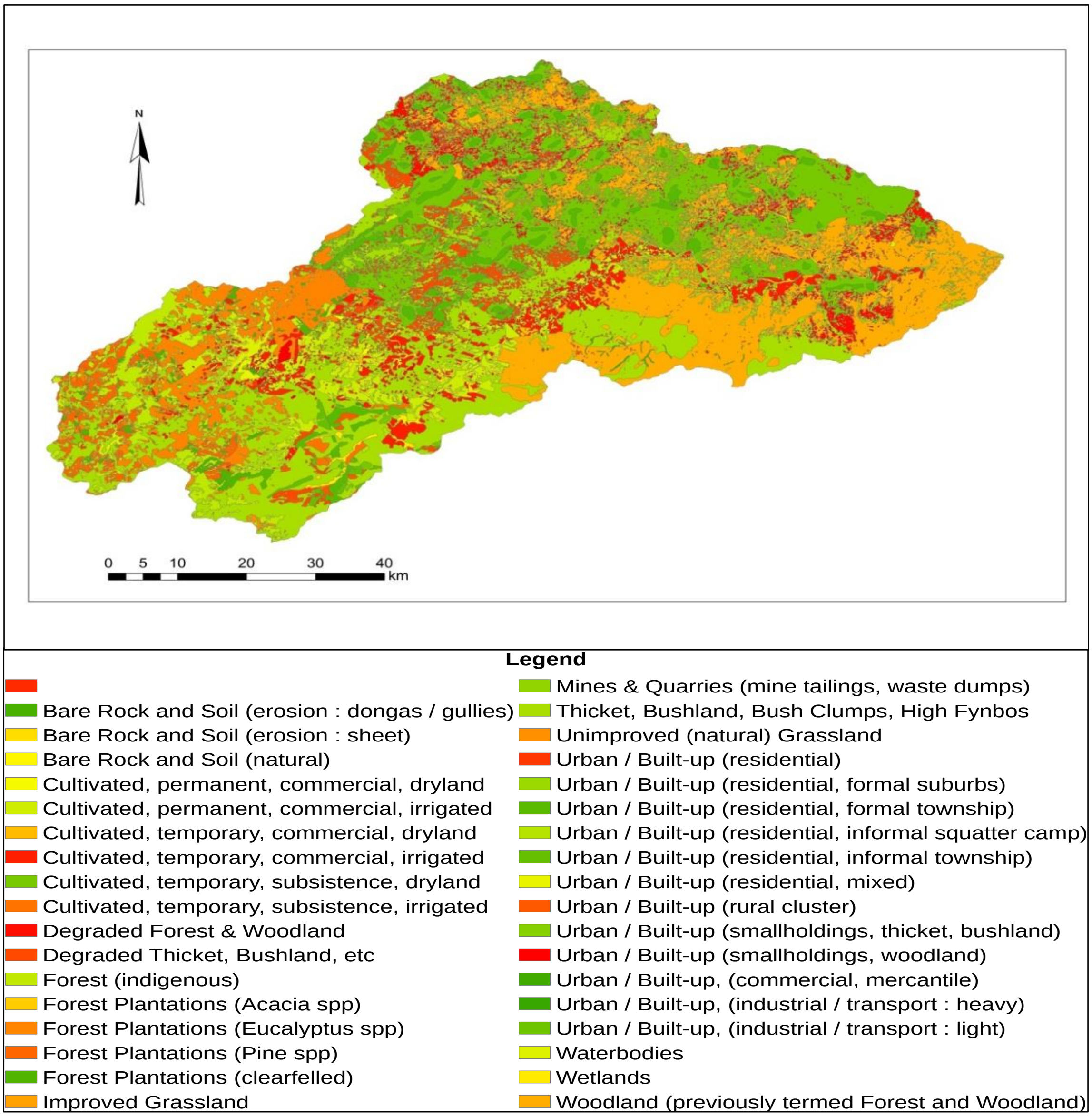


Figure 3.1: Great Letaba River Catchment Land Use Layout

In this research, conceptual models of surface water and groundwater systems were developed for the study area and then integrated to form a conjunctive water use conceptual model. The surface water and groundwater models were developed based on water balance and the law of conservation of mass. The water balance accounts for the difference in input and outputs in the system causing change in water storage of the systems.

The overall objective of conjunctive water use modelling was to develop water resources management tool that can manage utilization of both water sources in a sustainable (optimal) manner. It was assumed that surface water storage is fully utilized during normal wet period while recharging the aquifer storage system for use in dry periods. This means that in dry periods, groundwater supply is supplemented by surface water to meet total demands. This is because conjunctive use of water resources relies on the principle that by using surface water when it is plentiful, recharging aquifers and conserving groundwater supplies in wet periods, water will then be available for future pumping in the dry periods when surface water supplies are short (Mariño, 2001).

3.2.1 Surface Water Conceptual Model

In this sub-section, a conceptual representation of surface water hydrological processes components and water use pattern of a river basin is first presented. The overall surface water modelling effort was to determine hydrological components for estimating the net recharge that can be used as an input into groundwater model.

In general, surface water conceptual models are the representation and interpretation of the characteristics and dynamics of a surface water system which is based on an examination of all available hydrological data for a modelled area. This includes the external configuration of the system, water extraction rates, location and rates of inflow and outflow (Mandle, 2002). In modelling surface water system the following water balance components were considered: i) Precipitation; ii) Surface runoff; iii) Return Flows; iv) Evaporation; v) Surface water diversions (withdrawals); vi) Surface water infiltration; and vii) Surface water seepage. Figure 3.2 summarizes the surface water hydrological balance conceptual representation.

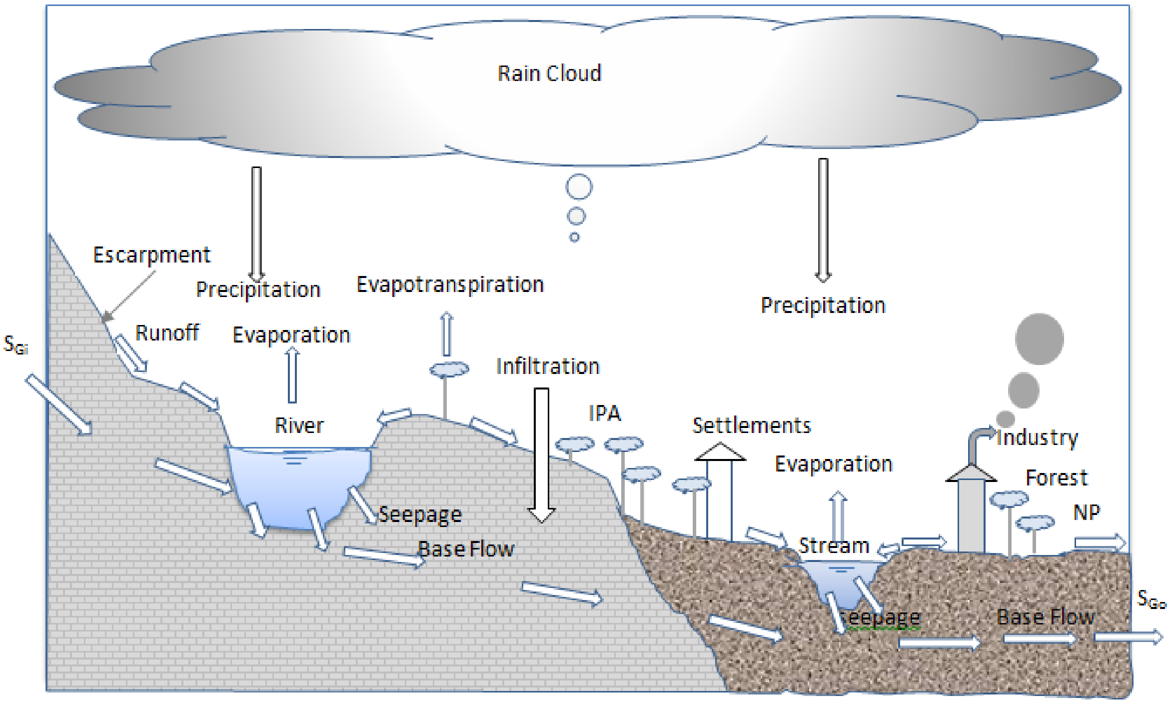


Figure 3.2: Surface Water Hydrological Balance Conceptual Representation

In Figure 3.2, S_{Gi} and S_{Go} represent the subsurface (groundwater) inflow and outflows respectively; IPA represents the Irrigation Plantations Area; and NP is the National Park. It should be understood that out of the infiltrated (net recharge) water into the soil, some percentage contributes to the base flow and the rest of it percolates into the aquifer system as groundwater recharge. Generally, surface water balance in the river basin can be defined as follows:

$$Q_{inf} - Q_{outf} = \pm \Delta S(p) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3.1)$$

where Q_{inf} is the total inflow; Q_{outf} is the total outflow; and $\pm \Delta S(p)$ is the change in surface water storage per given time period p.

Moreover, surface water systems governing continuity equations are defined as follows:

(i) The continuity equation for river/stream course system in each time step is defined as:

$$\mathrm{RST}(p)=\mathrm{RST}(p\text{-}1)+\begin{bmatrix} P(p)A_{RC}+(1-\mu)X_{ucw}(p)-\mathrm{E}_{0R}(p)\mathrm{A}_{RC}-\mathrm{X}_{DFout}(p) \\ +\mathrm{X}_{RERin}(p)\pm\sum_{i=1}^{TRR}\sum_{p=1}^{TP}X_{LR}(i,p)\ -\ \mathrm{X}_{RIVD}\ (p) \end{bmatrix}*\Delta p \quad ..(3.2)$$

where $\mathrm{RST}(\mathrm{p}-1), \mathrm{RST}(\mathrm{p})$ are the river/stream storage volume at the beginning and end of the time step p; $P(p)$ is the precipitation at time period p; μ is the percent of consumed water for different competing water users that is returned to the river/stream; $X_{ucw}\ (p)$ is the volume of water which is not consumed by various users and crops at time p (L^3/T); $E_{0R}(p)$ is the potential evaporation at time p (L/T); A_{RC} is the surface area of the river/stream course; $X_{DFout}\ (p)$ is the outflow to the downstream from the last reach of the river/stream course at time p (L^3/T); $\mathrm{X}_{\mathrm{RERin}}(\mathrm{p})$ is the inflow from surface water reservoir release to the first reach of the river/stream at time p (L^3/T); X_{LR} (i, p) is the surface water river/stream leakage to the river or from the river to the aquifer at river/stream reach i at time p (L^3/T); $X_{RIVD}\ (p)$ is the surface water diversions (withdrawal rate) to demand zones from river/stream (L^3/T) at time p; TRR is the number of river/ stream reaches; TP is the number of time period horizon and Δp is the time step (T).

(ii) The continuity equation for reservoir system in each time step is defined as follows:

$$\mathrm{REST}(p) = \mathrm{REST}(p\text{-}1) + \begin{bmatrix} \mathrm{X}_{NREin}(p) + \mathrm{P}(p)\mathrm{A}_{RE} - \mathrm{X}_{REout}(p) - \mathrm{X}_{RESD}(p) \pm \mathrm{X}_{REL}(p) - \\ X_{SPL}(p) - E_{0RE}(p)\mathrm{A}_{RE} \end{bmatrix}*\Delta p$$

....(3.3)

Where $\mathrm{REST}\,(\mathrm{p}-1)$, $\mathrm{REST}\,(\mathrm{p})$ is the reservoir (Dam) storage volume at beginning and end of the time step p (L^3); $X_{NREin}(p)$ is the net inflow to the reservoir (Dam) (L^3/T) at time period p; P(p) is the precipitation rate (L/T) at time period p; A_{RE} is the surface area of the reservoir (Dam) (L^2); $X_{REout}(p)$ is the reservoir (Dam) release to the first reach of the river or stream at time period p; $X_{RESD}(p)$ is the diversion rates from the reservoir (Dam) directly to the demand zones at time period p (L^3/T); $X_{REL}(p)$ is the leakage (seepage) to the reservoir (Dam) or from the reservoir (Dam) to the aquifer at time period p (L^3/T); $X_{SPL}(p)$ is the spilled water from the reservoir (dam) at time period p (L^3/T); $E_{0RE}(p)$ is the potential evaporation from reservoir at time period p (L/T); and A_{RE} is the surface area of the reservoir (Dam) (L^2).

It should be noted that, for a given surface water diversion withdrawal rates from all diversion (abstraction) points of the surface water storage systems (in this case, river course diversion withdrawal rates $X_{RIVD}(p)$) and reservoir diversion withdrawal rates $X_{RESD}(p)$) the total surface water diversion withdrawal rates in a given period, denoted by X_{sw} is defined as:

$$X_{sw} = X_{RIVD}(p) + X_{RESD}(p) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(3.4)}$$

where $X_{RIVD}(p)$, $X_{RESD}(p)$ and p are as previously defined.

Moreover, based on the continuity equations (3.2) and (3.3) with assumption that surface water system response is a linear function of the surface water withdrawal rates; the total surface water diversion withdrawals rates X_{sw} (from all identified potential surface water sources (in this case river/stream course and reservoir (Dam)) diversion points) for a given period can be derived as:

$$X_{sw} = [\Upsilon_1 \sum_{i=1}^{TRR} \sum_{p=1}^{TP} \left[X_{RRin}(i,p) - X_{RRout}(i,p) - E_{0R}(p)A_{RC} - (\frac{RSTV(p)-RSTV(p-1)}{\Delta p})\right] + \Upsilon_2 \sum_{n=1}^{TRED} \sum_{p=1}^{TP} [X_{REin}(n,p) - X_{REout}(n,p) - E_{0RE}(p)A_{RE} - \left[X_{REin}(n,p) - X_{REout}(n,p) - E_{0RE}(p)A_{RE} - (\frac{RESTV(p)-RESTV(p-1)}{\Delta p})\right]], i = 1, \dots, TRR, n = \ p = 1, \dots, TP, 1, \dots, TRED. \qquad (3.5)$$

Where:

- Υ_1 and Υ_2 are the weighting factors for indicating the allocation of surface water supplies from various surface water sources (in this case river course and reservoir) to different demand zones;
- TRR is total number of river/stream reaches;
- TP is the total time planning period horizon (T);
- X_{RRin} (i,p) is the inflow (L^3 /T) to the grid cell of the first river/stream reach i from the reservoir (dam) in time period p;
- X_{RRout} (i,p) is the outflow (L^3/T) from the grid cell of the last river/stream reach i downstream in time period p;
- $E_{0R}(p)$ is the potential evaporation (L/T) from the river/stream course in time period p;
- A_{RC} is the surface area of the river or stream course (L^2);
- $TRED$ is total number of surface water diversion points of the reservoirs (Dams);
- X_{REin} (p) is the inflow (L^3/T) to the reservoir (Dam) in time period p;
- X_{REout} (p) is the outflow (L^3/T) from the reservoir (Dam) to the river or stream in time period p;

- $E_{0RE}(p)$ is the potential evaporation (L/T) from the reservoir (Dam) in time period p;
- A_{RE} is the surface area of the reservoir(Dam);
- p is the time period;
- $RSTV(p)$ and $RSTV(p-1)$ are the river course storage volumes at the beginning and end of the time period $p,\ p-1$ respectively; and
- $RESTV(p)$ and $RESTV(p-1)$ are the reservoir (Dam) storage volumes at the beginning and end of the time period $p,\ p-1$ respectively.

In this study, river course storage volumes ($RSTV(p), RSTV(p-1)$) and reservoir (Dam) storage volumes ($RESTV(p), RESTV(p-1)$) at the beginning and end of the time period $p,\ p-1$, were estimated directly from changes in water levels of river course and reservoir by using rule curve storage technique. Rule curve storage technique can be used to estimate storage volume of river/stream course and reservoir (Basagaoglu *et. al.,* 1998).

3.2.2 Groundwater Conceptual Model

The overall objective of groundwater modelling was to develop groundwater conceptual model based on groundwater hydrological balance and the law of conservation of mass. This is the model that was integrated with the surface water model so as to determine sustainable (optimal) conjunctive water use strategy for implementation. In groundwater conceptual modelling sources and sinks of water, physical boundaries, distribution of hydraulic properties and groundwater use patterns were characterized.

Moreover, hydrological boundaries were identified and the mathematical boundary conditions was based on the identified hydrological boundaries i.e.,: i) Specified flow boundaries for Neumann conditions, in which derivative of head flux across the boundary is known; ii) Head dependent flow boundaries for Caunchy or mixed boundary condition. In this case, flow flux across the boundary is determined once a boundary head value is known; and iii) Specified head boundaries for Dirichlet condition whereby head is known. Usually, the best way to choose initial conditions is to consider it as a steady state head solution generated by a calibrated model. This is because the use of model generated head values ensures that the initial head data and the model hydrologic inputs and parameters are consistent (Bark, 1995). Figure 3.3 summarizes a Groundwater conceptual representation of hydrological and hydrogeological unit water balance components of aquifer system.

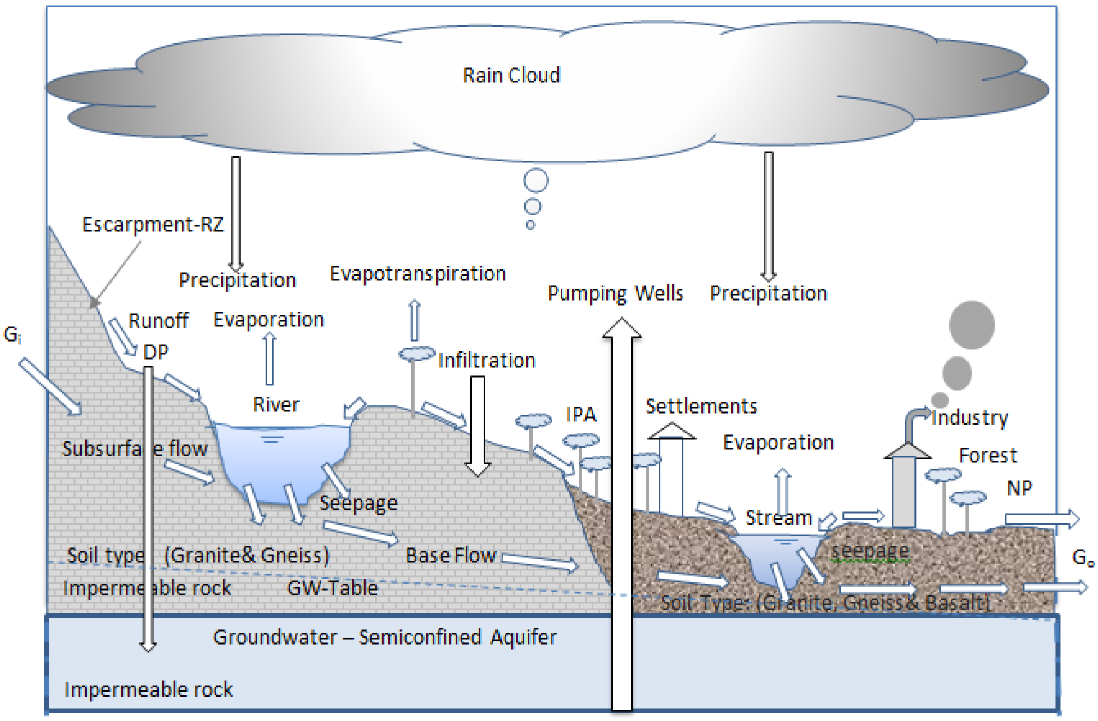


Figure 3.3: Groundwater Hydrological and Hydrogeological Unit Water Balance Conceptual Representation

In Figure 3.3, G_i , G_o are the Groundwater inflow and outflows respectively; NP is the National Parks; DP is the Deep Percolation; RZ is the Recharge Zone; GW-Table is the groundwater table; and IPA is the Irrigation Plantations Area. The following groundwater balance components were considered in estimating net recharge of the river basin: i) Recharge from Precipitation; ii) Surface runoff; iii) Actual Evapotranspiration; iv) Pumpage from wells; v) Recharge from canal distributaries; vi) Potential deep percolation from the unsaturated zone underlying an irrigated field to water table; vii) Recharge from water courses; viii) Evaporation from fallow/bare land; ix) Recharge from spilled water from dams (reservoirs); and x) Recharge from excess water not made available to crops or other competing water users.

Based on water balance of the hydrological and hydrogeological unit, the net recharge was determined and treated as input to groundwater model. The net recharge of the river basin area was estimated as:

$$Q_{net} = R_{FP} + DP_{FF} + R_{DDM} + R_{RWC} + R_{REWB} + R_{RSPL} + R_{IL} + R_{LC} + INFL_{AA} - S_{ROF} - ET_o - E_{FL} - \mathrm{X_{GPBW}} - \mathrm{X_{GPRW}} - \mathrm{SP_{WTSD}} \quad \ldots\ldots (3.6)$$

where Q_{net} is the net recharge to the aquifer under consideration; R_{FP} is the recharge from precipitation; DP_{FF} is the deep percolation from field; R_{DDM} is the recharge from diversion canals major and minors; R_{RWC} is the recharge from river or streams water courses (see page); R_{REWB} is the recharge from reservoirs (Dams) water bodies (seepage); R_{RSPL} is the recharge from reservoirs (Dam) spillway; R_{IL} is the recharge from irrigated land; R_{LC} is the recharge from leaking canals; $INFL_{AA}$ is the inflow from adjacent area; S_{ROF} is the surface runoff; ET_o is the potential evapotranspiration; E_{FL} is the evaporation from fallow land; $\mathrm{SP_{WTSD}}$ is the discharge (seepage) from water table to surface water river systems; $\mathrm{X_{GPBW}}$ is the groundwater pumpage by public owned wells; and $\mathrm{X_{GPRW}}$ is the groundwater pumpage of private owned wells.

There are various ways of categorising pumping wells; however, in this research groundwater pumping wells are grouped into two main categories, namely, private owned wells and public owned wells. Thus, the total groundwater pumping withdrawal rates (denoted by X_{gw}) from all identified potential operating pumping wells (in this case public wells and private wells denoted by $\mathrm{X_{GPBW}}$ and $\mathrm{X_{GPRW}}$, respectively), can be defined as:

$$X_{gw} = X_{GPBW} + X_{GPRW} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(3.7)}$$

It should be noted that the pumping withdrawals rates X_{gw} is one of the main components of the groundwater balance and net recharge estimation. In groundwater flow modelling, a partial differential governing continuity equation for a non-steady state, three – dimensional heterogeneous anisotropic case of the groundwater flow in saturated media, for a confined/unconfined aquifer system under interaction of surface water (recharge) and/or aquifer discharge is defined as (Tran, 2004; Todd & Mays, 2005; Safavi & Bahreini, 2009):

$$\frac{\partial}{\partial x}\left(K_{xx}\frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_{yy}\frac{\partial h}{\partial y}\right) + \frac{\partial}{\partial z}\left(K_{zz}\frac{\partial h}{\partial z}\right) \pm W = S_s\left(\frac{\partial h}{\partial t}\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(3.8)}$$

where K_{xx}, K_{yy} and K_{zz} are the hydraulic conductivities along the x, y, and z coordinates axes parallel to the major axes of hydraulic conductivities; h is the hydraulic potentiometric head; S_s is the specific storage of the porous medium; $\pm W$ is a volumetric flux per unit volume (it is a source or sink of water that is a function of space and time (x, y, z, t)); and t is time. Thus equation (3.8) is a Mathematical Model of groundwater system.

3.2.3 Conjunctive Water use Conceptual Model

Conjunctive water use conceptual model is a representation of dynamics of hydrologic processes and hydrogeologic settings, and response of all water source systems (in this case surface water and groundwater systems) to the external stresses. The conjunctive water use conceptual model was developed through integrating the surface water and groundwater models. Figure 3.4 summarises the conjunctive water use patterns, hydrological process and hydrogeological unit conceptual representation.

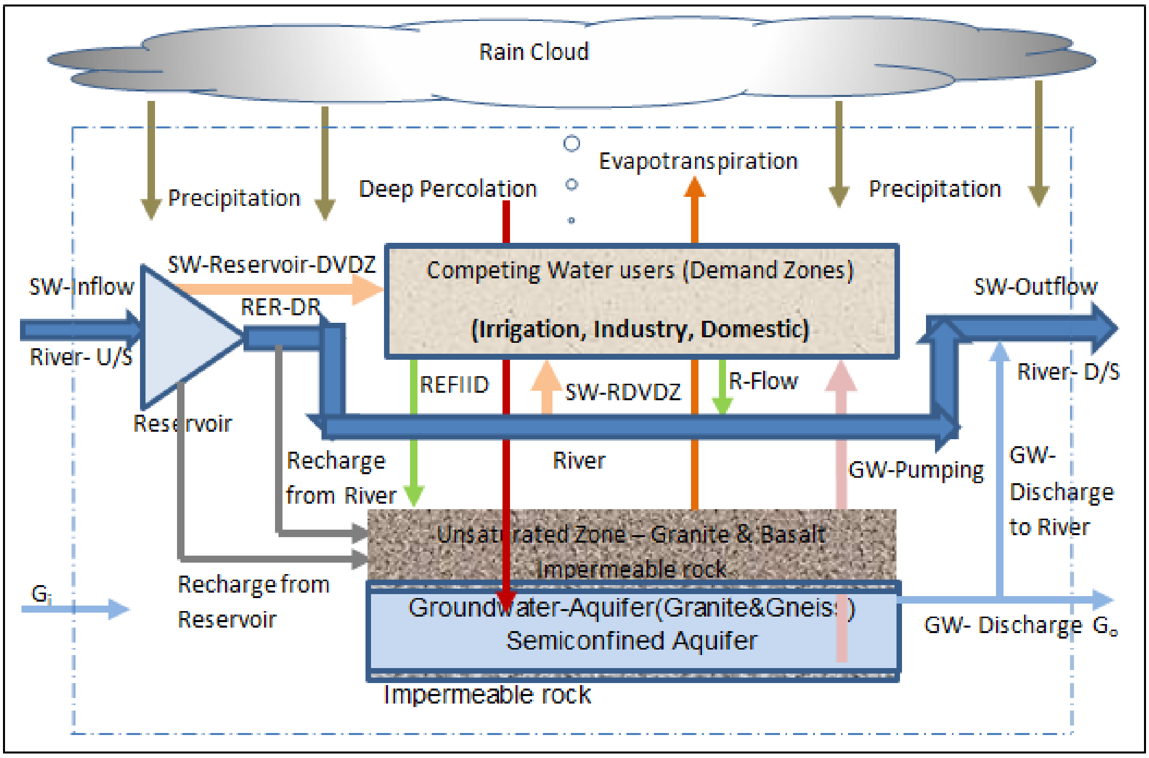


Figure 3.4: Conjunctive Water Use Patterns, Hydrological Process and Hydrogeological Unit Conceptual Representation

In Figure 3.4, SW-Inflow is the surface water inflow; SW-Outflow is the surface water outflow; SW-Reservoir-DVDZ is the surface water reservoir diversion (withdrawal) rates-direct to demand zones (which is referred to as X_{RESD} in surface water conceptual model); SW-RDVDZ is surface water river diversion (withdrawal) rates-direct to demand zones (which is referred to as X_{RIVD} in surface water conceptual model); GW-Pumping, is the groundwater pumping withdrawal rates (which is referred to as X_{gw} in groundwater conceptual model which is representing total pumping rates of public wells, X_{GPBW}, and private wells, X_{GPRW}); G_i, G_o are the Groundwater (GW) inflow and outflow, respectively; REFIID is the Recharge from water users; R-Flow is the return flow to the river course from water users; and RER-DR is the reservoir release direct to the river.

As it was discussed in subsections (3.1.2) and (3.1.3), variables X_{sw} and X_{gw}, represent the total surface water diversion (withdrawal) rates (i.e., river course diversion (withdrawal) rates X_{RIVD} and reservoir diversion (withdrawal) rates X_{RESD}) and total groundwater withdrawal rates (i.e., public wells X_{GPBW} and private wells X_{GPRW} withdrawal rates), respectively. Thus, from now on, total conjunctive water use withdrawal rates means the sum of surface water and groundwater withdrawal rates from identified potential surface water and groundwater sources and is defined as:

$$Z = X_{sw} + X_{gw} \quad \text{...} \quad (3.9)$$

where Z is the total conjunctive water use withdrawal rates from both surface water and groundwater sources. It should be noted that for sustainable conjunctive water use, it is imperative that the surface water and groundwater resources withdrawal rates must be optimal under prescribed sets of constraints. In this study, a simulation-optimization technique was adopted for development of conjunctive water use management model in which conjunctive water use simulation model is used to determine aquifer water system responses when the aquifer is subjected to external stresses. The responses from aquifer water system were used as coefficients of constraints which govern the groundwater drawdowns in the conjunctive water use management modelled domain.

Conjunctive Water Use Simulation Model

In conjunctive water use simulation, groundwater flow numerical model and river package (RIV) (herein refers as conjunctive use simulation model) were used to simulate surface water-groundwater flow interactions and to determine aquifer system responses (hydraulic heads/water table level drawdowns) due to external stresses. A mathematical groundwater flow partial differential equation governing continuity for a non-steady state three-dimensional heterogeneous anisotropic case in saturated media for a confined/unconfined aquifer system under interaction of surface water (recharge) and/or aquifer discharge for groundwater flow numerical simulation was used (for details see sub-section 3.1.3 equation (3.8)), however, for clarity of purpose, the expression is represented as shown equations 3.10 – 3.13 as follows:

$$\frac{\partial}{\partial x}\left(K_{xx}\frac{\partial h}{\partial x}\right)+\frac{\partial}{\partial y}\left(K_{yy}\frac{\partial h}{\partial y}\right)+\frac{\partial}{\partial z}\left(K_{zz}\frac{\partial h}{\partial z}\right)\pm W=S_s\left(\frac{\partial h}{\partial t}\right) \quad \ldots\ldots\ldots\ldots\ldots..\ldots\ldots\ldots\ldots\ldots \text{(3.10)}$$

All parameters are as previously defined.

It should be realized that for hydraulically connected surface water and groundwater systems, the effects of the flow (interactions) term ($\pm W = QRV$)

between the surface water and groundwater systems are defined as (Todd & Mays, 2005; Safavi & Bahreini, 2009):

$$QRV = CRV\left(HRV - H_{i,j,k}\right), \quad H_{i,j,k} \geq RVBOT \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(3.11)}$$

$$QRV = CRV(HRV - RVBOT), \quad H_{i,j,k} \leq RVBOT \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(3.12)}$$

in which CRV is the hydraulic conductance of the riverbed soil materials defined as KLB/M, where K is hydraulic conductivity of the riverbed soil materials, L is the length of the river reach, M is the thickness of the riverbed soil materials, and B is the width of the river reach. In the equations (3.11) and (3.12), the parameters, HRV is the head of river (i.e., river stage); $H_{i,j,k}$ is the head of aquifer water table underlying the river reach; and $RVBOT$ is the elevation of the bottom of the riverbed soil material layer.

Equation (3.11) indicates that hydraulic head (aquifer water level) is greater than or equal to the river/stream head (river/stream stage) which may result in water flowing from aquifer to the river/stream (a situation called gaining stream), while, equation (3.12) indicates a situation where head of aquifer water is less than or equal to the elevation of the bottom of the river/streambed layer which may result in water flowing from the river/stream to the aquifer water system (a situation called loosing stream).

This implies that interactions between surface water bodies and groundwater system depend on their head differences. These situations are common in alluvium aquifer systems in hydraulically connected surface water and groundwater systems. To ensure sustainability of the ecosystems and downstream water requirements, groundwater hydraulic heads near river/stream corridors were restricted not to drop below elevation of river/streambed bottom during the development of optimization models (management models).

It is important to realize that in modelling hydraulically connected surface water and groundwater systems, the interactions (flow $q_{bi,j,k}$) of water into and out of grid cell from external source (sink/source) with a known hydraulic head is considered to be directly proportional to the head difference of the model domain grid cell, $H_{i,j,k}$, and the associated head assigned to the external source, $H_{bi,j,k}$, and is defined as (Todd & Mays, 2005; Safavi & Bahreini, 2009):

$$q_{bi,j,k} = C_{bi,j,k}(H_{bi,j,k} - H_{i,j,k}) \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (3.13)$$

in which $C_{bi,j,k}$ is the hydraulic conductance of soil material between the external source and the grid cell (i, j, k).

Equations (3.10), (3.11), (3.12) and (3.13) together with associated specifications of aquifer properties (such as initial head conditions, hydraulic conductivity, storage, specific yield), boundary conditions of the aquifer system, and the river/stream parameters (such as river/stream stages, width, river bottom elevations, and river/streambed conductance) forms mathematical models for conjunctive water use simulation model of surface water and groundwater systems (Todd & Mays, 2005).

To solve these equations, numerical methods (such as finite difference or finite element) are usually used. In this research, the MODFLOW-2000 (Harbaugh *et al.,* 2000) code-a three dimensional finite-difference model of USGS, together with

River (RIV) Package supported in Visual MODFLOW Classic Interface (Software) was used to simulate the surface water-groundwater hydrogeological unit. In numerical modelling, the conjunctive water use model can be simulated in two ways deterministically and/or stochastically. In this research, simulation was done in both deterministic and stochastic frameworks.

Deterministic Simulation

Usually, after model calibration, surface water-groundwater conjunctive water use model can be simulated to determine hydraulic heads or water table levels in steady state and/or transient state conditions while assuming deterministic or stochastic input parameter(s) values. In this case, the groundwater numerical model (here referred to as surface water-groundwater conjunctive water use numerical model) was simulated deterministically. In deterministic simulation, aquifer system was assumed to be homogeneous while all model parameters were assumed deterministically fixed and accurately known.

Stochastic Simulation

In stochastic simulation, one needs to take into account uncertainty during simulation process. In surface water-groundwater conjunctive water use system, sources of uncertainty arise from both surface water and groundwater resource systems. In groundwater systems, uncertainty arises from aquifer parameters (such as hydraulic conductivity or transmissivity, recharge) and boundary conditions while in surface water resource systems, uncertainty arises from hydraulic conductivity/conductance of riverbed soil materials and natural variability of climatic inputs such as river flows, air temperature and precipitation (Safavi & Bahreini, 2009).

In this research, it was assumed that all parameters are adequately and precisely known except hydraulic conductivity of aquifer system, which is considered as the only source of uncertainty in conjunctive water use system. Thus, the stochastic process focused on spatial distribution of hydraulic conductivity of the aquifer system. This is because parameters such as hydraulic conductivities or riverbed hydraulic conductivity/conductance are recognised to be the most uncertain parameters in surface water-groundwater aquifer systems (Gelhars, 1993; Safavi & Bahreini, 2009).

In this study, simulation of hydraulically connected surface water-groundwater system was carried out to determine the response of the systems when the aquifer is subjected to the external stresses (such as surface water and groundwater withdrawal rates). This enables us to understand the physical behavior of the conjunctive water use system (surface water-groundwater system). This is because in a conjunctive water use system, simulation models account for the physical behavior of surface water-groundwater system (Safavi *et al.,* 2009).

However, it should be realized that simulation of surface water and groundwater systems alone can only enable the understanding of surface water-groundwater interactions and the behaviour of the systems when subjected to external stresses. Hence, in order to have an efficient management and adequate information of the systems behaviour, the best way is to link simulation and optimization models.

Optimization model assists to find out how best the water resource systems should be managed (i.e., the optimum strategy to be implemented).

Usually, conjunctive water use management optimization model has socio-economic and/or hydraulic constraints. However in this research, hydraulic constraints (such as maximum total allowable groundwater level drawdowns and river/stream stages depletion) were considered to satisfy the total demand (for irrigation, domestic, mining, and industrial) leading to the optimal water withdrawal strategy for both surface water and groundwater resources.

3.3 DEVELOPMENT OF CONJUNCTIVE WATER USE MANAGEMENT MODEL

Management models (in this case, simulation-optimization models) are developed to assist water resource managers and decision makers to solve water resources management problems. Normally, these simulation-optimization mathematical expression procedures are developed to describe a system and its response to the system inputs for various system excitations.

3.3.1 Deterministic Conjunctive Water Use Management Model

The overall goal of conjunctive water use management model designed in this research was to ensure that surface water and groundwater resources are used (withdrawn) in a sustainable manner. Thus utilization of surface water and groundwater resources should not exceed the allowable limits for sustainance of base flow. In this research, surface water and groundwater withdrawal rates, X_{sw}, X_{gw} , respectively, were derived based on water balance and continuity equations, and was considered to be a linear function relationship (see equations (3.5) and (3.7) in sub sections 3.2.1 and 3.2.2, respectively).

In developing conjunctive water use management model, optimization procedures were formulated as mathematical expressions to describe a system and its response to the system inputs for various parameters. Using response matrix approach, the unit responses (also referred to as an algebraic technological function) were numerically determined. The basic idea of response matrix approach is that the optimization procedures are externally linked with groundwater simulation model (in this case referred to as conjunctive water use simulation model). Simulation model generates the inputs (aquifer system responses) to the optimization model.

Numerically, to determine unit response of an aquifer system, a separate groundwater simulation was done for each pumping well. These generated responses are input to the optimization model as coefficients a_{ij} to the constraints (see inequality (3.16) in optimization problem formulation (3.15) through (3.21)) which restrict lowering of groundwater levels. Based on hydrogeological conditions of the aquifer system, it was assumed that responses (drawdowns) of aquifer system were linearly related to the groundwater pumping (withdrawal) rates, X_{gw}.

Now, recall equation (3.9) in sub-section 3.2.3, the total conjunctive water use of surface water and groundwater withdrawal rate as previously denoted as $Z,$ herein rewritten as:

$$Z = X_{sw} + X_{gw} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(3.14)}$$

All parameters are as previously defined.

It was assumed that the aquifer system was homogenous, semiconfined and that surface water and groundwater systems were hydraulically connected, and that surface water-groundwater (aquifer) system responses were linearly related to groundwater pumping rates. Hence, a deterministic linear conjunctive water use optimization problem was formulated as:

$$\underset{Z}{Maximize} \quad Z = \left[c^T x_{sw} + \tilde{c}^T x_{gw}\right] \qquad \text{(3.15)}$$

Subject to: $$\sum_{i=1}^{Ngcp} a_{ij}\, x_{gw,j} \leq b_i, \quad i = 1,2,\dots,N_{gcp}\,, \;\; j = 1,2,\dots,N_{gpw}, \qquad \text{(3.16)}$$

$$H_{h,r} \geq RVBOT_{h,r}, \quad h = 1,2,\dots,N_{scp}, \;\; r = 1,2,\dots\dots,N_{sdp} \qquad \text{(3.17)}$$

$$\left[\sum_{r=1}^{Nsdp} x_{sw,r} + \sum_{j=1}^{Ngpw} x_{gw,j}\right] \geq WD_T \qquad \text{(3.18)}$$

$$\sum_{j=1}^{N_{gpw}} x_{gw,j} \leq TR \qquad \text{(3.19)}$$

$$x_{gw,j} \geq 0, \quad j = 1,2,\dots,N_{gpw} \qquad \text{(3.20)}$$

$$x_{sw,r} \geq 0, \quad r = 1,2,\dots,N_{sdp} \qquad \text{(3.21)}$$

where Z is the objective function (i.e., the total conjunctive water use of surface water and groundwater resources); c and $\tilde{c}$ are the coefficients of the objective function; x_{sw} are the diverted surface water withdrawal rates from all surface water sources (in this case, reservoir and river or stream) (L^3/T); N_{scp} is the number of surface water control points; $H_{h,r}$ is the hydraulic head of the aquifer system underlying the river reach r at river control point h; $RVBOT_{h,r}$ is the elevation of the bottom of the riverbed soil material layer of river reach r at river control point h;

N_{sdp} is the number of surface water diversion (withdrawal) points; x_{gw} is the groundwater pumping rates from pumping well j, (L^3/T); N_{gcp} is the number of groundwater control points; a_{ij} is the response at control point i due to groundwater pumping in well j; b_i is the constraining value at control point i; N_{gpw} is the number of groundwater pumping wells; TR is the groundwater total net recharge (L^3/T); and WD_T is the total water demand.

Solution Methodology

To solve the deterministic conjunctive water use optimization problem (formulation (3.15) through (3.21)), the computer program executed the following main steps:

(1) From available data values (i.e., hydraulic conductivity and boundary conditions), read the input files necessary for groundwater and surface water simulation models (MODFLOW-USGS Groundwater Modular and RIVER Package (RIV)).

(2) Run MODFLOW simulation model (in this case conjunctive water use simulation model) for the two cases; i.e., without pumping wells and with pumping wells (in this case simulation was made for each pumping well with unit pumping rate at a time).

(3) From the output files of simulation model (i.e., the hydraulic heads output of step (2)) computation of responses of aquifer water system (i.e., hydraulic head drawdowns) at each specified control point location was done. The response coefficients were computed by finding the hydraulic heads differences of the two cases (i.e., without pumping and with pumping).

(4) Repeat steps (2) and (3) for each operating groundwater pumping well.

(5) Assemble response coefficients components produced in step (3) to form a response matrix of aquifer water system (response matrix represents aquifer system responses due to unit pumping rates from each operating groundwater pumping well).

(6) Based on the response matrix realized in step (5) and the objective function and constraints of the optimization management model, solve the problem using Active-Set (SQP standard core optimizer (algorithm)) implemented under the MATLAB environment. Figure 3.5 shows flow chart for the solution methodology.

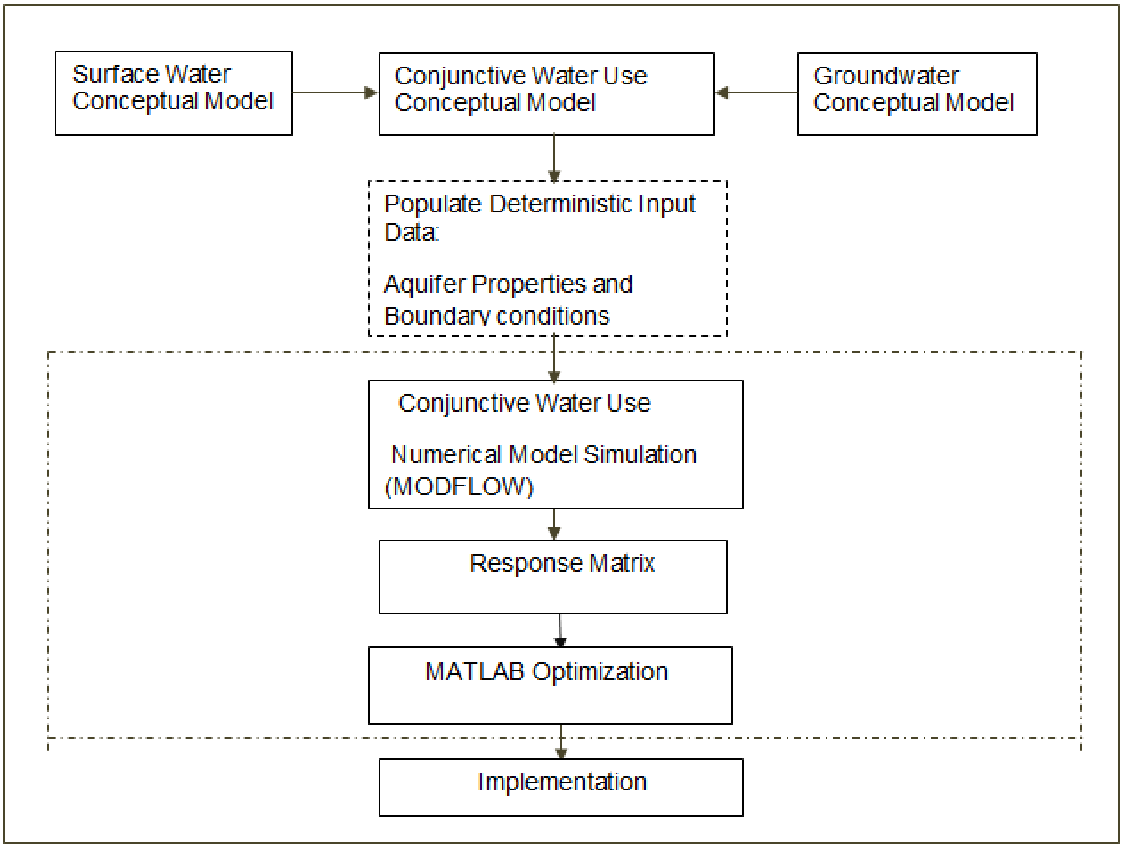


Figure 3.5: Flow Chart for Deterministic Management Conjunctive Water Use Model

The bottleneck of approaching an optimization problem in a deterministic way is the assumption that the input parameters (in this case hydraulic conductivities) are precisely known without error which is invalid in real world phenomena. Thus, in order to address uncertainties associated with the management model, sub-section 3.3.2 was designed to present a stochastic conjunctive water use management framework which was used to capture uncertainties, particularly, uncertainty brought about by scarcity of data (such as aquifer hydraulic conductivity) to fully characterize surface water-groundwater system.

3.3.2 Stochastic Conjunctive Water Use Management Model

In stochastic conjunctive water use management system, uncertainties arise from both surface water and groundwater systems. In this research, the goal for solving conjunctive water use optimization problem under uncertainty conditions was to understand how input parameter uncertainty affects the conjunctive water use management model.

In other words, the conjunctive water use management model was developed and solved under uncertainty condition so as to evaluate the impact of the uncertainty on the optimal conjunctive water use model. This uncertainty was included in the optimization formulation by sampling the stochastic field of continuous hydraulic conductivity field values of aquifer system. The sampling was carried out by considering a large set of realizations with every realization having a distinct value of the hydraulic conductivity for each grid cell of modelled domain.

Assuming that the uncertainty realizations of the hydraulic conductivity for aquifer system is denoted by ϖ, where ϖ = 1, 2, …., Ω, each realization ϖ will have different response matrices A_{ϖ} with response components $a_{ij,\varpi}$ and therefore, this may result in different optimal solutions. For the purpose of clarity, refer to the deterministic conjunctive water use optimization problem (formulations (3.15) – (3.21)) herein rewritten as:

$$\underset{Z}{Maximize} \quad Z = \left[c^T x_{sw} + \tilde{c}^T x_{gw}\right] \qquad \text{(3.22)}$$

Subject to: $$\sum_{i=1}^{Ngcp} a_{ij}\, x_{gw,j} \leq b_i, \quad i = 1,2,\dots,N_{gcp}\,, \; j = 1,2,\dots,N_{gpw}, \qquad \text{(3.23)}$$

$$H_{h,r} \geq RVBOT_{h,r}, \quad h = 1,2,\dots,N_{scp}, \; r = 1,2,\dots,N_{rc} \qquad \text{(3.24)}$$

$$\left[\sum_{r=1}^{Nsdp} x_{sw,r} + \sum_{j=1}^{Ngpw} x_{gw,j}\right] \geq WD_T \qquad \text{(3.25)}$$

$$\sum_{j=1}^{N_{gpw}} x_{gw,j} \leq TR \qquad \text{(3.26)}$$

$$x_{gw,j} \geq 0, \quad j = 1,2,\dots,N_{gpw} \qquad \text{(3.27)}$$

$$x_{sw,r} \geq 0, \quad r = 1,2,\dots,N_{sdp} \qquad \text{(3.28)}$$

All parameters are as previously defined. However, due to randomness in hydraulic conductivity field of the aquifer system, this randomness gives rise to different response values of the hydraulic head drawdown (the response parameters,a_{ij}).

Now, let uncertain aquifer system response coefficient elements be denoted by $a_{ij,\varpi}$. Replacing coefficient components a_{ij} in the deterministic constraint inequality (3.23) with uncertain coefficients responses, $a_{ij,\varpi}$ makes the conjunctive water use optimization problem (formulations (3.22) through (3.28)) to become stochastic under uncertainty realizations in hydraulic conductivity field. Therefore, the stochastic conjunctive water use optimization problem under uncertainty was formulated as:

$$\underset{Z}{Maximize} \quad Z = \left[c^T x_{sw} + \tilde{c}^T x_{gw}\right] \quad \text{……………..…………..... (3.29)}$$

Subject to: $\sum_{i=1}^{Ngcp} a_{ij,\varpi}\, x_{gw,j} \le b_i, \quad i = 1,2,\dots,N_{gcp}\,, \;\; j = 1,2,\dots,N_{gpw},$ …... (3.30)

$$H_{h,r} \ge RVBOT_{h,r}, \quad h = 1,2,\dots,N_{scp}, \;\; r = 1,2,\dots\dots,N_{sdp} \quad \text{…….. (3.31)}$$

$$\left[\sum_{r=1}^{Nsdp} x_{sw,r} + \sum_{j=1}^{Ngpw} x_{gw,j}\right] \ge WD_T \quad \text{………..…..…. (3.32)}$$

$$\sum_{j=1}^{N_{gpw}} x_{gw,j} \le TR \quad \text{…………………..……......……..…. (3.33)}$$

$$x_{gw,j} \ge 0, \quad j = 1,2,\dots,N_{gpw} \quad \text{…………..…...…. (3.34)}$$

$$x_{sw,r} \ge 0, \quad r = 1,2,\dots,N_{sdp} \quad \text{………….………….….…. (3.35)}$$

All parameters are as previously defined. The constraint (inequality (3.30)) is stochastic. It is stochastic because of its dependence on realization ϖ. Hence, formulations (3.29) through (3.35), is stochastic optimization problem because the responses $a_{ij,\varpi}$ are dependent on each realization ϖ of uncertain hydraulic conductivity field of groundwater (aquifer) system. Solution methodology for solving the above stochastic optimization problem was developed. In this research, a new (in the context of conjunctive water use management approach) approach – the Retrospective Optimization Approximation (ROA) method is proposed for solving such stochastic optimization problems and is discussed as follows:

Formulation of Conjunctive Water Use Management Using ROA Approach

As was highlighted in Chapter two, retrospective optimization approximation (ROA) method is a variant of Sample Average Approximation (SAA) class of methods which considers a sequence of increasing sample size of generated sample path sub-problems. In this case, the overall aim was to solve the stochastic conjunctive water use optimization problem with the objective of maximizing the groundwater and surface water withdrawal rates to meet total demand without compromising on the prescribed set of constraints (i.e., without lowering the hydraulic heads/water table levels and/or river flow water levels (stages) below some specified allowable maximum levels).

For the purpose of clarity, we recall the stochastic optimization problem formulations (3.29) through (3.35) herein represented as:

$$\underset{Z}{Maximize} \quad Z = \left[c^T x_{sw} + \tilde{c}^T x_{gw}\right] \quad \ldots\ldots \text{(3.36)}$$

Subject to: $$\sum_{i=1}^{Ngcp} a_{ij,\varpi}\, x_{gw,j} \leq b_i, \quad i = 1,2,\ldots,N_{gcp}\,, \;\; j = 1,2,\ldots,N_{gpw}, \quad \ldots\ldots \text{(3.37)}$$

$$H_{h,r} \geq RVBOT_{h,r}, \quad h = 1,2,\ldots,N_{scp}, \;\; r = 1,2,\ldots\ldots,N_{sdp} \quad \ldots\ldots \text{(3.38)}$$

$$\left[\sum_{r=1}^{Nsdp} x_{sw,r} + \sum_{j=1}^{Ngpw} x_{gw,j}\right] \geq WD_T \quad \ldots\ldots \text{(3.39)}$$

$$\sum_{j=1}^{N_{gpw}} x_{gw,j} \leq TR \quad \ldots\ldots \text{(3.40)}$$

$$x_{gw,j} \geq 0, \quad j = 1,2,\ldots,N_{gpw} \quad \ldots\ldots \text{(3.41)}$$

$$x_{sw,r} \geq 0, \quad r = 1,2,\ldots,N_{sdp} \quad \ldots\ldots \text{(3.42)}$$

in which all parameters are as previously defined. Formulations (3.36) through (3.42) are referred to as the True (original) Conjunctive water use Optimization Problem (TCOP).

To solve such an optimization problem to an exact solution is computationally impossible or prohibitively expensive. A simple way is to revert to Monte Carlo sampling based approximation methods. In this research, we consider Monte Carlo sampling based methods, particularly the ROA method which is a variant of Sample Average Approximation (SAA) class of methods to solve our stochastic conjunctive water use optimization problem.

Without loss of generality, a probability space (Ω, F, P) is considered to exist, where Ω is a set of all possible outcomes, F denotes a set of collection of all subsets of Ω possible outcomes, and P denotes a probability measure. Let X denote a set of decision variable vector in space $\mathbb{R}^n$ such that $n = n_{gw} + n_{sw}$ and that vector variables $x_{gw}, x_{sw} \in X$, in which x_{gw} is groundwater withdrawal rate decision variable (a vector in space $\mathbb{R}^{n_{gw}}$ of the first elements of X) and x_{sw} is surface water withdrawal rate decision variable (a vector in space $\mathbb{R}^{n_{sw}}$ of the remaining elements of X). Consider random response matrix denoted by $A_{\overline{\varpi}}$ with random responses components denoted by $a_{ij,\overline{\varpi}}$ (i.e., the responses denoted simply by $\overline{\varpi}$, that is the random hydraulic heads/water table level drawdowns). It was assume that randomness in hydraulic heads/water table level drawdowns is only due to uncertainty arising in hydraulic conductivity of aquifer system and that $a_{ij,\overline{\varpi}} \in A_{\overline{\varpi}}$ such that $A_{\overline{\varpi}} \in \overline{\Omega}$.

Moreover, P is assumed to be well defined with unknown distribution function but, what is known is expected mean values, standard deviation and/or covariance of the random responses $\overline{\varpi}$. A stochastic constraint function $G(x_{gw}, \overline{\varpi}) \in \overline{\Omega}$ was considered for the process such that $G(x_{gw}, \overline{\varpi}_i) = A_{\overline{\varpi}_i} x_{gw}$. To this end, the expected value of the function $G(x_{gw}, \overline{\varpi})$ is defined as (Kifanyi *et al.,* 2017):

$$\mathbb{E}[G(x_{gw}, \overline{\varpi})] = \mathbb{E}[A_{\overline{\varpi}} x_{gw}] \quad \ldots\ldots\ldots\ldots \text{(3.43)}$$

Thus, the constraint inequality (3.37) can be written as $\mathbb{E}[G(x_{gw}, \overline{\varpi})] \leq b$ such that the expectation $[G(x_{gw}, \overline{\varpi})] = \int_{\overline{\Omega}} G(x_{gw}, \overline{\varpi}) dP(\overline{\varpi})$ is the corresponding expected value function. Therefore, inequality (3.37) in the optimization problem (formulations (3.36) through (3.42)) can be estimated using Monte Carlo sampling based approximation methods (in this case, ROA method) by considering a sequence of finite sets of generated samples of random responses of N realizations of $\overline{\varpi} = \left\{A_{\overline{\varpi}_1}, \ldots, A_{\overline{\varpi}_{N_k}}\right\}$. The expected constraint function $\mathbb{E}[G(x_{gw}, \overline{\varpi}_{i_k})]$ was estimated as (Kifanyi *et al.,* 2017):

$$\mathbb{E}\left[G(x_{gw}, \overline{\varpi}_{i_k})\right] = \left[\frac{1}{N_k} \sum_i^{N_k} G(x_{gw}, \overline{\varpi}_{i_k})\right] \quad \ldots\ldots\ldots\ldots \text{(3.44)}$$

Thereafter, progressively evaluate resulting sample path optimization sub-problems of formulations (3.45) through (3.51) by using ROA method for $k = 1, 2, \ldots, N_{SP}$ (where N_{SP} is total number of sample path optimization sub-problems generated). Hence, by replacing the inequality constraint (3.37) with the estimates of expected constraint function, the corresponding Estimates Retrospective Conjunctive water use Sample path Optimization Problem (ERCSOP) can be formulated as:

$$\underset{Z_{N_k}}{Maximize} \quad Z_{N_k} = \left[c^T x_{sw} + \tilde{c}^T x_{gw}\right] \quad \ldots\ldots\ldots\ldots \text{(3.45)}$$

Subject to: $\left[\frac{1}{N_k}\sum_{i}^{N_k} G(x_{gw}, \varpi_{i_k})\right] \leq b_i\,, \quad i = 1,2,\ldots,N_k\,,\; k = 1,2,\ldots,N_{SP}$ .... (3.46)

$$H_{h,r} \geq RVBOT_{h,r}, \quad h = 1,2,\ldots,N_{scp}, \;\; r = 1,2,\ldots\ldots,N_{sdp} \qquad \text{...... (3.47)}$$

$$\left[\sum_{r=1}^{Nsdp} x_{sw,r} + \sum_{j=1}^{Ngpw} x_{gw,j}\right] \geq WD_T \qquad \text{.................... (3.48)}$$

$$\sum_{j=1}^{N_{gpw}} x_{gw,j} \leq TR \qquad \text{.. (3.49)}$$

$$x_{gw,j} \geq 0, \quad j = 1,2,\ldots,N_{gpw} \qquad \text{...................... (3.50)}$$

$$x_{sw,r} \geq 0, \quad r = 1,2,\ldots,N_{sdp} \qquad \text{................................ (3.51)}$$

It should be noted that formulations (3.45) through (3.51) are deterministic optimization problems which can be solved by any appropriate standard core deterministic optimization solver. In this research, the conjunctive water use optimization problems in the form of formulations (3.45) through (3.51) are referred to as the estimates retrospective conjunctive water use optimization problems.

The term $\frac{1}{\mathrm{N_k}}$ in the optimization problem inequality (3.46) defines the weight factor or probability associated with realizations $\varpi_{\mathrm{i_k}}$. Different sample sizes $\mathrm{N_k}$ (number of realizations) are considered for each sample path optimization sub-problem generated. The performance objective function considered is the expected total optimal conjunctive water use withdrawal rate. Decision variables considered are groundwater and surface water withdrawal rates (i.e., positive real values X, such that $X \in \mathbb{R}^n$, where n is the total number of groundwater pumping wells and surface water diversion points) which are spatially distributed within the model domain. In this case, it is assumed that the solution set X is closed and bounded, and hence the problem has finite number of feasible solutions.

Now, let $X \in \mathbb{R}^n$ where $n = n_{gw} + n_{sw}$, be a set of feasible solutions for a combined (conjunctive use) decision variable vector X_{gw} in space $\mathbb{R}^{n_{gw}}$ and decision variable vector X_{sw} in space $\mathbb{R}^{n_{sw}}$, of groundwater pumping rate and surface water diversion rate respectively; X^* be a true optimal solution of the objective function Z (i.e., the true (original) conjunctive water use optimization problem (TCOP) (formulation (3.36) through (3.42))); X_k, X_k^* be optimal solution and optimal value of the objective function Z_{N_k}, respectively (i.e., the k^{th} Estimates Retrospective Conjunctive water use Sample path Optimization Problem (ERCSOP) (formulation (3.45) through (3.51))); and that X^*, X_k , $X_k^* \in X$. Using ROA method, the ERCSOP optimization sub-problems were solved in a sequence of increasing number of realizations/sample size.

Note that the number of sample size (realizations) is increased from sub-problem to sub-problem and initial solution (guess) of the current sub-problem is basically the returned solution from the previous sub-problem. Hence, based on the Strong Law of Large Numbers (SLLN) and Central Limits Theorem (CLT), we claim that, for $k = 1, 2, \ldots, N_{SP}$, convergence can be achieved. This is because by considering SLLN and CLT it can be shown that as sample size increases, the optimal solution values of the ERCSOP optimization problem asymptotically approaches the optimal solution values of the TCOP, hence the optimal solution of $Z_{N_k} \longmapsto Z$ that is $X_k \longmapsto X^*$ as $N_k \longmapsto \infty$, where N_k is the number of realizations/sample sizes.

To solve the optimization problem, our ROA method procedure uses a standard core optimizer solver, the "Active-Set" algorithm implemented under MATLAB environment. In this case, ROA procedure works as follows: for a given set of realizations/sample sizes at k^{th} sample path sub-problem, the Active-Set optimization solver determines optimal solutions and optimal value based on maximization/minimization of objective function of an optimization problem and sequentially evaluates the expected total conjunctive water use optimal objective function values in a sequence of increasing number of realizations/sample sizes. It should be noted that the sequence of increasing number of realizations/sample sizes is determined heuristically (Wang *et al.,* 2012). In this research, *k*-means clustering sampling technique was used for the hydraulic conductivity field realizations mapping.

Solution Methodology using ROA

To solve the stochastic conjunctive water use optimization problem using Retrospective Optimization Approximation (ROA) method (i.e., to solve the ERCSOP problems formulation (3.45) through (3.51) for $k = 1,2, \ldots, N_{SP}$), the following elucidated steps were followed:

(1) For a given available data (in this case hydraulic conductivity field mean, standard deviation and correlation length) generate set of realizations of random aquifer hydraulic conductivity fields.

(2) For each given realizations of ϖ, run MODFLOW simulation model to generate random responses $a_{ij,\overline{\varpi}}$ (i.e., $\overline{\varpi}$ random responses of hydraulic heads/water table level drawdowns) due to a unit groundwater pumping rate (we consider existing maximum groundwater pumping rate to represent a unit pumping rate)).

(3) Generate a sequence of finite sets of independent identically distributed $(i.i.d.)$ samples of random aquifer response matrices A_{ϖ} of N realizations of $\{\varpi_{N_k}\} = \{A_{\varpi_1}, \dots, A_{\varpi_{N_k}}\}$, for $k = 1, 2, \dots, N_{SP}$.

(4) For a given sequence of finite sets of the generated samples of aquifer response matrices realizations use the ROA method to generate a sequence of retrospective sample path sub-problems Z_{N_k}, $k = 1,2, \dots, N_{SP}$.

(5) For each k^{th} sample path sub-problem apply Optimizer Solver to provide an optimal solution (or a nearly closer optimal solution) X_k of Z_{N_k}.

(6) Repeat step (5) for $k = 1, 2, \dots, N_{SP}$ until the optimal solution X_k converges to true optimization problem optimal solution X^*. Figure 3.6 summarizes flow chart for the ROA solution methodology.

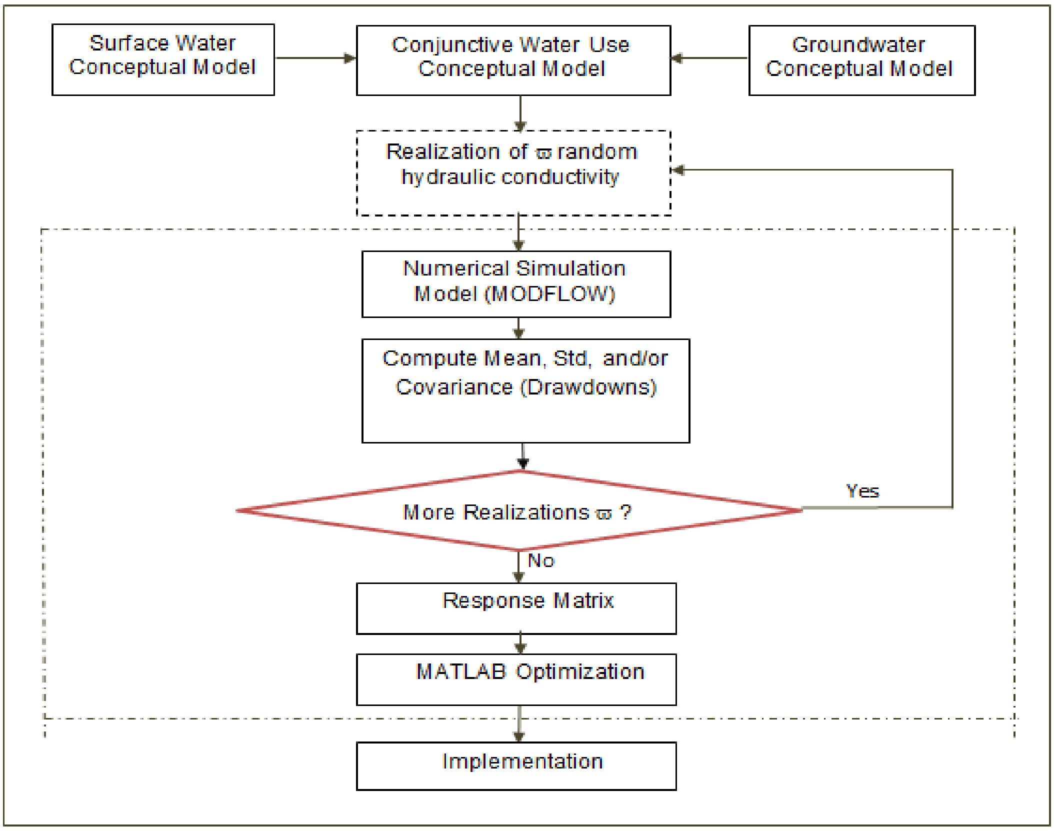


Figure 3.6: Flow Chart for Stochastic Optimization Model Using ROA Method

CHAPTER FOUR: APPLICATION OF METHODOLOGY TO HYPOTHETICAL EXAMPLE

4.1 INTRODUCTION

The proposed methodologies were applied to hypothetical aquifer water system modified after Tyagi *et al.* (1995) to illustrate a quantitative conjunctive water use management problem. The hypothetical aquifer consisted of a river system, surface water diversions, groundwater pumping wells, and agricultural fields. The model was formulated to represent realistic hydrology and hydrogeology of aquifer water resource systems. The following sub-section presents conceptual models of the hypothetical aquifer water system.

4.2 CONCEPTUAL MODELS OF HYPOTHETICAL AQUIFER WATER SYSTEM

In this sub-section, a general surface water, groundwater, and conjunctive water use systems conceptual models are presented in a simplified schematic way to show the most relevant hydrological and hydrogeological features of the hypothetical aquifer.

4.2.1 Surface Water Conceptual Model

In this example, the river flows from northeast to southwestern part of the aquifer system. The river flow is sustained by upstream inflows and flow from a single layer alluvial unconfined aquifer. The main river was discretized into five (5) reaches. Figure 4.1 shows the discretized conceptual model of the system.

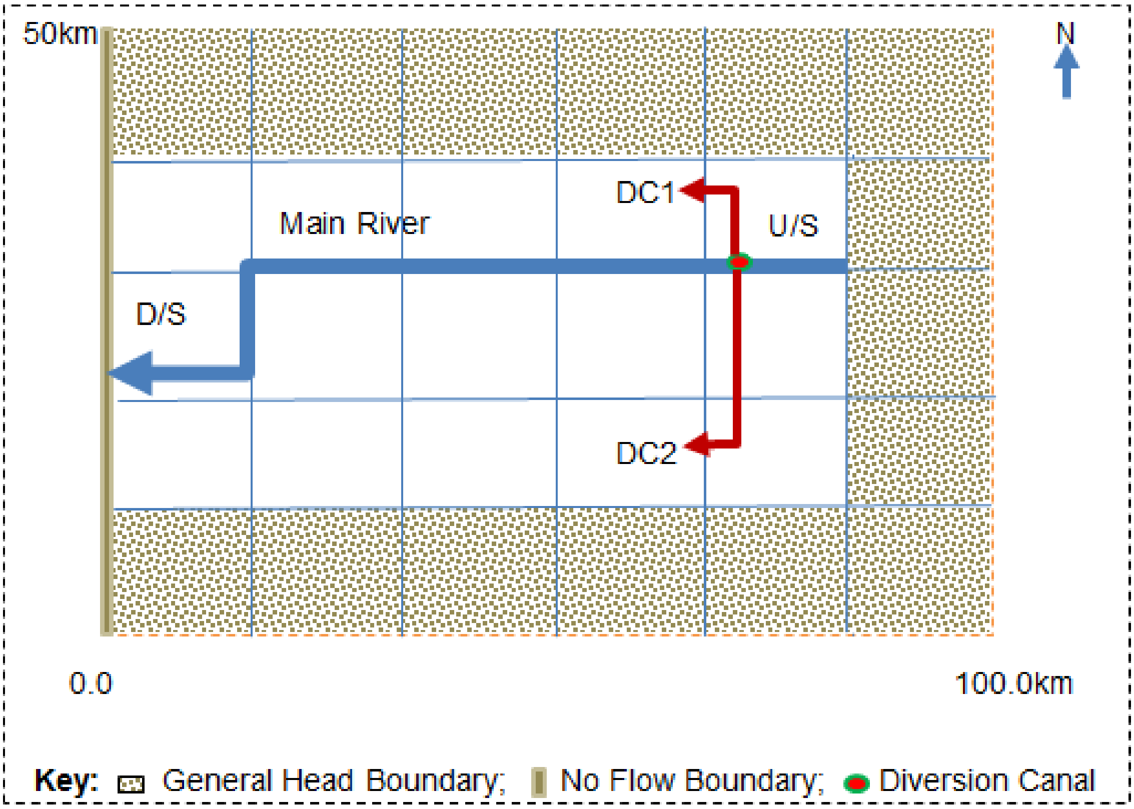


Figure 4.1: A Discretized Surface Water Conceptual Model Layout Schematic Diagram

The river was assumed to be of excellent saturated hydraulic connection with the aquifer. The river channel was assumed to be a natural rectangular channel with an average width ranging from 80 to 100m and a riverbed depth ranging from 2 to 3m. The water users were allowed to divert water for irrigation from the main river at upstream (U/S) location point (diversion canal DC1 and DC2) as shown in Figure 4.1. Agricultural fields occur on both sides of the river banks.

4.2.2 Groundwater Conceptual Model

The hypothetical aquifer has a rectangular areal extent of 100km by 50km which was uniformly discretized into 30 grid cells in 5 rows and 6 columns with an equal grid spacing of 16.667km in the x - direction and 10.0km in the y – direction. Finite difference method was used to discretize the area. Figure 4.2 shows finite difference groundwater conceptual model layout.

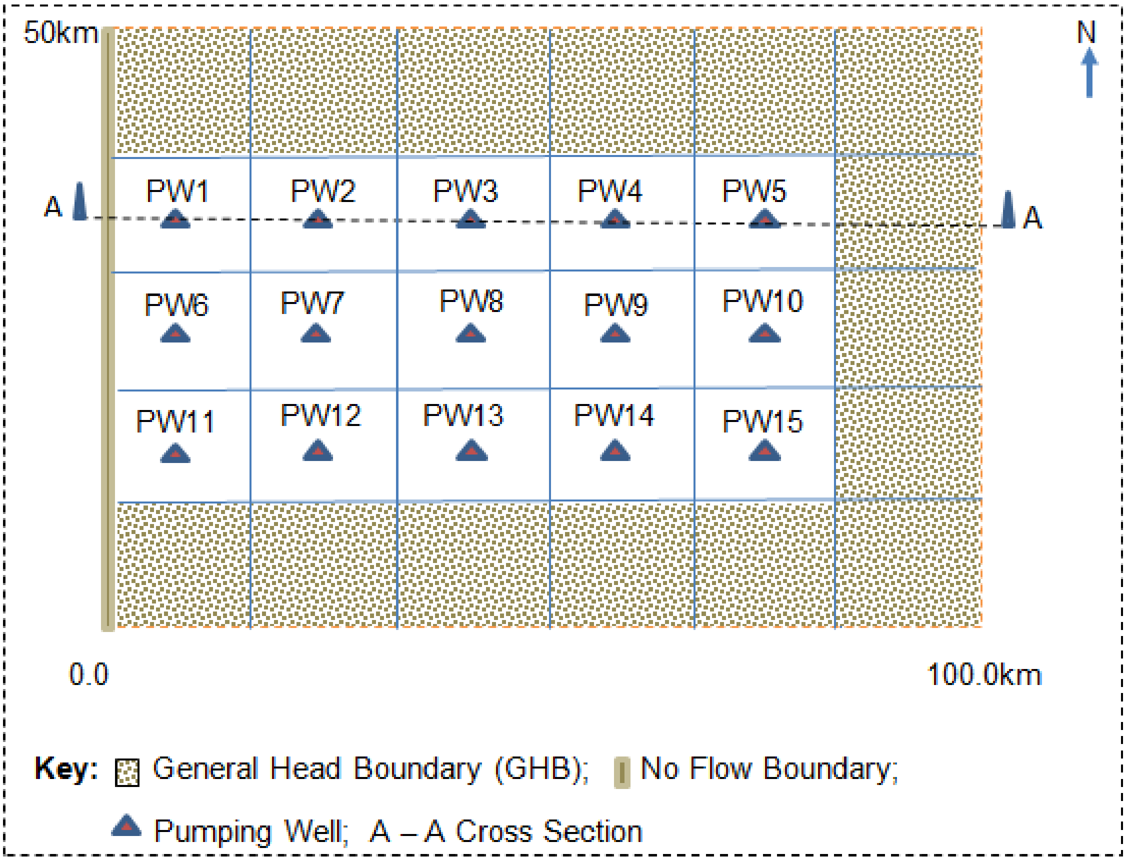


Figure 4.2: Finite Difference Groundwater Conceptual Model Layout

It was assumed that the aquifer has no-flow boundary conditions on the western part. On the eastern, northern and southern parts of the model domain boundaries provide a controlled general head flow. The potential groundwater pumping wells are located in 15 internal active model domain grid cells as shown in Figure 4.2.

Aquifer heads at these potential groundwater pumping wells were restricted by not allowing them to fall below 50 percent of the specified saturated aquifer thicknesses. The aquifer system has an average saturated thickness ranging from 93.50m to 135.20m. Figure 4.3 shows groundwater conceptual model hydrogeological unit cross –section A – A (see figure 4.2).

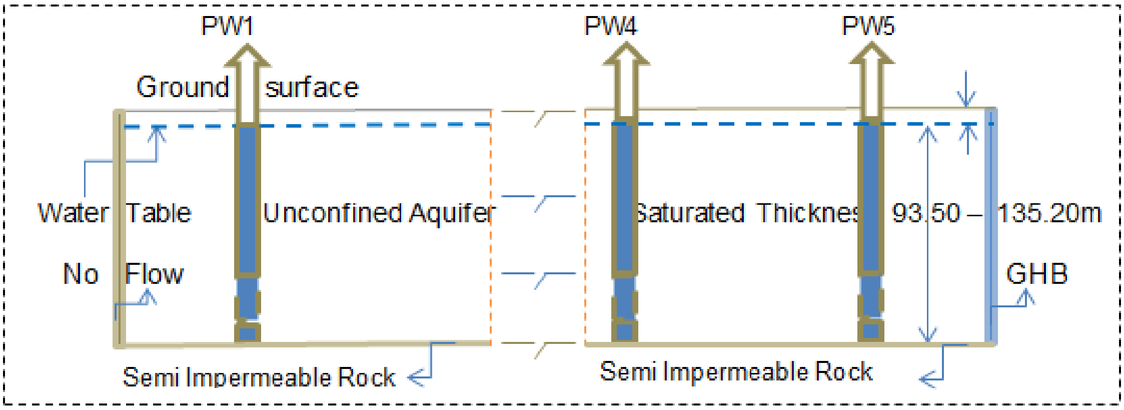


Figure 4.3: Groundwater Conceptual Model Hydrogeological Unit Cross-Section

4.2.3 Conjunctive Water Use Conceptual Model

The surface water and groundwater conceptual models were integrated to build up surface water-groundwater conjunctive water use model. Figure 4.4 shows a simplified conjunctive water use conceptual model of the system.

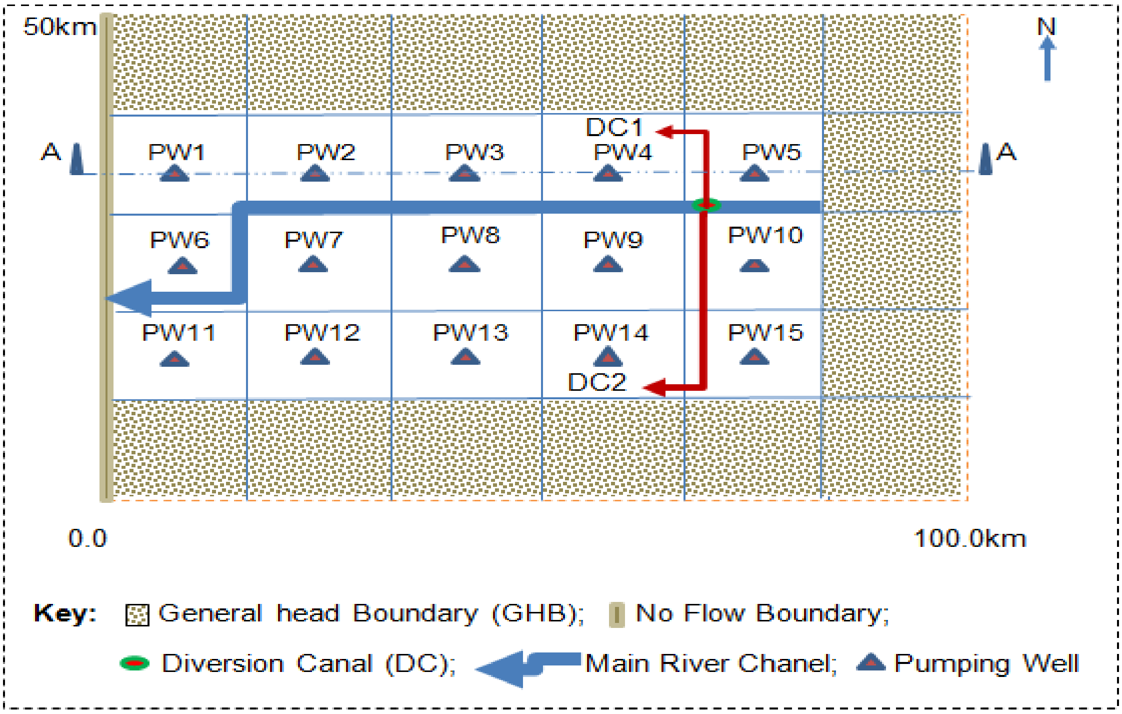


Figure 4.4: Conjunctive Water Use Conceptual Model of the System

Figure 4.5 shows a schematized model showing the conjunctive water use components.

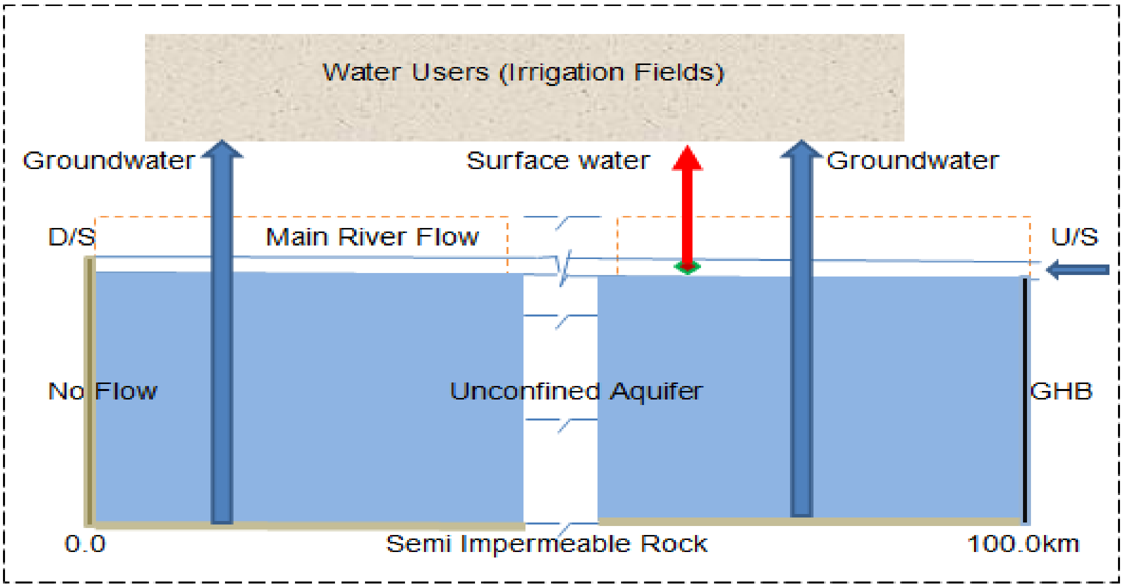


Figure 4.5: Schematized Model Showing the Conjunctive Water Use Components

In this example, it was assumed that irrigation is the only main water user. The irrigated area receives water from the surface water diversions as well as groundwater pumping wells. Recharge to the groundwater body from the fields was adopted as 7.5 percent of application losses and total net recharge was estimated at a rate of 106mm/year (Tyagi *et al.,* 1995). Recharge from canal conveyance losses were assumed to be negligible because water flows through new pipe and lined concrete canal.

4.3 CONJUNCTIVE WATER USE MANAGEMENT MODEL

In this sub-section, management (optimization) model results of the hypothetical example solved through standard deterministic simulation-optimization approach and the proposed stochastic methodology (i.e., Retrospective Optimization Approximation (ROA) method framework) discussed in Chapter three are presented.

The overall optimization problem was to establish: (1) optimal conjunctive water use withdrawal strategy satisfying all (or nearly all) the pre-specified constraints; and (2) optimal groundwater pumping rates (i.e., when groundwater source was considered as the only source of water supply) satisfying all (or nearly all) the prescribed set of constraints.

4.3.1 Objective

The objective was to maximize surface water and groundwater production rates so as to meet total water demands without violating the prescribed set of constraints.

4.3.2 Constraints

To ensure sustainable utilisation of surface water and groundwater resources, the optimization problem was subjected to the following constraints:

(i) For sustenance of surface water river flow for downstream ecological requirements, the surface water level (river/stream stages) at the specified control river/stream reaches were not allowed to fall below 2.5m above sea level.

(ii) To avoid excessive drawdown of the aquifer (groundwater) water system along rivers/stream corridors and hence excessive depletion of river/stream flow, aquifer heads in grid cells next to the river/stream course were not allowed to fall below river/streambed bottom elevations.

(iii) Surface water and groundwater withdrawal rates from each potential surface water diversion point and pumping well was limited to a maximum withdrawal rate.

(iv) To ensure existing water demands are met, groundwater/surface water withdrawal rates should be greater than or equal to the current withdrawal rates.

(v) Total groundwater pumping rate should be less than or equal to total aquifer recharge.

Aquifer Hydraulic Conductivity:

The heterogeneous aquifer system property (hydraulic conductivities) of the model domain is characterised by five (5) zones as shown in Figure 4.6

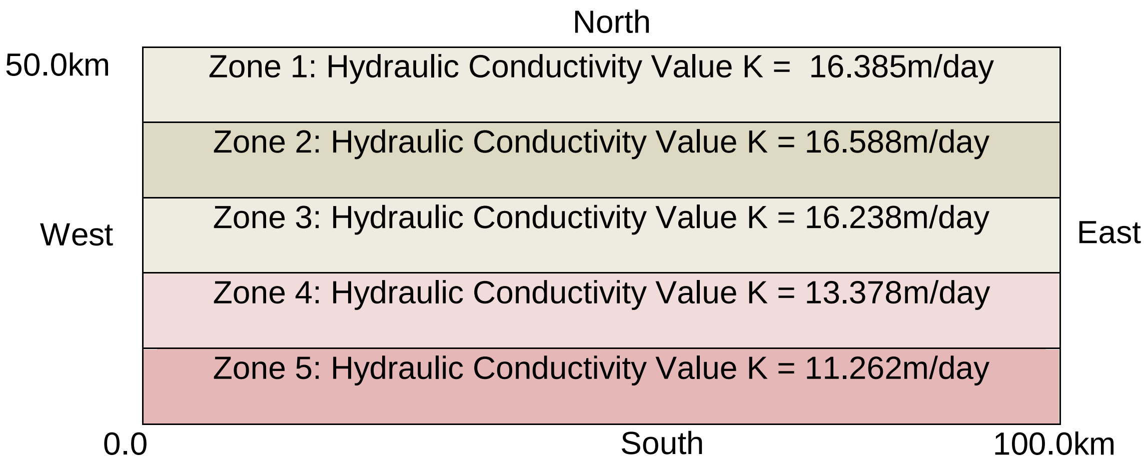


Figure 4.6: Heterogeneous Aquifer Hydraulic Conductivity Zones

4.3.3 Deterministic Conjunctive Water use Management

In deterministic conjunctive water use management, the hydraulic conductivity and hence transmissivities were considered spatially invariant. It was assumed that field measurements were adequate, precisely measured and hence all the associated parameters were set fixed. Table 4.1 and 4.2 present river and aquifer systems properties parameter values after Tyagi *et al.* (1995), which were used for simulation-optimization problems analysis.

Table 4.1: River System Model Inputs Parameters

R/Cell	River Stage Elev. AMSL (m)	River Bottom Elev. AMSL (m)
2,1	202.25	200.75
2,2	203.68	201.68
2,3	208.53	206.53
2,4	216.98	214.98
2,5	224.15	222.65
3,1	197.86	195.86
3,5	219.42	216.42
4,1	198.70	195.70
4,2	209.10	206.10
4,3	211.00	208.00
4,4	212.90	209.90
4,5	216.83	214.83

Table 4.2: River/Stream-Aquifer System Properties Model Inputs Parameters

Item	River/Stream - Aquifer System Property	Parameter value
1	Average River/Streambed Hydraulic Conductivity	0.2 m/day
2	Average River Width ranges	80.0 m – 100.0 m
3	Average Riverbed Thickness ranges	2.0 m – 3.0 m
4	Average Aquifer Saturated Thickness ranges	93.50 m – 135.20 m
5	Average Aquifer Hydraulic Conductivity	14.77 m/day
6	Aquifer specific yield	0.1
7	Average Total Groundwater Net Recharge	106 mm/annum

Discussion of Model Results

It should be noted that in this research, management model for groundwater as sole source of water supply is only applied to this example for the purpose of comparison of the results with the optimal groundwater pumping rates scheme determined by Tyagi *et al.* (1995) through embedded approach.

In this example, the simulation optimization model was solved by considering the maximum withdrawal rate limits (i.e., the constraining upper bounds) equal to n times the current groundwater pumping rates, where the derived "n" multiple values were assumed to be equal to the ratios (proportion values) of the optimal groundwater pumping rate values to the current (existing) groundwater pumping rates. The derived n multiple values with their corresponding current and the example of optimal groundwater pumping rates determined by Tyagi *et al.* (1995) are presented in Table 4.3.

The existing un-optimized groundwater pumping rates ranged from a minimum of 2988.00 m^3/day to a maximum of 59400 m^3/day. The optimal groundwater pumping rates determined by Tyagi *et al.* (1995) ranged from a minimum of 9000.00 m^3/day to a maximum of 305280.00 m^3/day. Table 4.3 shows the existing un-optimized and the example of optimal groundwater pumping rates after Tyagi *et al*. (1995) with corresponding derived n multiple values.

Table 4.3: Derived n Multiple Values with Corresponding Current and Example Optimal Groundwater Pumping Rates

P.Well	**Current Pumping Rate*(m^3/day)**	**Example Optimal Pumping Rate* (m^3/day)**	**n Multiple Value**
PW1	11448.00	93240.00	8
PW2	13428.00	285480.00	21
PW3	16488.00	305280.00	19
PW4	59400.00	291600.00	5
PW5	47520.00	86040.00	2
PW6	12456.00	87120.00	7
PW7	12168.00	72720.00	6
PW8	7668.00	30600.00	4
PW9	8352.00	47520.00	6
PW10	17496.00	59400.00	3
PW11	4968.00	68400.00	14
PW12	3960.00	9000.00	2
PW13	3204.00	54000.00	17
PW14	2988.00	48240.00	16
PW15	27720.00	50040.00	2

* Current and Example optimal groundwater pumping rates (10hrs pumping period per day) derived after Tyagi *et al.* (1995).

It should be noted that in conjunctive water use of surface water and groundwater, the total current (existing) surface water diversion (withdrawal) rates at two diversion points located at (see Figure 4.1 DC1 and DC2) the upstream of the main river was assumed to be equal to 72,000.00m^3/day and 90,000.00m^3/day, respectively. In which, it was assumed that the maximum surface water withdrawal rate constraining limits (i.e., the upper bounds) was set equals to the current surface water withdrawal rate times an N multiple number. In this example, two cases were considered for simulation optimization problems analysis as follows:

Case 1: Groundwater as only Source of Water Supply

Table 4.4 shows optimal groundwater pumping rates solution with corresponding saturated aquifer thicknesses.

Table 4.4: Optimal Pumping Rates Solution with Corresponding Saturated Aquifer Thicknesses

P.Well	**Optimal Pumping Rate (m^3/day)**	**Saturated Thickness* (m)**
PW1	93240.00	106.66
PW2	285480.00	108.53
PW3	305280.00	102.32
PW4	291600.00	93.55
PW5	86040.00	118.76
PW6	87120.00	112.17
PW7	72720.00	100.82
PW8	30600.00	94.11
PW9	47520.00	99.64
PW10	59400.00	109.85
PW11	68400.00	135.21
PW12	9000.00	127.35
PW13	54000.00	112.89
PW14	48240.00	107.72
PW15	50040.00	118.19

*Saturated thicknesses after Tyagi *et al.* (1995).

Based on the results presented in Table 4.4, the optimal groundwater pumping rate solution strategy obtained ranges from a minimum of 9000.00 m^3/day (in pumping well PW12) to maximum of 305280.00 m^3/day (in pumping well PW3). This optimal groundwater pumping rate strategy resulted in an optimal objective function value of 1,588,680.00 m^3/day. The optimal pumping rates solution obtained from this example, were verified through the pumping rates determined by Tyagi *et al.* (1995). Figure 4.7 shows comparison of the optimal groundwater pumping rate solution with the example of optimal pumping rate solution determined by Tyagi *et al.* (1995).

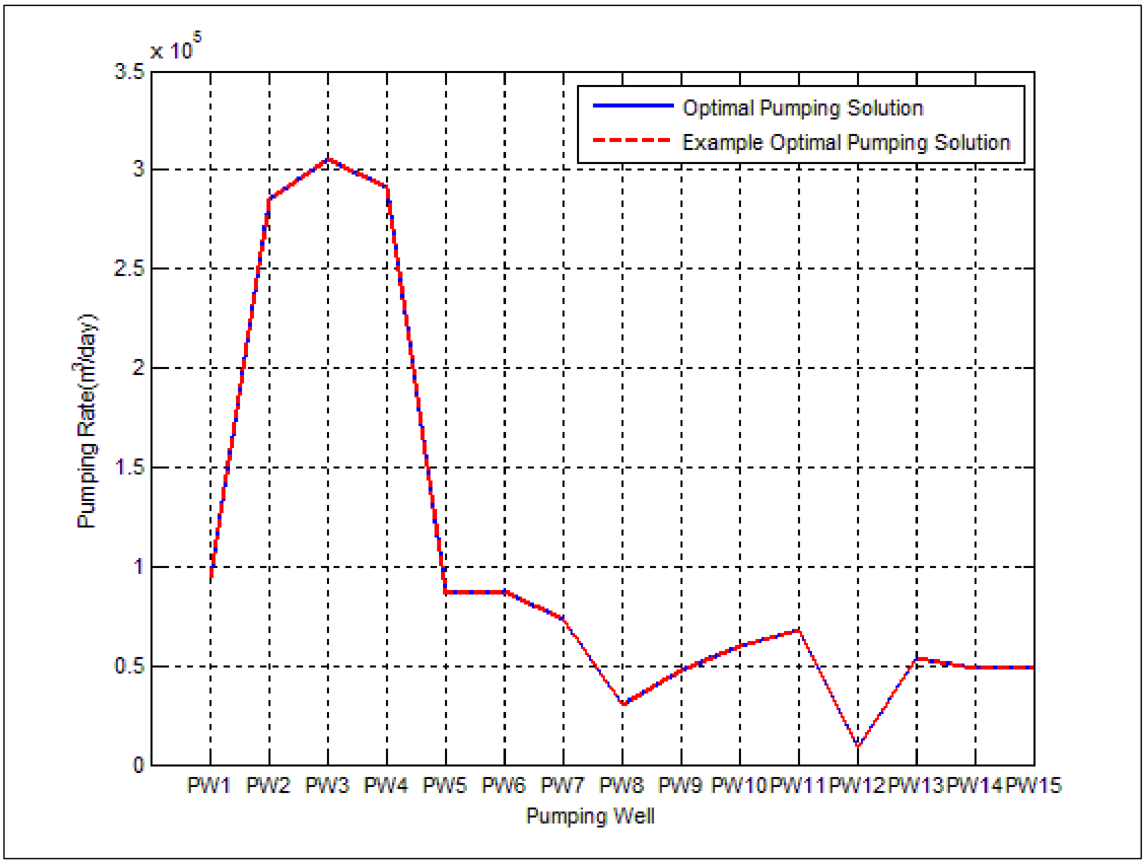


Figure 4.7: Comparison of Optimal Pumping Rate Solution with Example Optimal Pumping Rate Solution

As it can be seen from Figure 4.7 the two graphs coincide. The optimal groundwater pumping rates calculated in this case were almost equal to the pumping rates determined by Tyagi *et al.* (1995). This implies that ROA method can be relied upon to give accurate results as any other tested method in use. Figures 4.8 and 4.9 present the results of this case in 2Dimension and 3Dimension views, respectively.

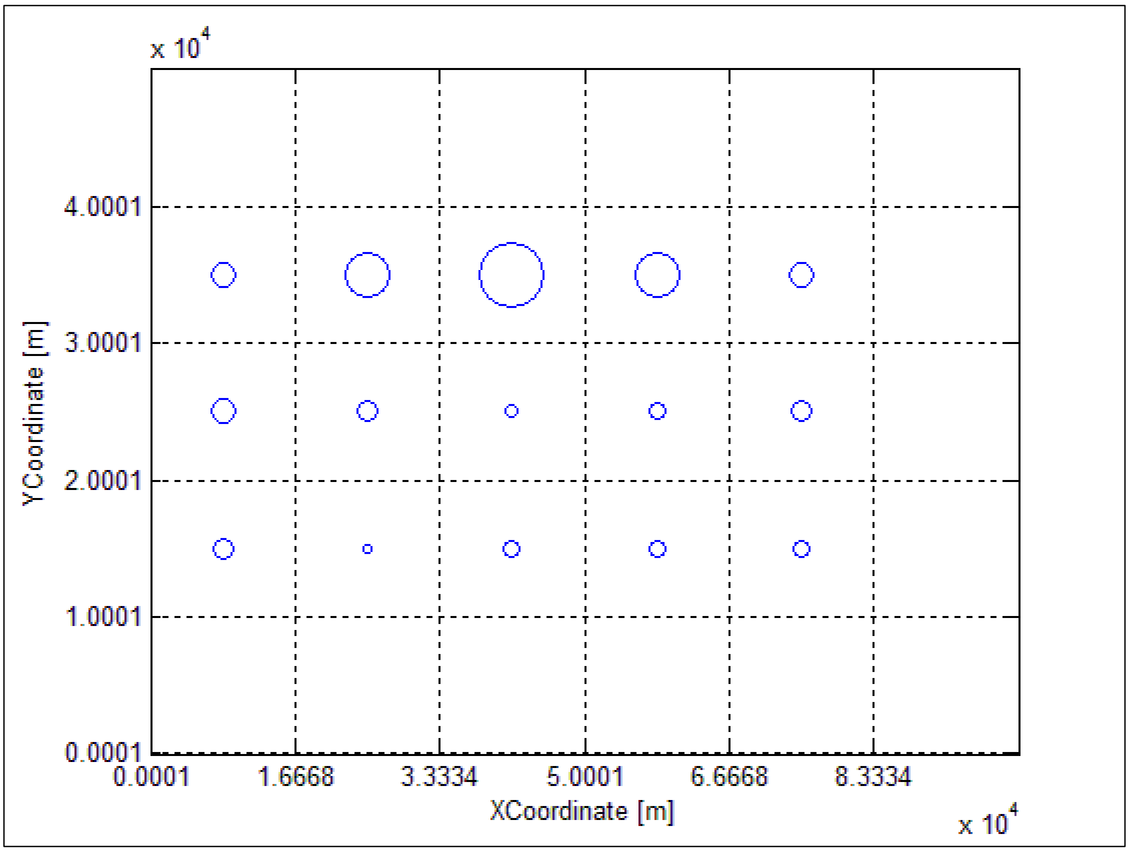


Figure 4.8: Optimal 2D Bubble Chart Groundwater Pumping Rates Strategy

In Figure 4.8, the size of bubble circle indicates the magnitude of the groundwater pumping rates. Thus, the biggest bubble circle indicates groundwater well of the highest pumping rate, and vice versa. As expected, pumping rate magnitudes of the pumping wells varied in accordance to their location in the model domain. This is due to the aquifer properties and boundary conditions differences of the model domain. For example, pumping wells located closer to river boundary have relatively higher pumping rates compared to those which are far from the main river course because pumping wells which are closer to river course may have additional recharge opportunity compared to those which are far from the river course. Also it depends on boundary conditions (such as no-flow boundary) and the magnitude of hydraulic conductivity/transmissivity values (see Figures 4.4, 4.6 and 4.8). In Figure 4.9, a 3D spatial distribution of pumping wells is presented, in which the highest height stem represents the pumping well of the highest groundwater pumping rate, and vice versa.

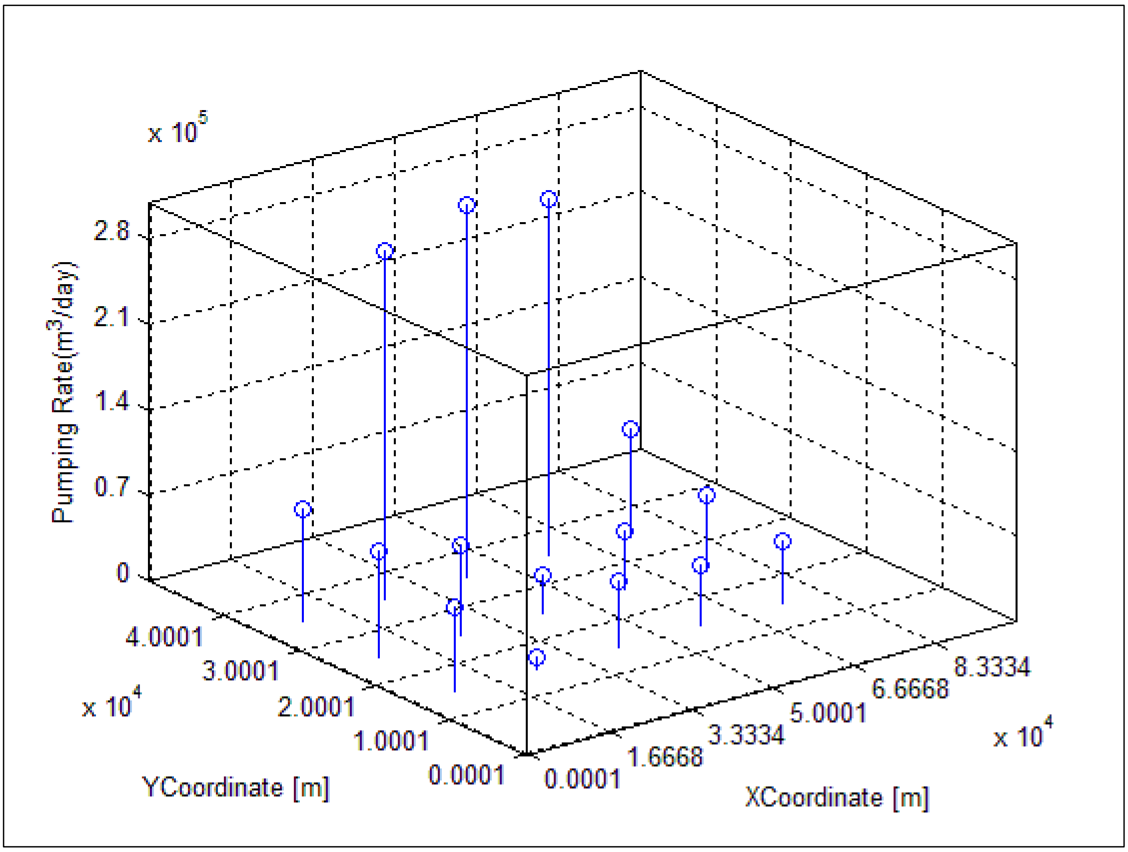


Figure 4.9: Optimal 3D Stem View of Groundwater Pumping Strategy

From Figures 4.8 and 4.9, pumping wells PW3 and PW12 have maximum and minimum groundwater pumping rates, respectively. The high difference in pumping rate is due to high variation in groundwater (aquifer) recharge opportunity as well as variations in aquifer hydraulic conductivity zones (see Figure 4.6). Pumping well PW3 is closer to the main river course compared to pumping well PW12 (refer to Figure 4.4) which is located far from the main river course. Pumping well PW3 receives recharge from the river compared to pumping wells which are relatively far from the river.

Higher pumping rates are also reflected in pumping wells PW2 and PW4 which are also closer to the main river course. Aquifer recharge contributes to aquifer storage as well as to the groundwater pumping yield. It should be noted that even though pumping wells PW1, PW5, and PW6 are closer to the main river course, their pumping rates are relatively low compared to those of pumping wells PW2, PW3, and PW4. This may be attributed to differences in water table-elevations, saturated thickness and hydraulic conductivity/transmissivity values. Pumping wells PW1, PW6 and PW11 are closer to no-flow boundary hence; higher pumping rates would mean violation of the maximum allowable drawdown constraints.

It was observed that when this maximum groundwater pumping rate solution was used as simulation model inputs, pumping well PW6 became dry. This implies that some constrains in the simulation optimization problem were violated. This means that the feasible optimal solution strategy obtained under this environment satisfies all groundwater pumping wells constrains, except pumping well PW6.

Case 2: Conjunctive Water use

In the case of conjunctive water use, it was assumed that in critical dry periods, groundwater supply is supplemented by surface water supply. The conjunctive water use management optimization problem was solved with the overall objective of maximizing the water production rates subject to the same constrains as imposed in groundwater model but, with additional surface water constrains which restricted river flow from not falling below prescribed base flow level. Table 4.5 presents optimal conjunctive water use withdrawal rates strategy.

Table 4.5: Optimal Conjunctive water use Withdrawal Rates Strategy

PW/SWD	Optimal Conjunctive use Withdrawal Rate (m^3/day)
PW1	93240.00
PW2	285480.00
PW3	305280.00
PW4	291600.00
PW5	86040.00
PW6	87120.00
PW7	72720.00
PW8	30600.00
PW9	47520.00
PW10	59400.00
PW11	68400.00
PW12	9000.00
PW13	54000.00
PW14	48240.00
PW15	50040.00
SWD01	224800.00
SWD02	261200.00
Σ	**2072607.00**

In Table 4.5, it can be observed that conjunctive water use optimal solution strategy obtained had a maximum withdrawal rate of 305280.00 m^3/day and minimum withdrawal rate of 9000.00 m^3/day with objective function volume rate of 2,072,607.00 m^3/day. Figure 4.10 shows histogram of the conjunctive water use optimal withdrawal rates.

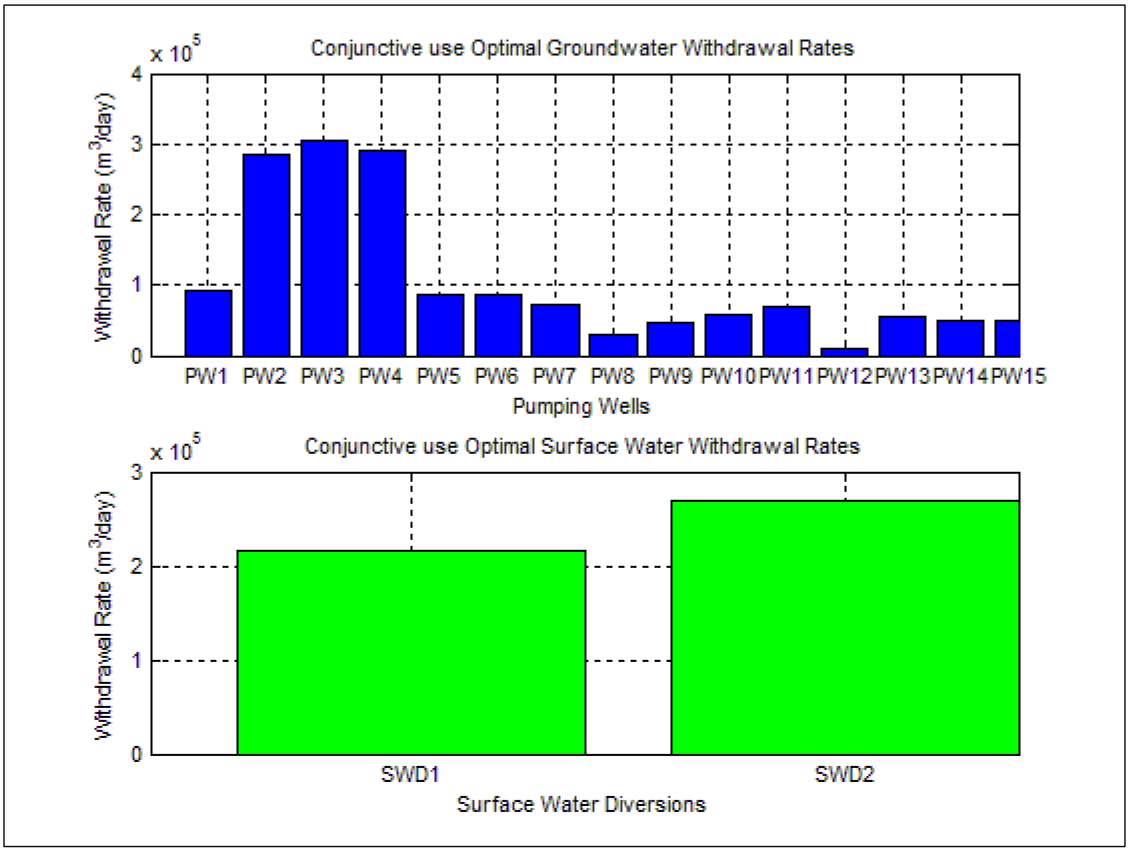


Figure 4.10: Conjunctive Water Use Optimal Groundwater and Surface Water Withdrawal Rates

As was expected in Figure 4.10, groundwater maximum and minimum withdrawal rates occurred at pumping wells PW3 and PW12, respectively. This is mainly due to differences in hydraulic conductivity and boundary condition variations in the model domain.

In this example, surface water source contributed a total withdrawal volume rate of 486000.00 m^3/day which is about 23.45 percent of the total withdrawal rate with the remaining 76.55 percent being contributed by groundwater aquifer. Figure 4.11 presents the overall percentages of contribution of surface water and groundwater sources to the total optimal conjunctive water use withdrawal rate in a pie chart view.

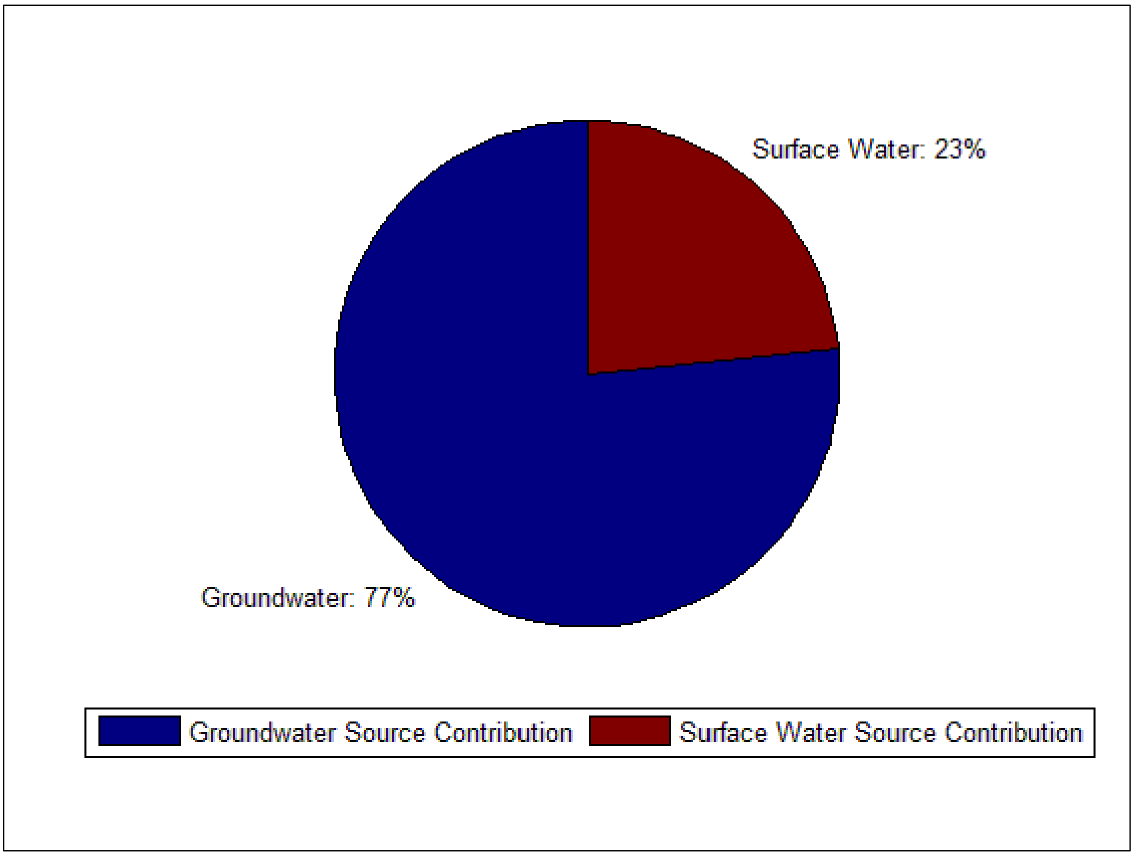


Figure 4.11: Overall Percentages of Contribution of Surface Water and Groundwater Sources to the Total Optimal Conjunctive Use Withdrawal Rate

In Figure 4.11, it emerged that groundwater source contributed higher percentage of the total optimal conjunctive water use than surface water source. This tends to suggest that the aquifer system has more water storage for supply than the surface water storage system. Figure 4.12 compares the existing un-optimized conjunctive water use scheme with the optimal conjunctive water use withdrawal rate scheme.

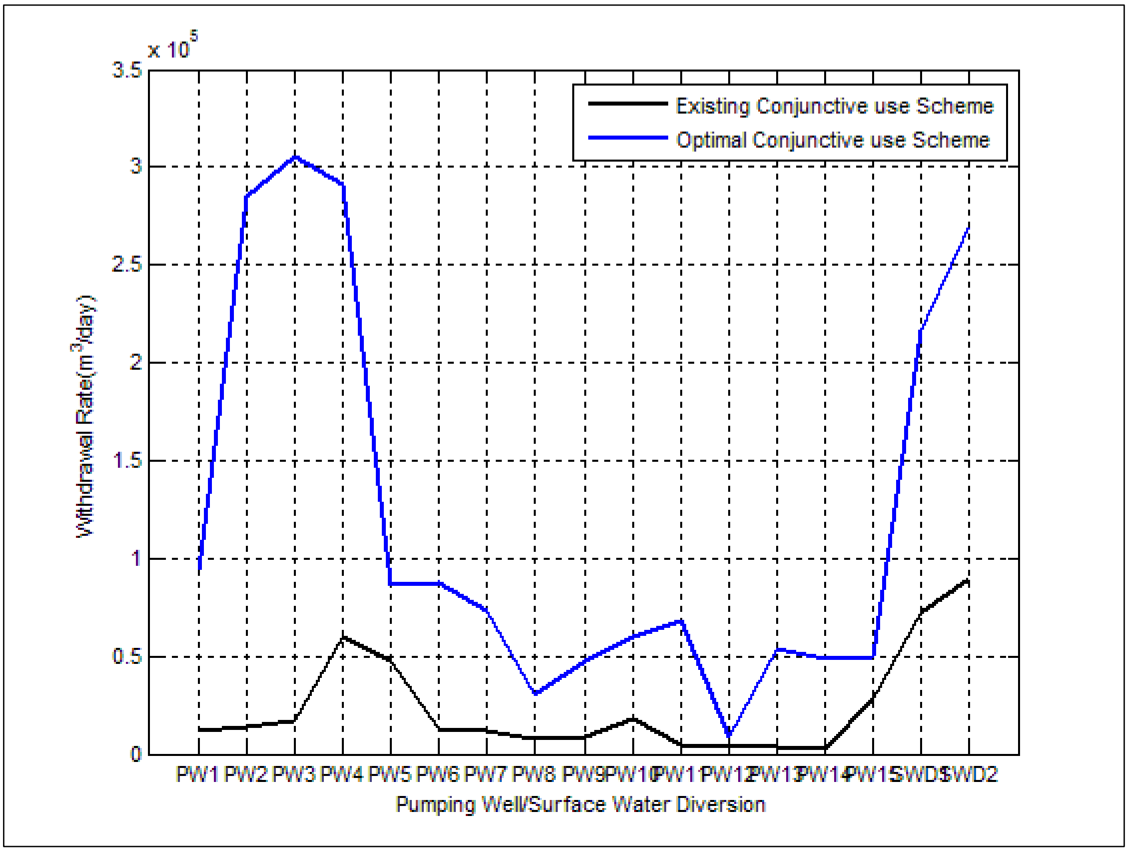


Figure 4.12: Comparison of Existing Un-optimized versus Optimized Deterministic Optimal Conjunctive Water Use Withdrawal Rate Schemes

From Figure 4.12, it can be observed that pumping wells which had higher groundwater withdrawal rates in existing (un-optimized) pumping scheme also have higher withdrawal rates in optimized scheme. In Figure 4.12, optimal conjunctive water use scheme withdrawal rates are higher than the existing un-optimized conjunctive water use scheme withdrawal rates. In all two conjunctive water use schemes the lowest groundwater withdrawal rate occurred at pumping well PW12. High magnitude of withdrawal rate variations occurred in pumping wells PW2, PW3, and PW4. As mentioned in previous case, the main reason behind this is due to the fact that aquifer properties and boundary conditions as well as water table level variations control the groundwater pumping rates and consequently the conjunctive use withdrawal rates. Pumping well PW12 is located far from main river course and in a lower hydraulic conductivity value zone than pumping wells PW2, PW3, and PW4.

Concluding Remarks

It can be concluded that the deterministic methodology proposed gives similar results as those by Tyagi *et al.* (1995). This indicates that the proposed deterministic models (i.e., groundwater as sole source of water supply management model and its extension, which is conjunctive water use management model) are capable to manage quantitative conjunctive water use/groundwater resource optimization problems within the deterministic limitations. Hence, this implies that ROA approach can be relied upon to give accurate results as any other tested method in use.

4.3.4 Stochastic Conjunctive Water use Management

In stochastic conjunctive water use management, the optimization problem was solved while assuming heterogeneous aquifer properties (hydraulic conductivity property). Five (5) heterogeneous zones of the hydraulic conductivity fields were identified in the hydrogeological unit as shown in Figure 4.6. The stochastic conjunctive water use optimization problem was solved and evaluated through Retrospective Optimization Approximation (ROA) method framework (i.e., the methodology developed in Chapter three).

Description of the stochastic optimization problem

In this example, a total of 500 realizations of uncertain hydraulic conductivity fields were generated from the heterogeneous aquifer system. A correlation length of 100,000m by 50,000m in a 2-Dimensional x-, y-direction, respectively, was considered sufficient enough to capture significant representation of input parameter uncertainties.

Fifteen (15) groundwater monitoring wells (control points) and two (2) surface water gauging stations (control points) were identified active and the attributes used account for variations in aquifer properties. It should be noted that the same control points and pumping wells locations are used to measure responses of the aquifer system (aquifer heads/water table level drawdowns) when subjected to external stresses (in this case, a unit pumping rate) for every realization of hydraulic conductivity field.

Assemblage of aquifer system responses due to the 500 realizations of hydraulic conductivity fields resulted in a total of 7502 response matrix rows (observations). Hence, in total, the constraining response matrices of 7502 by 15, and 7502 by 17 were generated for groundwater as only source of water supply management model, and conjunctive water use management model, respectively. These response matrices were used to generate (sample) ten (10) sample path optimisation sub-problems of different sample sizes (observation rows) for each of the respective management models. Sample sizes were determined heuristically. Table 4.6 presents the description of formulation of sample path optimization sub-problems generated for the ROA method framework evaluation.

Table 4.6: Descriptions of Sample Path Optimization Sub-Problems

Sample Path Sub-Problem	# Realizations	Response Matrix (# Rows/Constraints)	#Columns (#Decision Variables*)
SOSP1	1	17	15/17
SOSP2	5	77	15/17
SOSP3	10	152	15/17
SOSP4	20	302	15/17
SOSP5	30	452	15/17
SOSP6	50	752	15/17
SOSP7	100	1502	15/17
SOSP8	150	2252	15/17
SOSP9	200	3002	15/17
SOSP10	500	7502	15/17

*Decision variables for groundwater and conjunctive use withdrawal rates

The sequence of 1, 5, 10, 20, 30, 50, 100, 150, 200, 500 realizations of hydraulic conductivity fields generated a sequence of 17, 77, 152, 302, 452, 752, 1502, 2252, 3002, and 7502 of constraints, respectively (i.e., sample rows/observation rows, excluding total recharge constrain). This sequence of constraints generated the corresponding ten (10) sample path optimization sub-problems in a sequence of increasing number of rows (including aquifer total recharge constrain) of 18, 78, 153, 303, 453, 753, 1503, 2253, 3003, and 7503 (excluding lower and upper bounds, and nonnegative bounds constraints).

Discussion of Model Results

To investigate how parameter uncertainty (in this case, uncertainty due to spatial variations of aquifer system hydraulic conductivity fields) impacts on the management model performance, the stochastic optimization problem was solved and evaluated using ROA method framework by considering two cases. In the first case, groundwater was considered as the only source of water supply and then as conjunctive water use.

In order to compare results obtained in these schemes with those of deterministic schemes, withdrawal rates limits (i.e., minimum and maximum limits (i.e., the lower and upper bounds constraining values)) constraining values of these cases were set equal to those of the optimal deterministic schemes.

Case 1: Groundwater as only Source of Water Supply

In this case, groundwater model application results of the sample path optimization sub-problems (SOSPs) corresponding to the 500 realizations of hydraulic conductivity fields generated is discussed. Table 4.7 presents the groundwater sample path optimization sub-problems (SOSPs) solutions.

Table 4.7: Groundwater Sample Path Optimization Sub-Problems Solutions

PW	SOSP1R1	SOSP2R5	SOSP3R10	SOSP4R20	SOSP5R30	SOSP6R50	SOSP7R100	SOSP8R150	SOSP9R200	SOSP10R500
PW1	93240.00	139860.00	170940.00	194250.00	212898.00	228438.00	241758.00	253413.00	263773.00	273097.00
PW2	285480.00	428220.00	523380.00	594750.00	651846.00	699426.00	740208.86	775893.86	807613.86	836161.86
PW3	305280.00	457920.00	559680.00	636000.00	697056.00	747936.00	791547.43	829707.43	863627.43	894155.43
PW4	291600.00	437400.00	534600.00	607500.00	665820.00	714420.00	756077.14	792527.14	824927.14	854087.14
PW5	86040.00	129060.00	157740.00	179250.00	196458.00	210798.00	223089.43	233844.43	243404.43	252008.43
PW6	87120.00	130680.00	159720.00	181500.00	198924.00	213444.00	225889.71	236779.71	246459.71	255171.71
PW7	72720.00	109080.00	133320.00	151500.00	166044.00	178164.00	188552.57	197642.57	205722.57	212994.57
PW8	30600.00	45900.00	56100.00	63750.00	69870.00	74970.00	79341.43	83166.43	86566.43	89626.43
PW9	47520.00	71280.00	87120.00	99000.00	108504.00	116424.00	123212.57	129152.57	134432.57	139184.57
PW10	59400.00	89100.00	108900.00	123750.00	135630.00	145530.00	154015.71	161440.71	168040.71	173980.71
PW11	68400.00	102600.00	125400.00	142500.00	156180.00	167580.00	177351.43	185901.43	193501.43	200341.43
PW12	9000.00	13500.00	16500.00	18750.00	20550.00	22050.00	23335.71	24460.71	25460.71	26360.71
PW13	54000.00	81000.00	99000.00	112500.00	123300.00	132300.00	140014.29	146764.29	152764.29	158164.29
PW14	48240.00	72360.00	88440.00	100500.00	110148.00	118188.00	125079.43	131109.43	136469.43	141293.43
PW15	50040.00	75060.00	91740.00	104250.00	114258.00	122598.00	129746.57	136001.57	141561.57	146565.57
Σ	**1588680.00**	**2383020.00**	**2912580.00**	**3309750.00**	**3627486.00**	**3892266.00**	**4119220.29**	**4317805.29**	**4494325.29**	**4653193.29**

Based on results shown in Table 4.7, the optimal solutions corresponding to the sequence of groundwater sample path optimization sub-problems generated yields optimal objective function values ranging from a minimum of 1588680.00 m^3/day to a maximum of 4653193.29 m^3/day. From Table 4.7, it can be observed that the optimal pumping rate solutions corresponding to the 500 hydraulic conductivity realizations generated are different from one another as well as those from deterministic approach, except when realization/sample size is equal to 1.

This is because the optimal solutions depend on the outcomes of realizations of hydraulic conductivity fields. It should be realized that the optimal pumping rate solution corresponding to realization/sample size equal to 1 are exactly the same as deterministic optimal solution strategy determined by Tyagi *et al.* (1995), as can be seen in Figure 4.13, and Tables 4.4 and 4.7 column SOSP1R1. This further proves that ROA method can be relied upon to give accurate results as any other tested method in use. Figure 4.13 presents groundwater sample path optimization sub-problems optimal solutions and the example of optimal pumping rate solution.

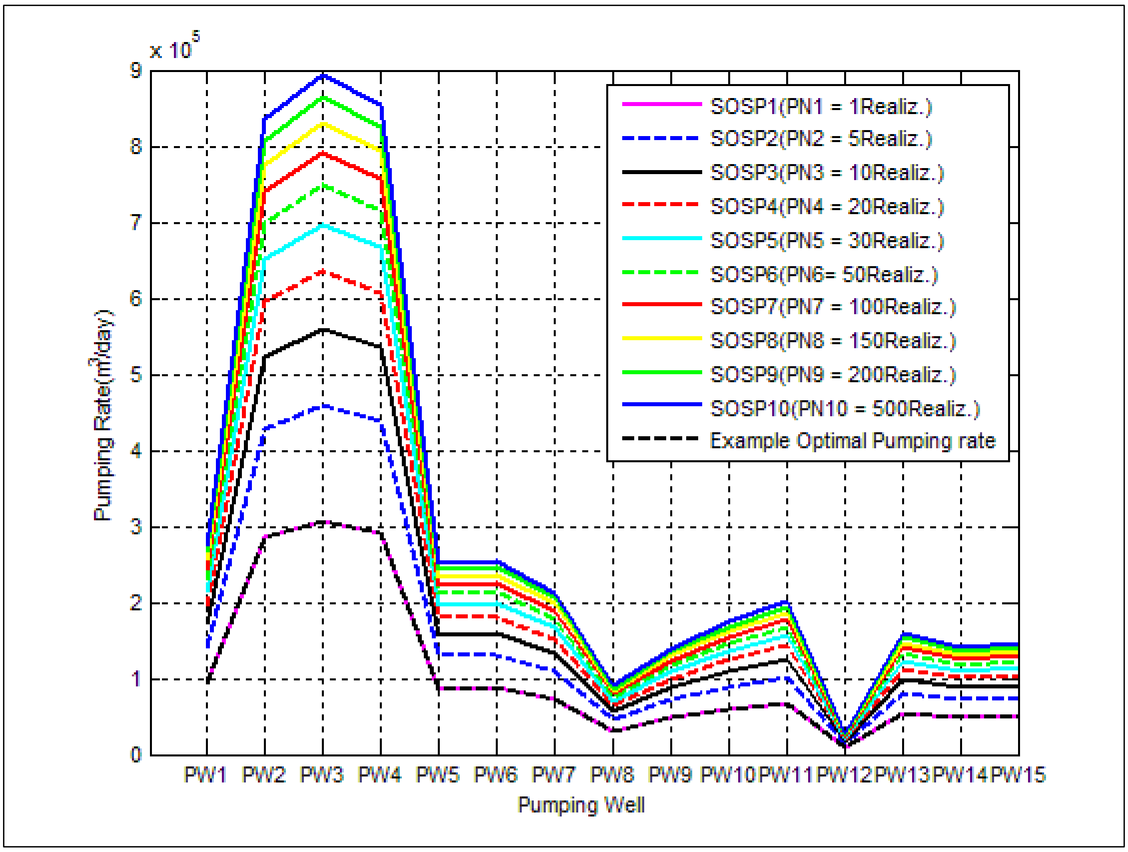


Figure 4.13: Groundwater Sample Path Sub-Problems Optimal Solutions

From Figure 4.13, the optimal solution corresponding to the sample path optimization sub-problem (SOSP1) with realization/sample size equal to 1 are almost the same as the example optimal solution deterministically obtained by Tyagi *et al.* (1995). The graphs of optimization sub-problem (SOSP1) optimal solution and the example of optimal solution fit each other (see Figure 4.13). In Figure 4.13, the sample path optimization sub-problems optimal solutions converge towards the true groundwater optimization optimum solution as the sample size increases.

The ninth optimal solution (SOSP9) is very close to the true optimization problem (SOSP10). The tenth sample path optimization problem (SOSP10) which considers all the 500 realizations generated, converges in a relatively few iterations because its initial solution guess (which is solution of SOSP9) is probably nearly equal to the true groundwater optimization problem (i.e., SOSP10) optimal solution. In order to evaluate the performance of the ROA framework, the optimization problems were solved with different initial solution (guesses) for three runs, which resulted in different optimal solutions and, therefore, different objective function values. The results for the expected total pumping rates evaluated over 500 realizations for three runs were then averaged. Figure 4.14 shows the performance of the ROA framework with cluster sampling for the groundwater management hypothetical example evaluated over the 500 realizations.

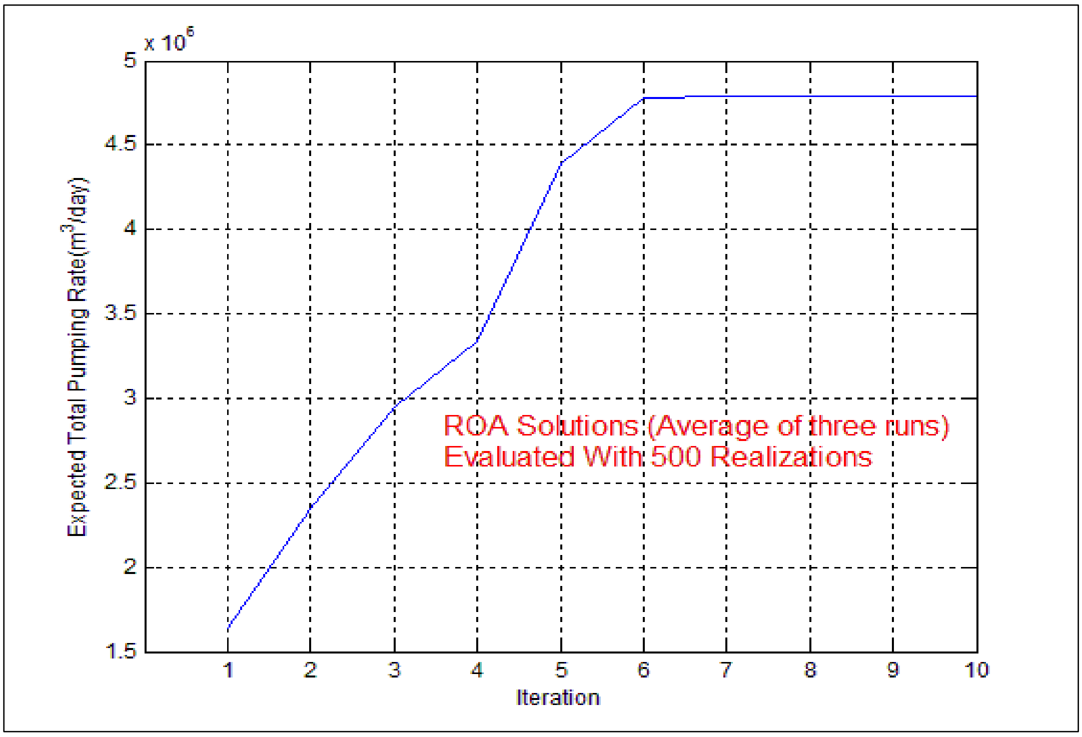


Figure 4.14: Performance of ROA with Cluster Sampling for the Groundwater Management Hypothetical Example

From Figure 4.14, the ROA expected total pumping rate (average of three runs) converged to its maximum value of about 4.7 x 10^6 m^3/day in about 6 iterations (evaluated over the 500 realizations of hydraulic conductivity fields).

Case 2: Conjunctive Water Use

In this case, the same sample size sequence was considered as the one used in the groundwater management model analysis, except that the number of decision variables were increased from 15 to 17 to represent additional surface water diversion points (decision variables) from river surface water source. The Conjunctive water use Sample path Optimization Sub-Problems (SOSPs) were solved and evaluated through ROA framework. Results of the ROA method framework for the conjunctive water use case are summarized in Table 4.8.

Table 4.8: Conjunctive Water Use Sample Path Optimization Sub-Problems Optimal Solutions

PW/SWD	SOSP1R1	SOSP2R5	SOSP3R10	SOSP4R20	SOSP5R30	SOSP6R50	SOSP7R100	SOSP8R150	SOSP9R200	SOSP10R500
PW1	93240.00	139860.00	170940.00	194250.00	212898.00	228438.00	241758.00	253413.00	263773.00	273097.00
PW2	285480.00	428220.00	523380.00	594750.00	651846.00	699426.00	740208.86	775893.86	807613.86	836161.86
PW3	305280.00	457920.00	559680.00	636000.00	697056.00	747936.00	791547.43	829707.43	863627.43	894155.43
PW4	291600.00	437400.00	534600.00	607500.00	665820.00	714420.00	756077.14	792527.14	824927.14	854087.14
PW5	86040.00	129060.00	157740.00	179250.00	196458.00	210798.00	223089.43	233844.43	243404.43	252008.43
PW6	87120.00	130680.00	159720.00	181500.00	198924.00	213444.00	225889.71	236779.71	246459.71	255171.71
PW7	72720.00	109080.00	133320.00	151500.00	166044.00	178164.00	188552.57	197642.57	205722.57	212994.57
PW8	30600.00	45900.00	56100.00	63750.00	69870.00	74970.00	79341.43	83166.43	86566.43	89626.43
PW9	47520.00	71280.00	87120.00	99000.00	108504.00	116424.00	123212.57	129152.57	134432.57	139184.57
PW10	59400.00	89100.00	108900.00	123750.00	135630.00	145530.00	154015.71	161440.71	168040.71	173980.71
PW11	68400.00	102600.00	125400.00	142500.00	156180.00	167580.00	177351.43	185901.43	193501.43	200341.43
PW12	9000.00	13500.00	16500.00	18750.00	20550.00	22050.00	23335.71	24460.71	25460.71	26360.71
PW13	54000.00	81000.00	99000.00	112500.00	123300.00	132300.00	140014.29	146764.29	152764.29	158164.29
PW14	48240.00	72360.00	88440.00	100500.00	110148.00	118188.00	125079.43	131109.43	136469.43	141293.43
PW15	50040.00	75060.00	91740.00	104250.00	114258.00	122598.00	129746.57	136001.57	141561.57	146565.57
SWD1	224800.00	540000.00	660000.00	750000.00	822000.00	882000.00	933428.57	978428.57	1018428.57	1054428.57
SWD2	261200.00	675000.00	825000.00	937500.00	1027500.00	1102500.00	1166785.71	1223035.71	1273035.71	1318035.71
Σ	**2072607.00**	**3598020.00**	**4397580.00**	**4997250.00**	**5476986.00**	**5876766.00**	**6219434.57**	**6519269.57**	**6785789.57**	**7025657.57**

From Table 4.8, it can be observed that optimal objective functions values range from a minimum of 2072607.00m^3/day to a maximum of 7025657.57m^3/day. It is also interesting to note that the optimal conjunctive water use withdrawal rate solutions corresponding to the 500 hydraulic conductivity realizations are different from one another as well as from the deterministic solution, except when realization/sample size is equal to 1 (see Table 4.8 column SOSP1R1 and Table 4.5 results). This is because the optimal withdrawal rates depend on the outcomes of the uncertainty realizations. Figure 4.15 shows conjunctive water use sample path optimization sub-problems solutions.

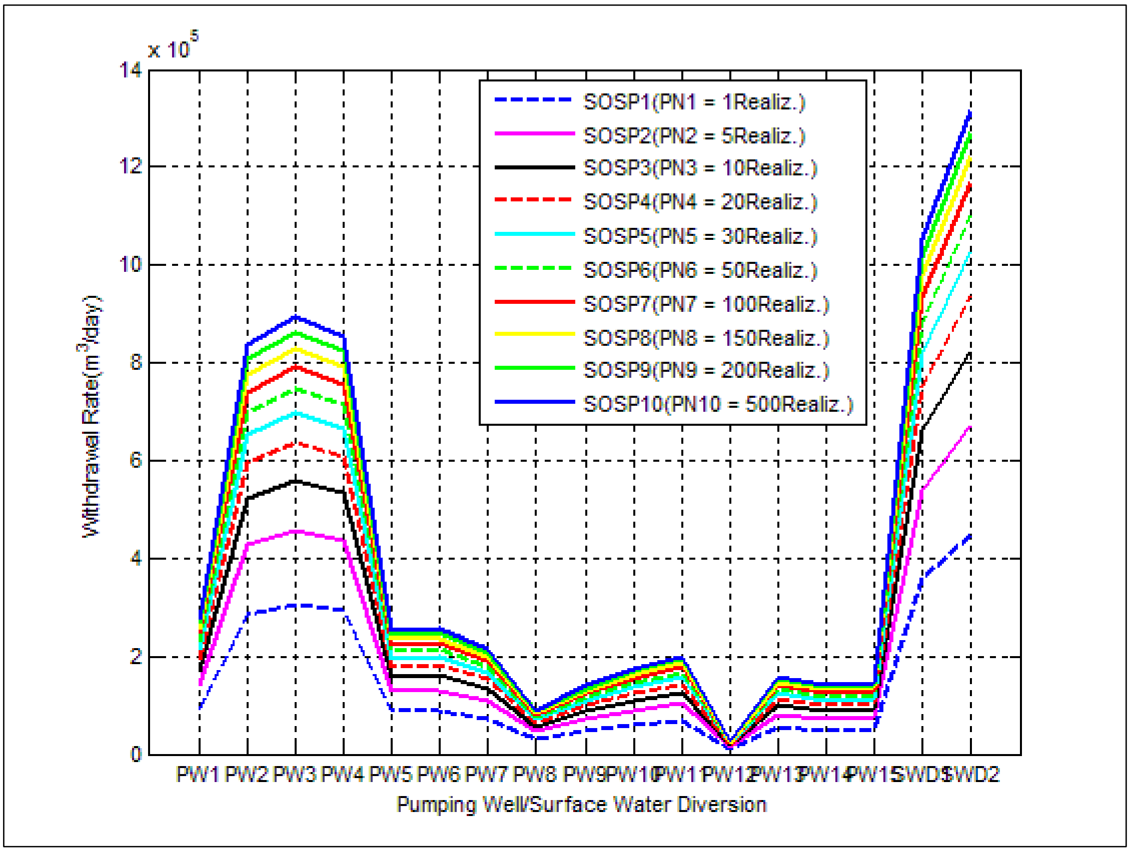


Figure 4.15: Conjunctive Water Use Sample Path Optimization Sub-Problems Solutions

Figure 4.15 indicates that the sample path optimization sub-problems solutions converge to the true conjunctive water use optimization problem (i.e., SOSP10) as the sample size increases. It should be noted that the first sample path optimization problem (SOSP1) is inexpensive because computing every optimal solution or objective function requires only one function evaluation. The tenth sample path optimization problem (SOSP10) which considers all the 500 realizations generated, converges in a relatively few iterations because its initial solution (which is solution of problem SOSP9) is probably nearly equal to the true optimization problem (i.e., SOSP10) optimal solution. Figure 4.16 presents histogram diagram for the true optimization problem (SOSP10) optimal conjunctive water use withdrawal rates.

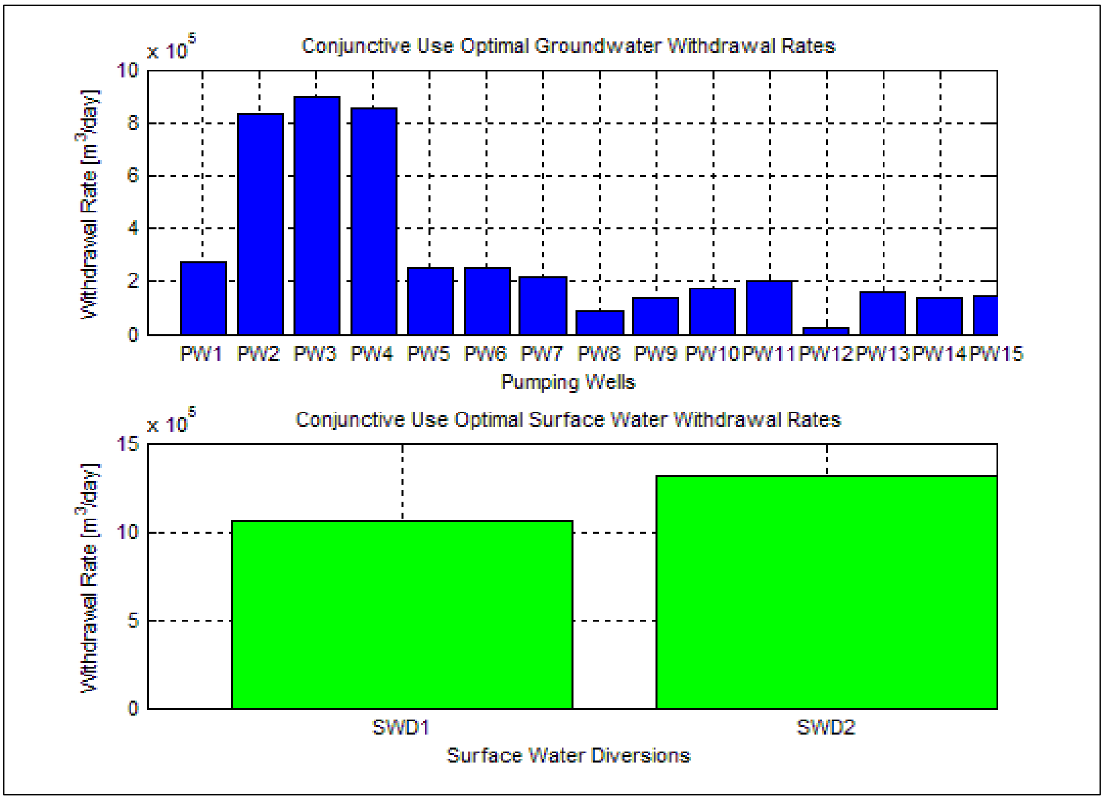


Figure 4.16: Conjunctive Water Use Optimized Surface Water and Groundwater Withdrawal Rates

From Figures 4.15 and 4.16, the highest and lowest groundwater pumping rates occurred at the same pumping wells PW3 and PW12, respectively. It is also interesting to see that even though pumping wells appeared to have high pumping rates compared to the deterministic case, re-simulation using this optimal solution scheme indicates that no pumping well became dry. This implies that the feasible optimal solution obtained satisfy all defined constraints. This change effect may suggest that uncertainty realization outcomes of hydraulic conductivity fields dictate groundwater pumping rates and consequently the conjunctive water use withdrawal rates.

To evaluate the overall performance of the ROA framework for conjunctive water use management, the sample path optimization problems were solved with different initial solutions (guesses) for three runs, which consequently resulted in different optimal solutions and, therefore, different expected objective function values. The results for the expected total conjunctive water use withdrawal rate evaluated over 500 realizations for three runs were then averaged. Figure 4.17 shows overall performance of the ROA framework for the conjunctive use management model with cluster sampling evaluated over 500 realizations.

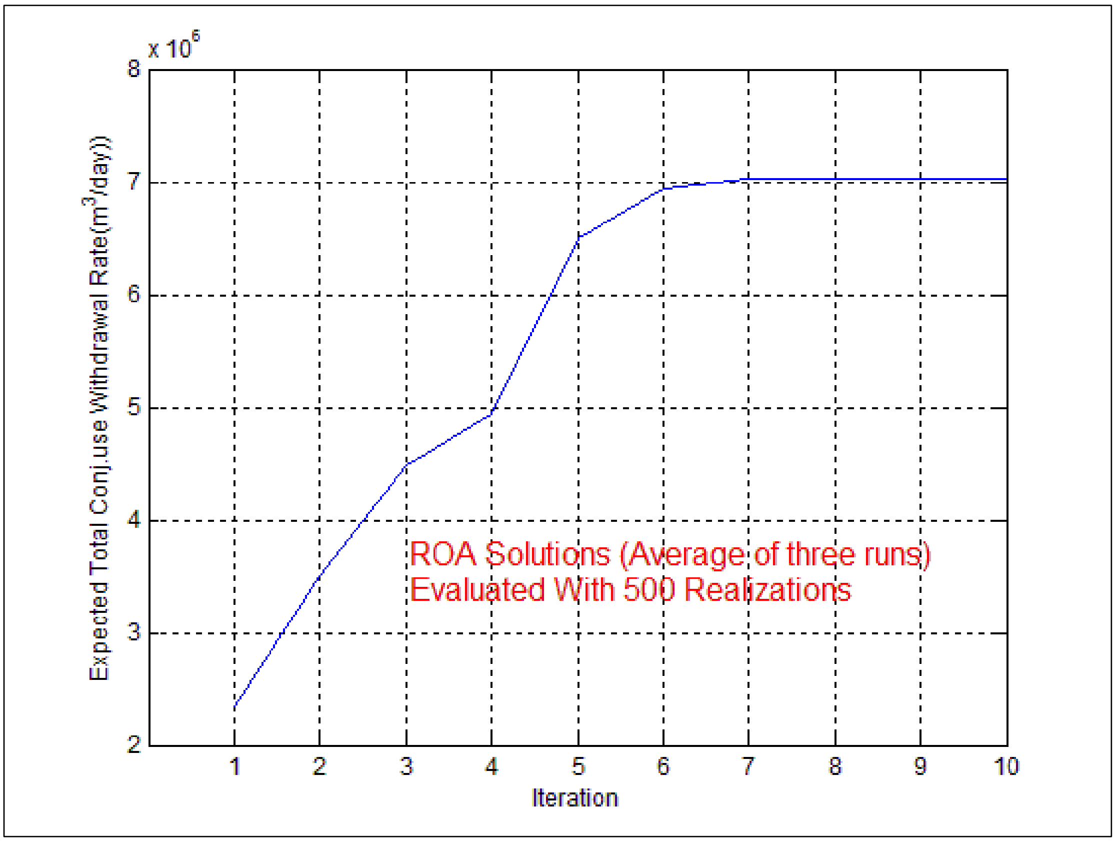


Figure 4.17: Overall Performance of ROA with Cluster Sampling for Conjunctive Water Use Hypothetical Example

In Figure 4.17, the ROA expected total conjunctive water use withdrawal rate (average of three runs) converged to its maximum value of about 7.0 x 10^6 m^3/day within 6 to 7 iterations (evaluated over the 500 realizations of hydraulic conductivity fields). Figure 4.18 presents overall percentages of surface water and groundwater sources contributions to the ROA total optimal conjunctive water use withdrawal rate in a pie chart view.

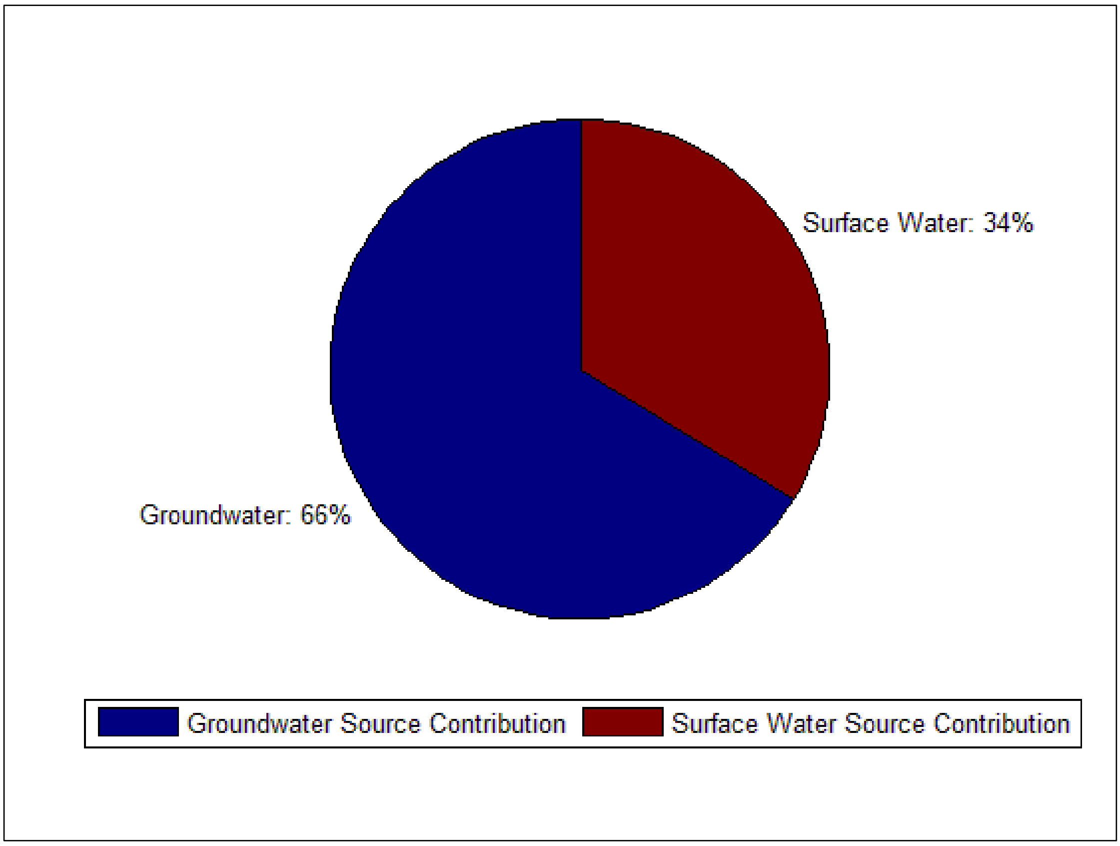


Figure 4.18: Overall Percentages of Surface Water and Groundwater sources Contribution to the ROA Total Optimal Conjunctive Water Use Withdrawal Rate

In Figure 4.18, surface water source contributed about 34 percent of the total water required, while the remaining 66 percent was contributed by groundwater source.

Concluding Remarks

It emerged that optimal withdrawal volume rates strategies designed based on deterministic optimization approach have lower withdrawal volume rates values than those designed based on ROA approach (stochastic optimization approach). This suggests that the deterministic approach provides unrealistic solution results and underestimates pumping volume rates. In all maximized pumping rate schemes (i.e., groundwater as only source of water supply model and conjunctive water use model) it has been revealed that the difference between optimized withdrawal rate schemes (deterministic and ROA optimal withdrawal rates) and the existing un-optimized withdrawal rate scheme is high. This suggests further that there is potential volume of groundwater resource storage in the aquifer system which is undeveloped.

CHAPTER FIVE: APPLICATION OF METHODOLOGY TO OLIFANTS RIVER BASIN

5.1 INTRODUCTION

In Chapters one, two and three, the main objective, concepts and methodological framework of this study were presented and discussed. The overall modelling motivation of this research was to develop a methodology which is capable of optimizing the conjunctive water use of surface water and groundwater resources of the Olifants River basin taking into account uncertainty arising due to scarcity of data in the study area.

5.2 THE STUDY AREA

The study was carried out in the Great Letaba River catchment (also known as the Groot Letaba River catchment) area in the primary drainage region B of the Olifants River basin Water Management Area (WMA) situated in the Limpopo Province of South Africa. The Great Letaba River is one of the major tributaries of the Olifants River. Figure 5.1 shows the study area location map.

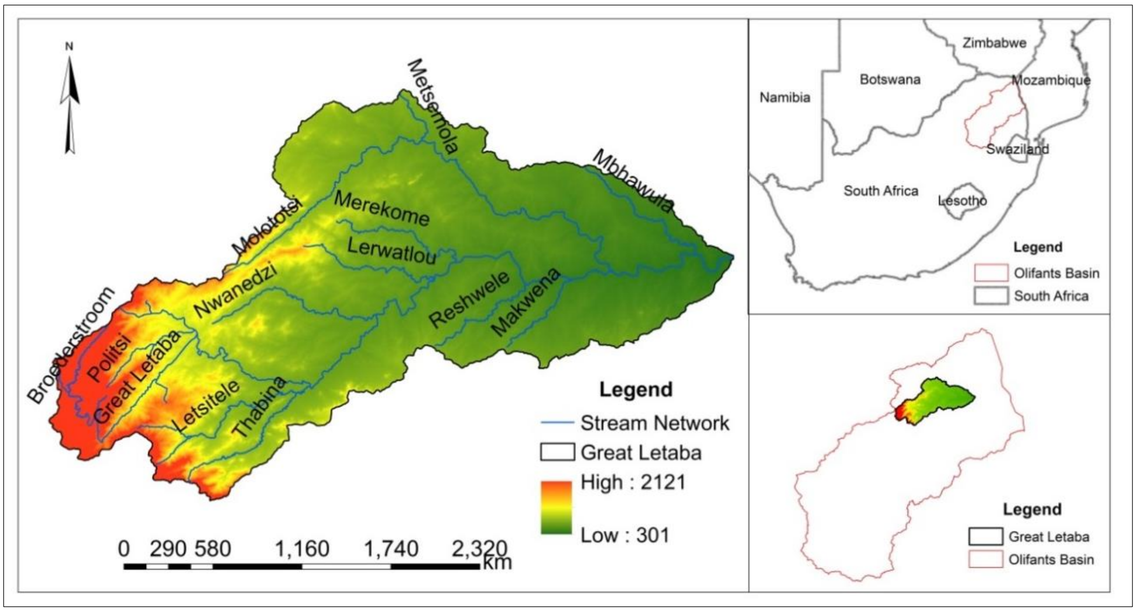


Figure 5.1: Study Area Location Map

The study area River catchment is characterised by high elevation differences between 301 and 2121m (AMSL). It should clearly be noted that in many decades most of studies (DWAF, 1990, 1994, 1998, 2003, 2004, 2006a, 2006b, 2006c, etc.) consider the study area River catchment as part of Limpopo River basin. This is because previously the Great (Groot) Letaba River catchment area was known to be within the then Luvuvhu/Letaba Water Management Area (WMA) which was falling in the Limpopo River basin, until recently (20th August 2012) when it was officially declared by the Government of South Africa as being part of the Olifants River basin (Thomas, 2015).

The study area covers a surface area of approximately 4952 km^2 (DWAF, 2006c). The main urban areas in the catchment are Tzaneen and Nkawakowa. The Kruger National Park (KNP) is located at the eastern lower end of the catchment to the Mozambique border.

In the study area River catchment along the Letsitele River tributary, a new dam (N'wamitwa Dam) has been proposed for construction so as to meet ever increasing water demand. However, improved operating techniques appear to offer a more favourable option compared to the construction of new dams for the present. Hence, implementation of new water resources techniques such as conjunctive water use is of paramount importance for the present water shortages (DWAF, 2006a).

5.3 CONCEPTUAL MODELS OF THE STUDY AREA

This sub-section presents surface water, groundwater, and conjunctive water use systems conceptual models in a simplified manner to show the most relevant hydrological and hydrogeological features.

5.3.1 Surface Water Conceptual Model

The study area is mainly drained by the Great Letaba River and its tributaries (see Figure 5.2). From the confluence of the Great Letaba and Klein Letaba Rivers, the Great Letaba River flows to the eastward through the Kruger National Park (KNP) until it joins with the Olifants River near the border to Mozambique. Figure 5.2 shows the study area River system and flow gauging stations networks.

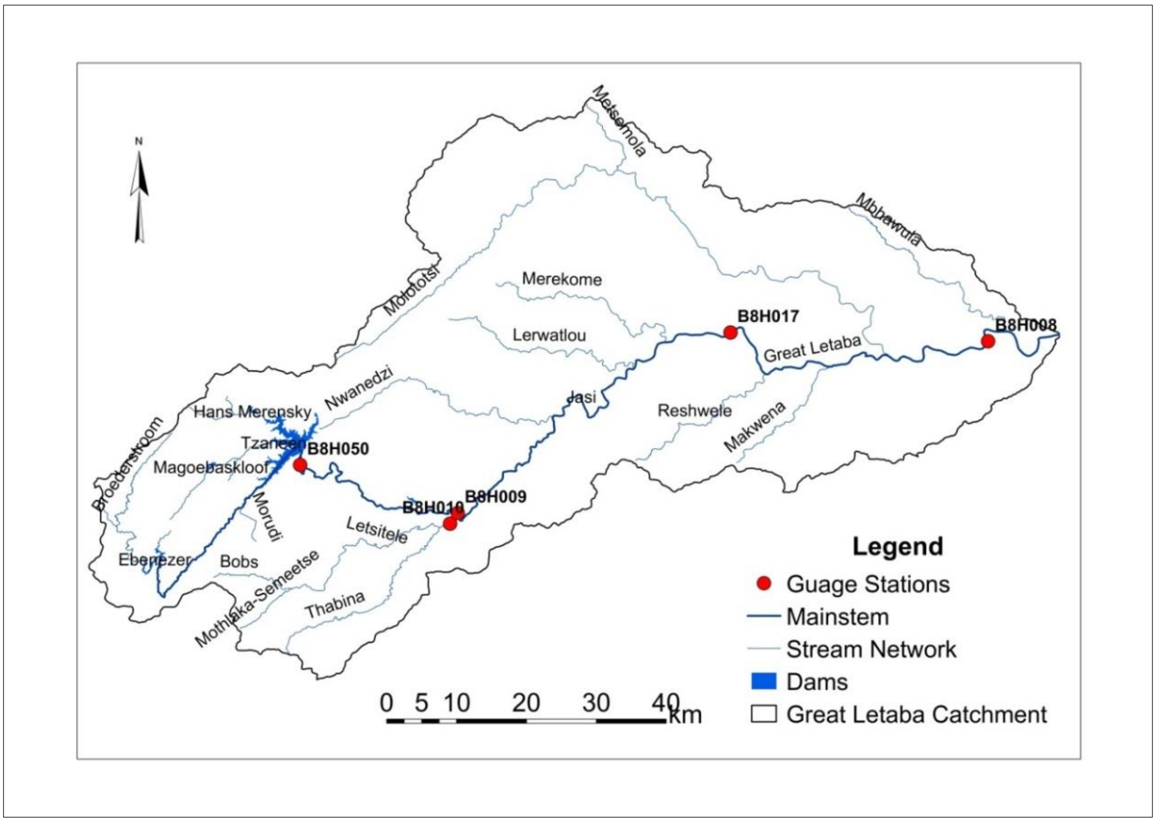


Figure 5.2: Study Area River System and Flow Gauging Stations Networks

It should be noted that flow gauging station network in the study area is poor. Most of the stations are concentrated in the upper part of the catchment (DWAF, 2006b). The study area regional water supply scheme uses water from the Great Letaba River and its tributaries to supply water to a number of towns including Polokwane (Pietersburg), Tzaneen, Duiwelskloof, Haenertsburg and to various villages. Also extensive irrigation activities occurring within the catchment is supplied with water from this water supply system (DWAF, 2003). Figure 5.3 shows the regional water supply scheme components of the study area in a schematized layout.

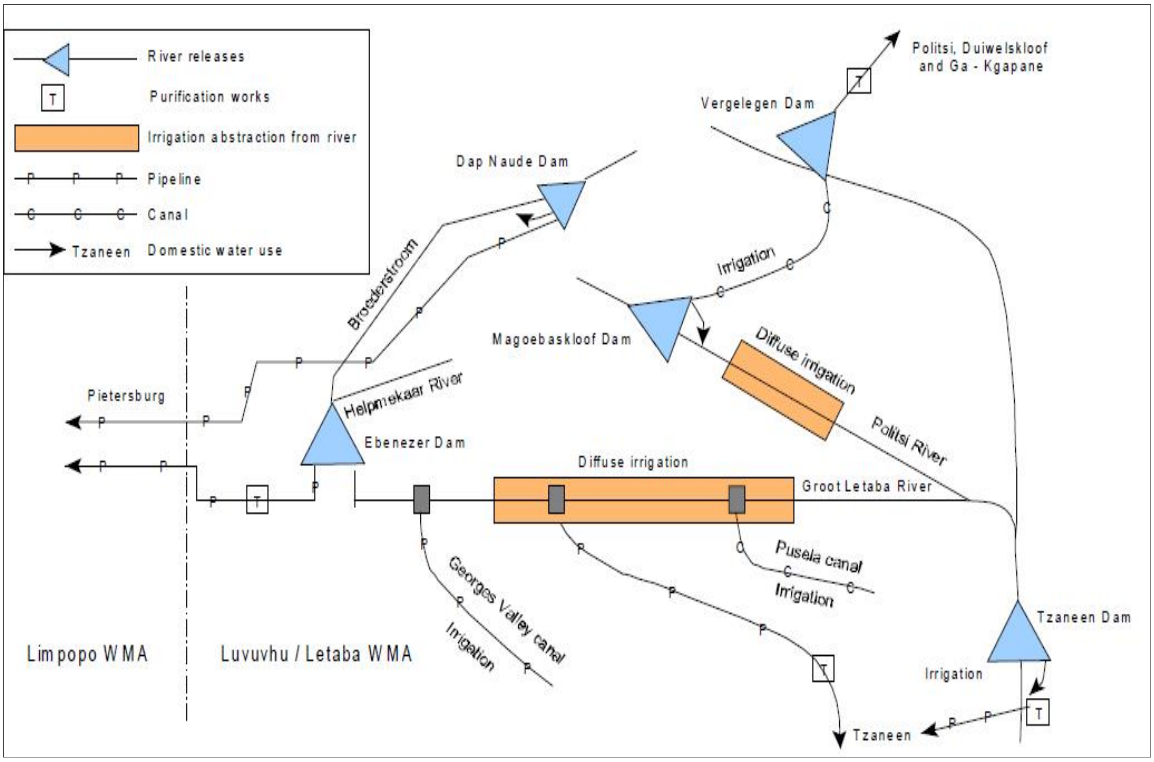


Figure 5.3: Study Area Regional Water Supply Scheme Components

Source: Adopted after (DWAF, 2003)

Based on limited hydrological and water use information available, surface water conceptual water balance model of the river system was developed and schematized in a simple manner. Figure 5.4 shows the study area surface water conceptual model schematized layout.

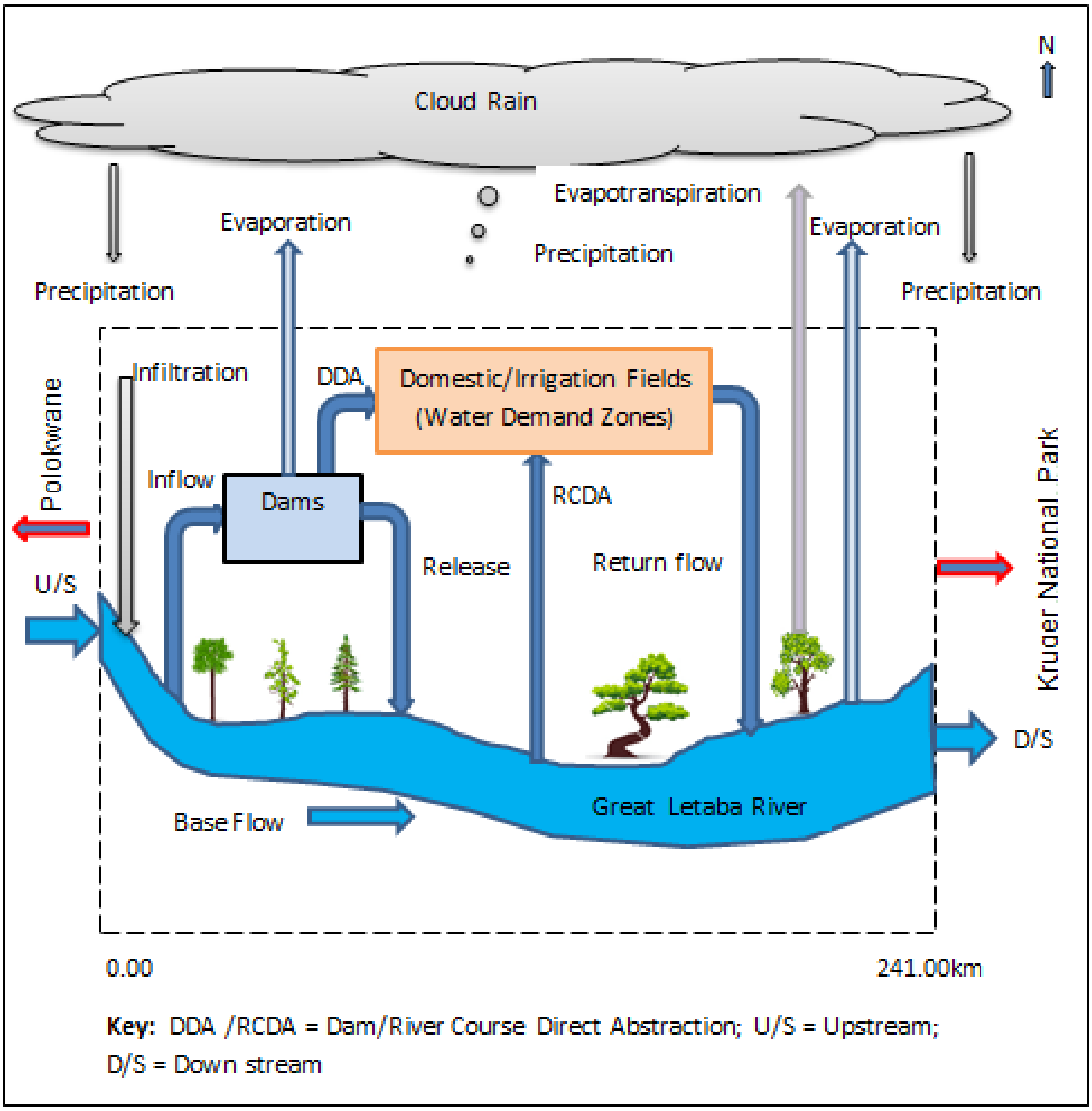


Figure 5.4: Study Area Surface Water Conceptual Model Schematized Layout

5.3.2 Groundwater Conceptual Model

The study area River catchment aquifers are predominantly secondary with exception of alluvial deposits along the main river system. Intergranular aquifers (ranging from unconsolidated to semi-consolidated materials with primary porosity) occur in the study area. Figure 5.5 shows the hydrogeological regions of the aquifer system.

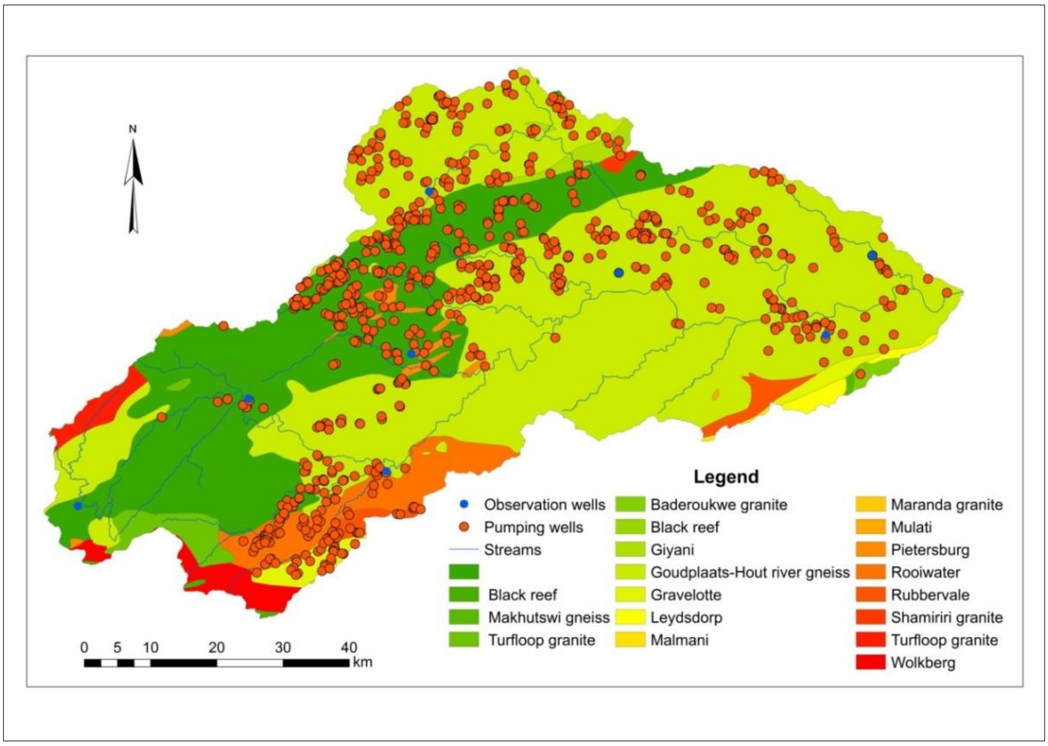


Figure 5.5: Hydrogeological Regions of the Study Area Aquifer System

In the study area, a total of eight (8) groundwater monitoring (control points) wells have been installed by the Government for groundwater level measurements (see Figure 5.5 blue dots). The existence of hot springs within the regional aquifer system suggests that the aquifer is confined except along the major rivers where localized alluvium aquifers occur including in the eastern end of the catchment within the KNP where unconfined aquifers occur. The study area aquifer system is characterised by relatively low magnitude hydraulic conductivity/transmissivity values. Figure 5.6 shows the transmissivity values for the aquifer system.

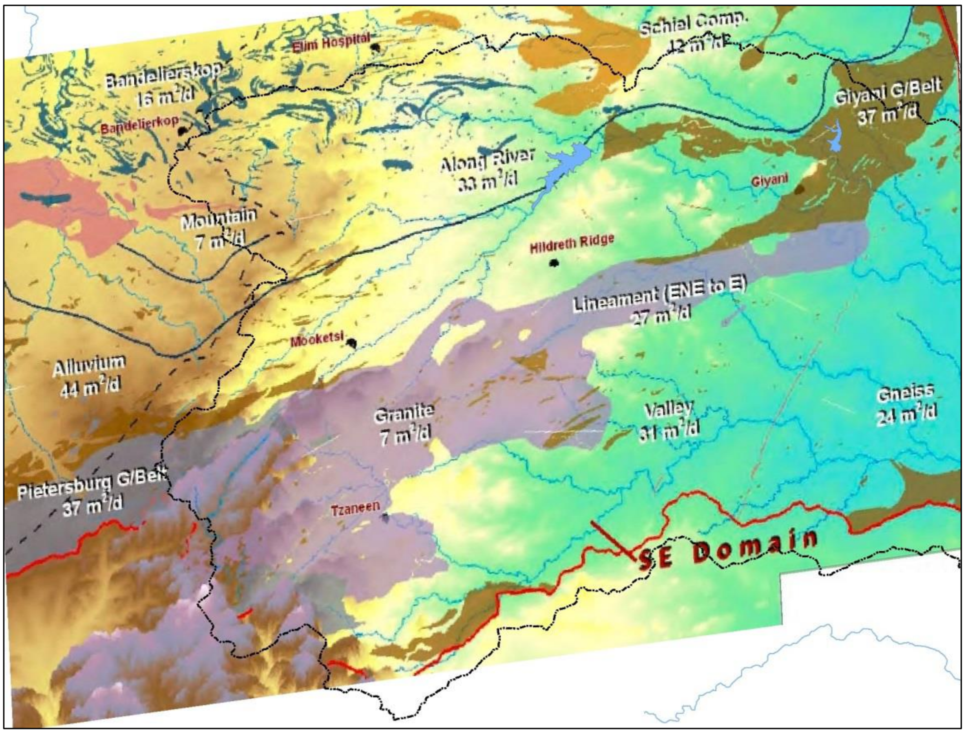


Figure 5.6: Transmissivity Values and Faults for the Study Area Aquifer System

Source (Holland, 2011)

It can be observed that in the aquifer system, transmissivity values range from a minimum of 7 m^2/day to a maximum of 31 m^2/day. Recharge is mainly realised within the high elevation areas of the Great Escarpment (the Drakensburg Escarpment) mountains where rainfall is high above 1000mm/annum and in the alluvium aquifers along major rivers. Total net recharge is about 126 mm/annum (DWAF, 2003). Figure 5.7 shows the groundwater conceptual water balance model of the aquifer system.

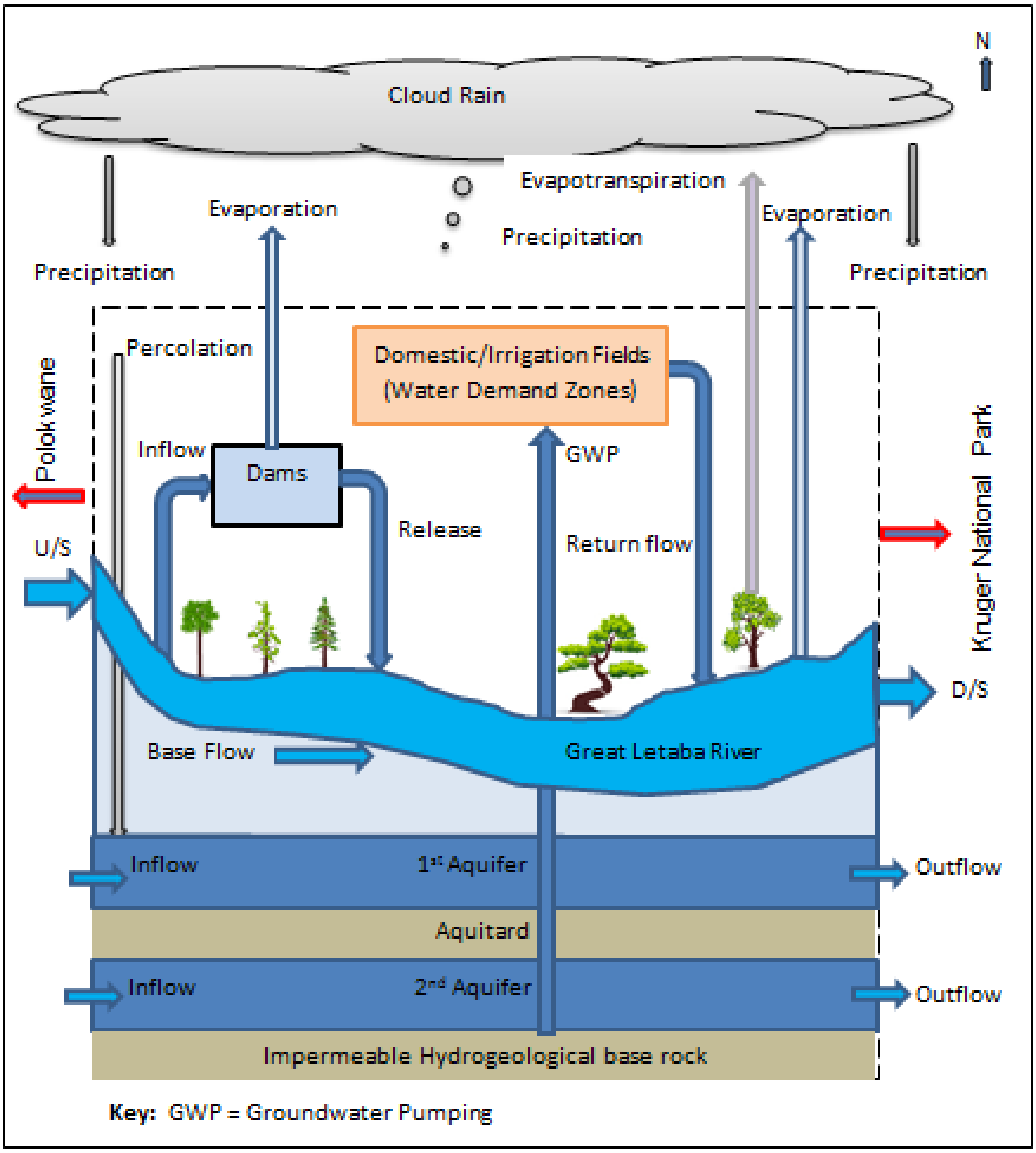


Figure 5.7: Groundwater Conceptual Model of the Study Area

5.3.3 Conjunctive Water Use Conceptual Model

The conjunctive water use system was schematised in a simplified manner as shown in Figure 5.8.

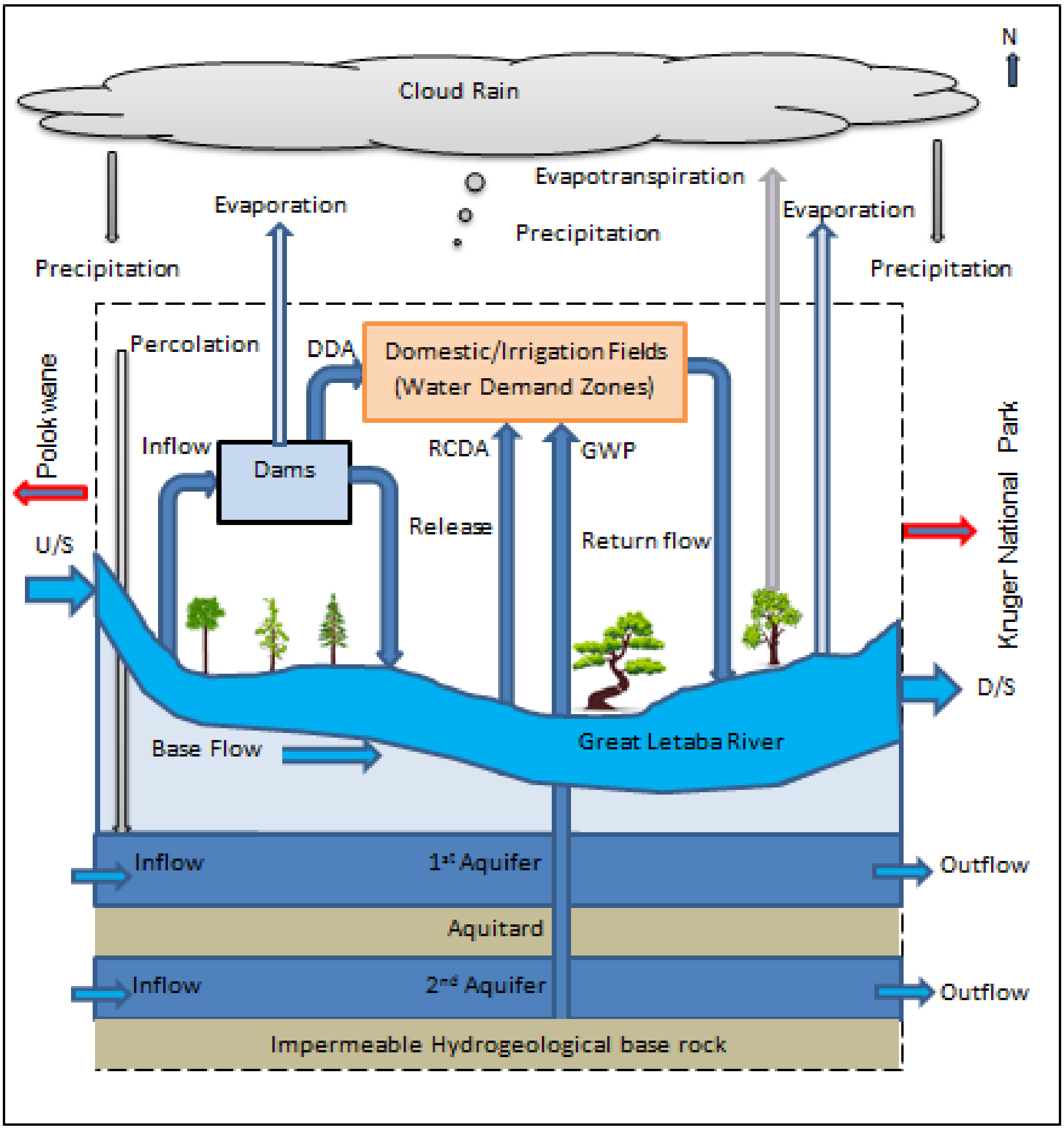


Figure 5.8: Conjunctive Water Use Conceptual Model of the Study Area

5.3.4 Model of the Study Area

To delineate the study area regional aquifer system from local aquifers, the whole area of study was discretized into 150 X 100 cells, each in a model domain of a grid cell dimensional size of 1500m X 1500m. The discretized modelled area of interest is shown in Figure 5.9. Figure 5.9 shows finite difference groundwater flow numerical simulation model.

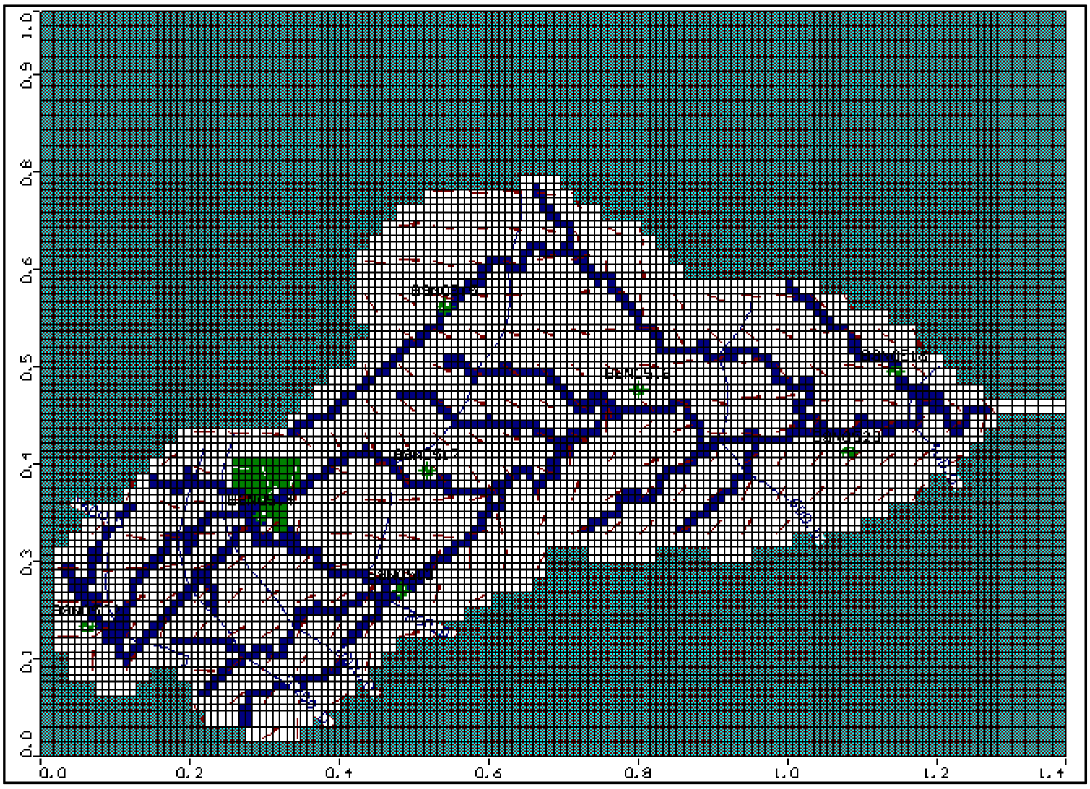


Figure 5.9: Finite Difference Model of the Study Area

It was realized that in the modelled area, no-flow boundary appears at the northern and southern parts of the catchment, while constant flux boundary conditions exist for the two boundaries located at the north-west and south-west parts (these are areas bounded by the Great Escarpment (i.e., the Drakensburg Escarpment mountainous regions).

Moreover, specific head boundary condition (also referred to as Dirichlet condition) for which head is considered to be independent of time was specified for the surface water bodies (i.e., rivers and Dams) and eastern boundary of the modelled domain. Other potential surface water bodies (such as wetlands and springs) were also included in the modelled area.

In this study, steady-state simulations have been considered and hence, initial condition is not a major concern. This is because in any groundwater steady-state numerical simulations, the focus is to determine aquifer drawdown in response to externally imposed excitations/stresses such as groundwater pumping whereby relative heads as measured with respects to drawdown responses are of great importance rather than absolute head values. Due to limited availability of water demands data, the existing (current) water pumping (withdrawal) rates were considered as the minimum pumping (withdrawal) rate limits (i.e., lower bounds pumping (withdrawal) rates) which were required to satisfy the minimum water requirements of the competing water users. Both deterministic and stochastic simulations were carried out.

5.3.5 Study Area Conjunctive Water Use Management Model

The overall objective was maximization of total conjunctive water use of surface water and groundwater withdrawal rates. Because of the water shortages of the study area for both irrigation and domestic use, surplus water from any water storage source can be used for irrigation and/or domestic. Hence, this objective seeks to maximize the amount of total conjunctive water use which can sustainably be withdrawn from both groundwater (aquifer) and surface water storage systems subjected to the same conjunctive water use management model constrains as described in Chapter four.

5.4 DISCUSSION OF RESULTS

In the study area, currently there exist 809 boreholes which are operational and another 294 that are blocked or abandoned due to various reasons such as poor water quality. Data on boreholes were obtained from GRIP Database (GRIP Limpopo, 2013). From 515 boreholes with known capacity, the overall average daily abstraction capacity ranges from 62 – 70 m^3/day. Table 5.2 presents quaternary catchments names, named variables (combined pumping wells (CPW) withdrawal rates) and surface water diversion named variable (combined surface water diversion (CSWD) withdrawal rate) with their corresponding number of wells/abstraction points, and quaternary catchment polygon surface areas, which were used for the optimization problem analysis.

Table 5.1: Quaternary Catchment (QC) Names, Named Variables, Number of Wells/ Abstraction Points and Quaternary Catchment (QC) Surface Areas

S/No	QC Name*	Named Variable	# of Wells/Abstraction Points	QC Area* (km^2)
1	B81G	CPW1	122	513
2	B81H	CPW2	83	668
3	B81A	-	0	169
4	B81B	CPW3	5	481
5	B81C	CPW4	10	208
6	B81E	CPW5	65	665
7	B81F	CPW6	112	1201
8	B81J	CPW7	38	568
9	B81D	CPW8	80	479
10	-	CSWD	20	-
Σ	**-**	**-**	**535**	**4952**

*Source (DWAF, 2006c)

A total of 515 pumping wells and 20 surface water abstraction points (i.e., from river course and dams abstraction points with average daily abstraction capacity ranges from 500 – 14500 m^3/day) were used. Hence, a total of 535 abstraction points (i.e., decision variables excluding slack variables) from groundwater and surface water sources were used in simulation-optimization problem analysis. Figure 5.10 shows overall percentages contributions of the existing (un-optimized) conjunctive water use of surface water and groundwater sources in a pie chart view.

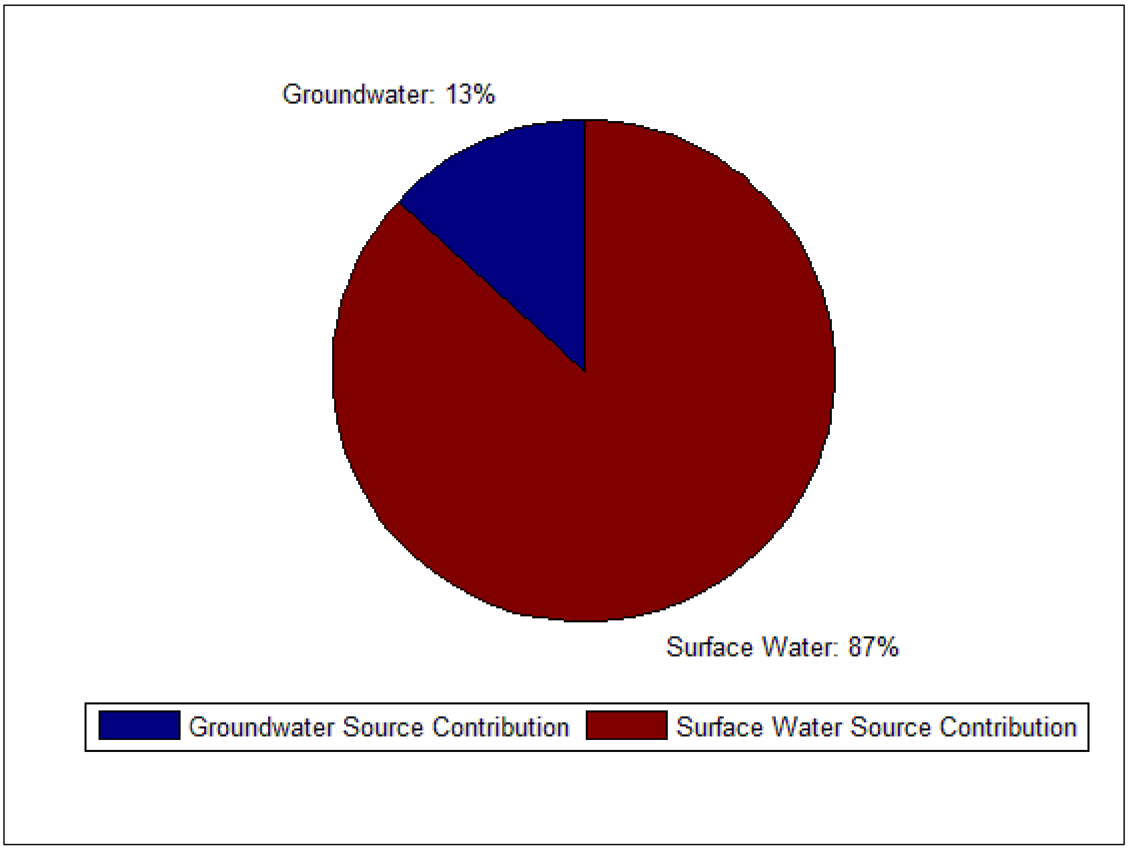


Figure 5.10: Existing un-optimized surface water and groundwater sources overall percentages contribution pie chart view

Currently, surface water source contributes about 87 percent of the total un-optimized conjunctive water use and the remaining 13 percent is contributed by groundwater source (see Figure 5.10). Table 5.2 presents river and aquifer systems properties model input parameter values which were used for the simulation optimization problems analysis.

Table 5.2: River/Stream - Aquifer System Properties Model Inputs Parameters

Item	River/Stream - Aquifer System Property	Parameter value
1	Average River/Streambed Hydraulic Conductivity	0.2 m/day
2	Average River Width ranges	10 m – 100 m
3	Average Riverbed Thickness ranges	0.5 m – 2.5 m
4	Average River flow depth (stage) ranges	0.05 m – 2.5 m
5	Average Aquifer Saturated Thickness ranges	7.0 m – 110.50 m
6	Average Aquifer Hydraulic Conductivity value	1.45 m/day
7	Aquifer specific yield	0.05
8	Total net groundwater Recharge	126 mm/year

5.4.1 Deterministic Conjunctive Water use Management

In this management approach, the hydraulic conductivity and hence transmissivities were considered spatially invariant. It was assumed that field measurements were adequate, precisely measured and hence all the associated parameters were set fixed. The maximum abstraction rate limits (i.e., the constraining upper bound values) were set equal to n times the current (existing) withdrawal rates. The multiple values “n” were assumed equal to the ratio (proportion value) of the maximum recommended withdrawal rates (i.e., the maximum abstraction rates recommended by the Government of South Africa) values to the current (existing) withdrawal rates.

In order to present the optimal conjunctive water use results in various graphical formats, the withdrawal volume rates were sorted out according to their locations in the model domain and their associated quaternary catchments (refer Table 5.1) polygons and then for every quaternary catchment's pumping wells, a geometrical mean of optimal conjunctive water use withdrawal rate was computed. Table 5.3 presents existing un-optimized as well as computed optimal conjunctive water use quaternary catchment (QC) mean withdrawal rates for the study area.

Table 5.3: Optimal Conjunctive water use mean Withdrawal Rates

Named Variable	Existing Un-Optimized Mean Withdrawal Rate (m^3/day)	Optimal Mean Withdrawal Rate (m^3/day)
CPW1	20.44	333.60
CPW2	37.00	281.31
CPW3	18.66	184.82
CPW4	4.11	66.24
CPW5	11.08	139.96
CPW6	22.16	173.03
CPW7	28.97	385.75
CPW8	33.94	409.20
CSWD	6335.70	11040.17
Σ	**6512.04**	**13014.08**

The total optimal conjunctive mean water use withdrawal rate value was 13014.08 m^3/day. Groundwater contributed a total mean volume rate of 1973.91m^3/day which is about 15.17 percent. The remaining total mean volume withdrawal rate of 11040.08m^3/day was supplement by surface water source (equivalent to 84.83

percent). Figure 5.11 shows the overall percentages of contribution of surface water and groundwater sources in pie chart view.

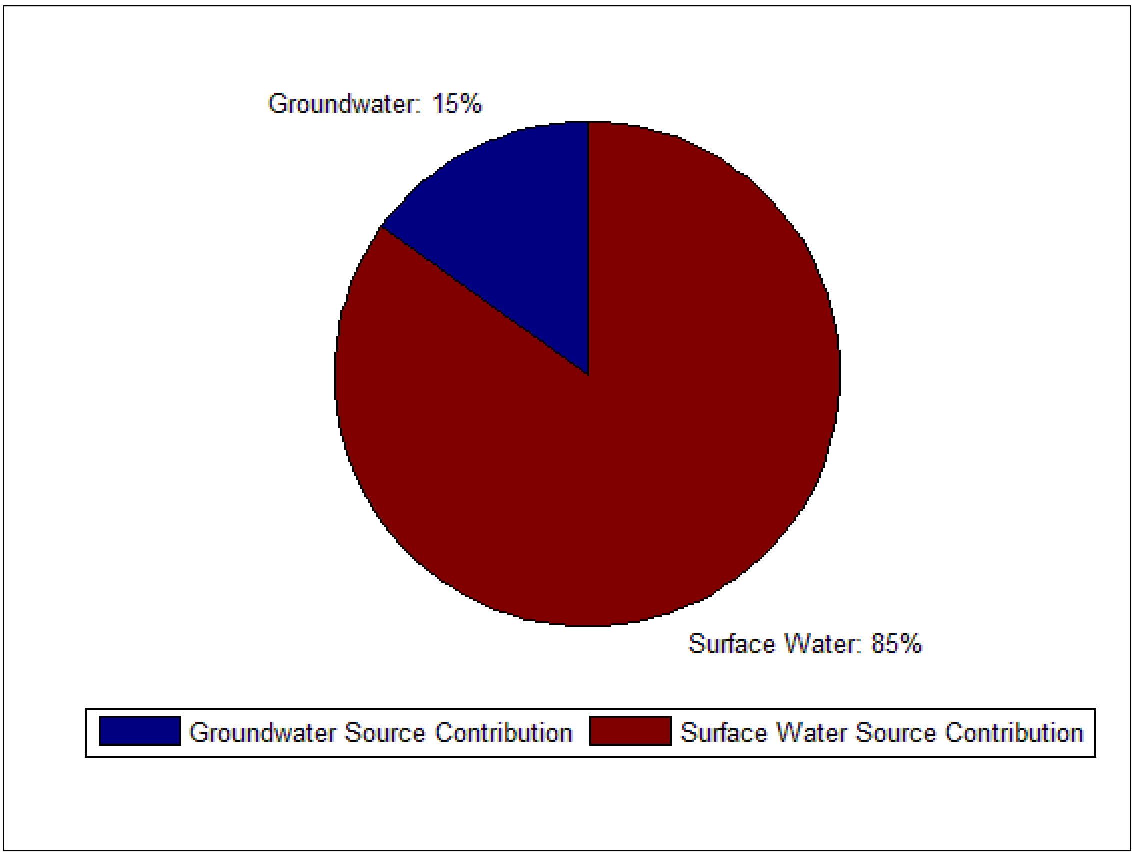
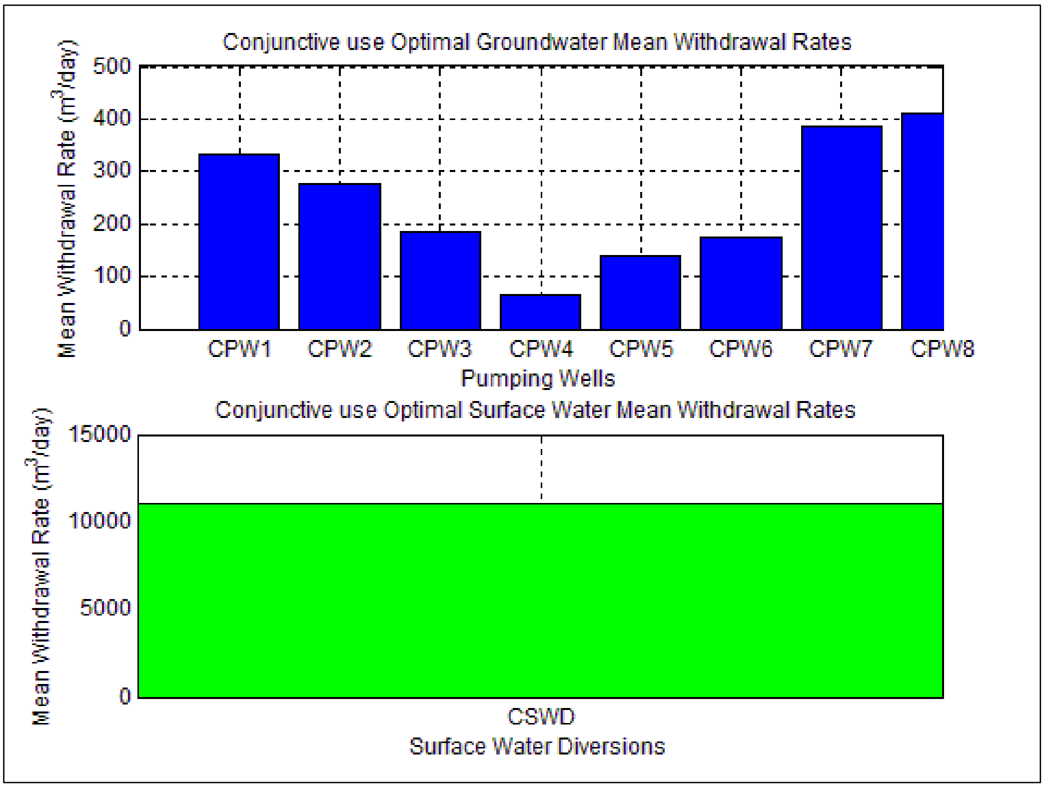


Figure 5.11: Overall Percentages of Contribution of Surface Water and Groundwater Sources to the Total Optimal Conjunctive Water Use Withdrawal Rate

Figure 5.11 indicates that groundwater source contributed less percentage compared to surface water. This suggests that the aquifer storage system has little water to supply than the surface water storage system. Figure 5.12 presents histogram for the optimal conjunctive water use mean withdrawal rates.



.

Figure 5.12: Optimal Conjunctive Water Use Mean Withdrawal Rates

From Figure 5.12, high groundwater withdrawal rates occurred at quaternary catchments B81G, B81H, B81J and B81D (see Table 5.1), with combined pumping wells CPW1, CPW2, CPW7 and CPW8, respectively. This is because these pumping wells fall within quaternary catchments which are characterized by dense river networks and located within a relatively high magnitude of hydraulic conductivity values. River networks and high magnitudes of hydraulic conductivity values may contribute to an increase in groundwater pumping rates. Low groundwater withdrawal rate appeared at quaternary catchment B81C with combined pumping well CPW4 which falls within a relatively low magnitude of hydraulic conductivity values. It should be noted that, due to high difference between groundwater and surface water withdrawal rate values, it was important to scale down surface water withdrawal rates values by 1:10 scaling factor so as to have a proportionate graphical appearance. Figure 5.13 compares the existing un-optimized conjunctive water use with optimized conjunctive water use mean withdrawal rates.

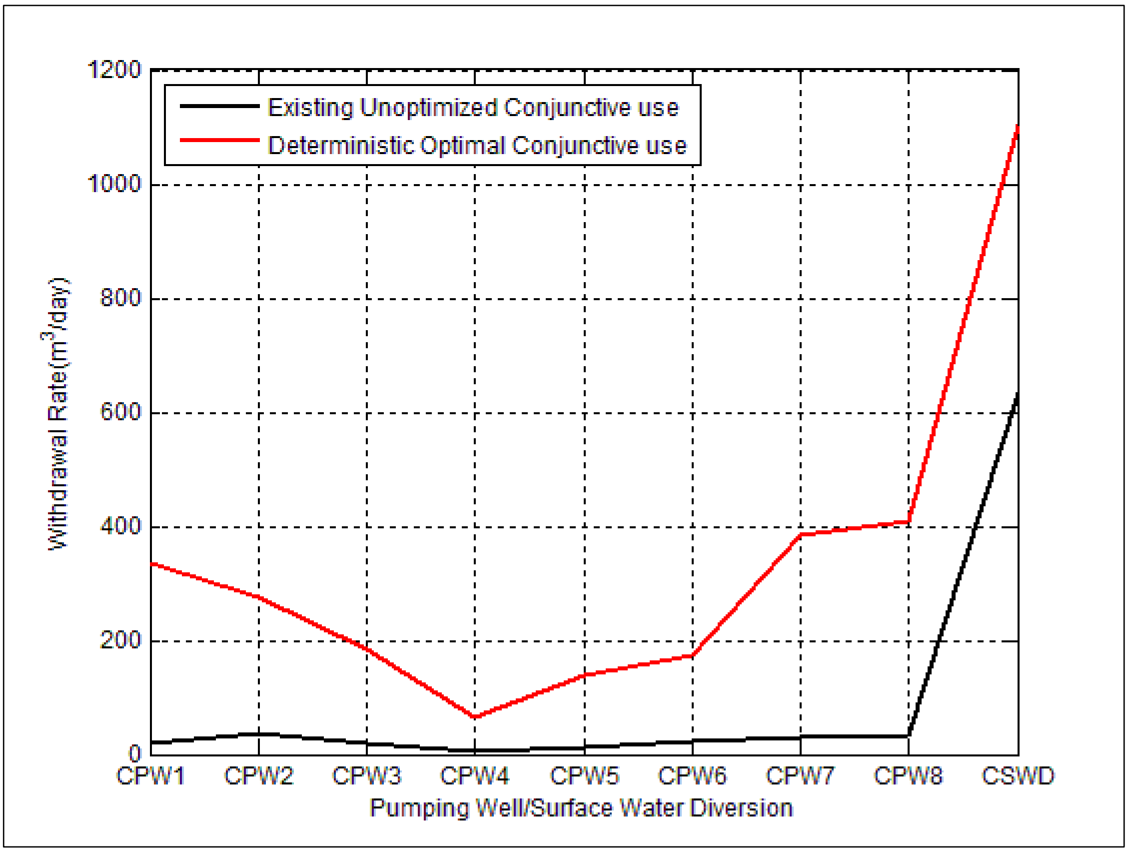


Figure 5.13: Comparison of existing un-optimized conjunctive water use and Deterministic Optimal conjunctive water use mean withdrawal rates

From Figure 5.13, it can be observed that pumping wells which had higher groundwater withdrawal rates in existing (un-optimized) pumping also have higher withdrawal rates in optimized conjunctive water use scheme. The optimized scheme mean withdrawal rates are higher than the existing un-optimized mean withdrawal rates. This indicates that there is potential volume of groundwater resource which is undeveloped within the quaternary catchments which have high magnitude of groundwater pumping rates variations.

In Figure 5.13, it can be noted that the lowest groundwater withdrawal rate occurred at quaternary catchment B81C with combined pumping well CPW4. High magnitude of withdrawal rate variations occurred in quaternary catchments B81G, B81H, B81J and B81D with combined pumping wells CPW1, CPW2, CPW7 and CPW8, respectively. This is because of variation in aquifer hydraulic conductivity properties and boundary conditions which govern groundwater pumping rates and consequently the conjunctive water use withdrawal rates. It should be realized that quaternary catchment with combined pumping well CPW4 falls within a relatively low magnitude of hydraulic conductivity value (within Tzaneen area see Figure 5.6) while quaternary catchments with combined pumping wells CPW1, CPW2, CPW7 and CPW8 fall within a relatively high magnitude of hydraulic conductivity value.

Concluding Remarks

It was revealed that the difference between the optimized withdrawal rates and the existing un-optimized withdrawal rate scheme is high. This tends to indicate that there is potential volume of groundwater resource in the study area aquifer water system (particularly, within quaternary catchments B81G, B81H, B81J and B81D) which is undeveloped. Further, it was observed that surface water source contributed higher percentage to the total optimal conjunctive water use withdrawal volume rate than groundwater source. However, the main drawback of tackling such optimization problem deterministically is the assumption that data are adequately available and precisely known. This assumption is practically invalid because in real world, management systems always have uncertainties.

5.4.2 Stochastic Conjunctive Water use Management

To address uncertainty of the study area for conjunctive water use management, the optimization problem was solved while assuming heterogeneous aquifer properties were uncertain. The stochastic conjunctive water use optimisation problem was solved and evaluated through Retrospective Optimization Approximation (ROA) framework (i.e., the methodology developed in Chapter three).

Description of the stochastic optimization problem

In this research, a total of 500 realizations of uncertain hydraulic conductivity fields (which leads to different aquifer system responses (i.e., hydraulic heads/water table level drawdowns)) were generated from the heterogeneous aquifer system. A correlation length of 100000m by 50000m in a 2-Dimensional x-, y-direction, respectively, was considered sufficient enough to capture significant representation of input parameter uncertainties.

In the aquifer system, eight (8) groundwater control points were identified active. The water heads at these control points were controlled not to fall below certain prescribed maximum allowable limiting values. Twenty (20) abstractions points from river system network were identified active and surface water levels (stages) at these abstraction points were controlled not to fall below riverbed bottom elevations.

Assemblage of aquifer system responses due to the 500 realizations of hydraulic conductivity fields resulted in a total of 4020 response matrix rows (observations rows). Hence, in total, a constraining response matrix of 4020 by 535 was generated. This response matrix was used to generate ten (10) sample path optimization sub-problems of different sample sizes (observation rows) for the management model. Sample sizes were also determined heuristically. Table 5.4 presents the description of formulation of sample path optimization sub-problems generated for the ROA method framework evaluation.

Table 5.4: Descriptions of Sample Path Optimization Sub-Problems

Sample Path Sub-Problem	**#Realizations**	**Response Matrix (#Rows/Constraints)**	**#Columns (#Decision Variables*)**
SOSP1	20	180	535
SOSP2	30	260	535
SOSP3	40	340	535
SOSP4	60	500	535
SOSP5	80	660	335
SOSP6	100	820	535
SOSP7	150	1220	535
SOSP8	200	1620	535
SOSP9	250	2020	535
SOSP10	500	4020	535

*Decision variables for conjunctive water use withdrawal rates (in which, the first 515 decision variables/columns are for groundwater withdrawal rates and the remaining 20 decision variables /columns are for surface water withdrawal rates).

The sequence of 20, 30, 40, 60, 80, 100, 150, 200, 250, 500 realizations of hydraulic conductivity fields generated a sequence of 180, 260, 340, 500, 660, 820, 1220, 1620, 2020, and 4020 of constraints, respectively (i.e., sample rows/observation rows, excluding total recharge constrain). This sequence of constraints generated the corresponding ten (10) sample path optimization sub-problems in a sequence of increasing number of rows (including aquifer total recharge constrain) of 181, 261, 341, 501, 661, 821, 1221, 1621, 2021, and 4021 (excluding lower and upper bounds, and nonnegative bounds constraints). The last sample path optimization sub-problem (i.e., SOSP10) is considered to be the true conjunctive water use optimization problem.

To enable the comparison between the deterministic and stochastic model results within the same constraints, the constraining values were set equal to those of the optimal deterministic scheme. In order to present the results in different graphical formats, the optimal conjunctive water use groundwater withdrawal rates solutions obtained were sorted out according to their locations in the model domain and their associated quaternary catchments, and then for each quaternary catchment polygon a geometrical mean of the optimal withdrawal rates was computed. Table 5.5 summarizes the ROA conjunctive water use sample path optimization sub-problems optimal mean withdrawal rates.

Table 5.5: ROA Conjunctive Water Use Sample Path Optimization Sub-Problems Optimal Mean Withdrawal Rates

Named Variable	**SOSP1R20**	**SOSP2R30**	**SOSP3R40**	**SOSP4R60**	**SOSP5R80**	**SOSP6R100**	**SOSP7R150**	**SOSP8R200**	**SOSP9R250**	**SOSP10R500**
CPW1	352.17	528.25	645.64	733.68	804.11	862.81	913.12	957.14	996.27	1031.49
CPW2	510.49	765.73	935.89	1063.52	1165.61	1250.69	1323.62	1387.43	1444.15	1495.20
CPW3	184.82	277.23	338.84	385.05	422.01	452.82	479.22	502.32	522.86	541.34
CPW4	78.58	117.88	144.07	163.72	179.43	192.53	203.76	213.58	222.31	230.17
CPW5	157.73	236.59	289.17	328.60	360.14	386.43	408.96	428.68	446.20	461.98
CPW6	289.42	434.13	530.60	602.96	660.84	709.08	750.43	786.60	818.76	847.70
CPW7	385.75	578.62	707.20	803.64	880.79	945.08	1000.19	1048.40	1091.26	1129.84
CPW8	464.06	696.09	850.78	966.79	1059.61	1136.95	1203.25	1261.25	1312.82	1359.22
CSWD	12412.70	18619.05	22756.62	25859.79	28342.33	30411.11	32184.36	33735.94	35115.13	36356.40
Σ	**14835.72**	**22253.57**	**27198.81**	**30907.74**	**33874.88**	**36347.50**	**38466.89**	**40321.36**	**41969.77**	**43453.36**

From Table 5.5, it can be observed that optimal conjunctive water use objective functions mean values range from a minimum of 14835.72m^3/day to a maximum of 43453.36 m^3/day. It is interesting to note that the optimal conjunctive water use mean withdrawal volume rate solutions corresponding to the 500 hydraulic conductivity realizations are different from one another as well as from the deterministic solution values (see Tables 5.3 and 5.5 results). This is because the optimal withdrawal volume rates designed based on ROA approach depend on the outcomes of the uncertainty realizations. Figure 5.14 shows the stochastic conjunctive water use sample path optimization sub-problems optimal solutions.

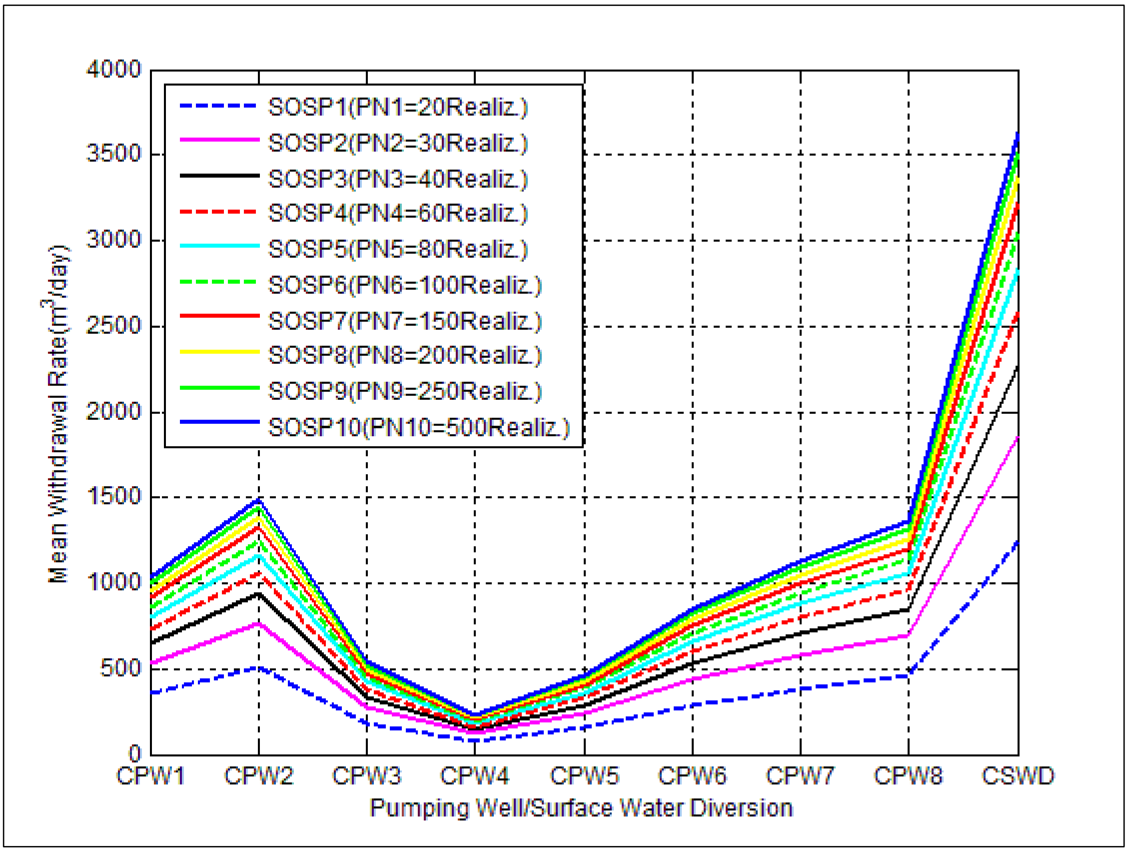


Figure 5.14: Stochastic Conjunctive Water Use Sample Path Optimization Sub-Problems Optimal Solutions

Figure 5.14 indicates that the sample path optimization sub-problems solutions converge to the true conjunctive water use optimization problem (i.e., SOSP10) as the sample size increases. As it can be observed from Figure 5.14 that after 30 realizations/sample sizes the sample path optimization sub-problems solutions are very close to true optimization problem solution. This is because statistically, sample sizes below 30 are considered as the small, hence, beyond which the convergence gains momentum and therefore, through ROA approach it converges rapidly because the optimization process takes the advantage of “warm start” technique whereby initial solution of subsequent sub-problem is the solution of the current sub-problem.

As it was expected from Figure 5.14, it can be seen that the ninth optimal solution (SOSP9) is almost equal to the true (original) optimization problem (SOSP10). Thus, the tenth sample path optimization problem (SOSP10) (whereby all the 500 realizations generated were considered), converged with a relatively few number of iterations. This is because the initial solution (guess) of problem (SOSP10) is the solution of optimization sub-problem SOSP9 which is probably nearly equal to the true optimization problem (SOSP10) optimal solution. Figure 5.15 presents histogram diagram for the true optimization problem (SOSP10) solution.

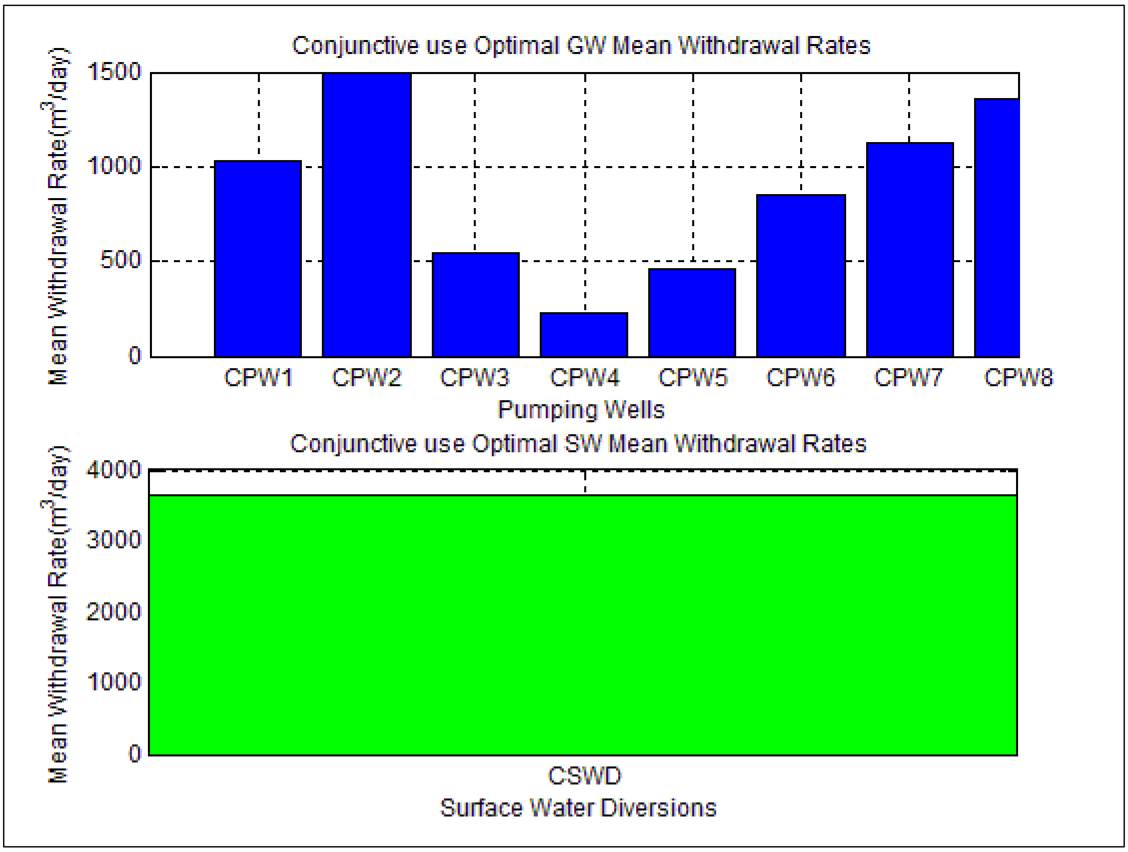


Figure 5.15: Optimal Conjunctive Water Use Surface Water and Groundwater Withdrawal Rates

In Figures 5.14 and 5.15, the highest groundwater withdrawal rates occurred within the quaternary catchments B81G, B81H, B81J, B81D with combined pumping wells CPW1, CPW2, CPW7, CPW8 (see Table 5.1) while the lowest volume rate occurred at quaternary catchment B81C with combined pumping well CPW4. High differences in groundwater pumping rates are due to variations in aquifer hydraulic conductivity values and boundary conditions of the model domain.

In order to evaluate the overall performance of the ROA method framework for the study area, the sample path optimization sub-problems were solved with different initial solutions (guesses) for three runs, which consequently resulted in different optimal solutions and, therefore, different expected objective function values. The results for the expected total optimal conjunctive water use withdrawal rates evaluated over 500 realizations for three runs were then averaged. Figure 5.16 presents the overall performance of the ROA method framework with cluster sampling for the study area evaluated over 500 realizations.

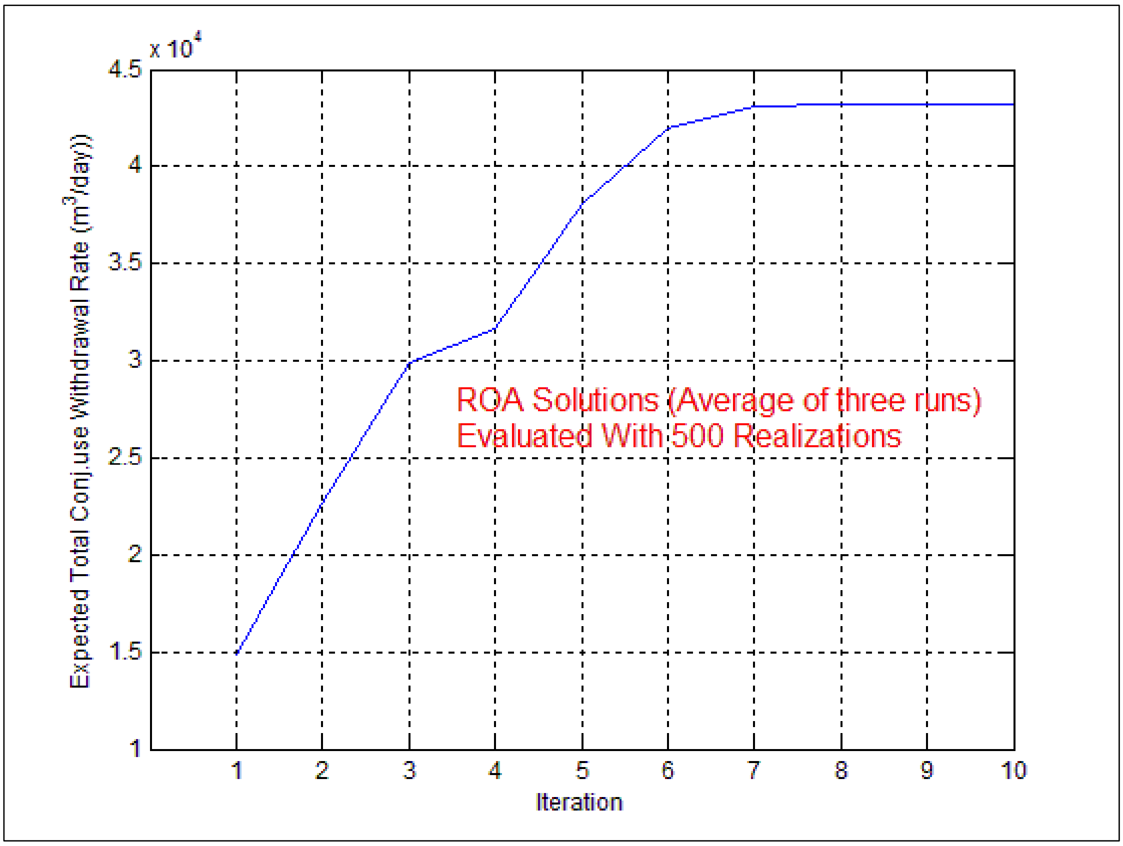


Figure 5.16: Overall Performance of ROA with Cluster Sampling for the Study Area Conjunctive Water Use Management

From Figure 5.16, the ROA expected total optimal conjunctive water use withdrawal volume rate (average of three runs) converged to its maximum mean value of about 4.35 x 10^4 m^3/day within 7 to 8 iterations. Figure 5.17 shows ROA overall percentages contribution of surface water and groundwater to the optimal conjunctive water use withdrawal rate in a pie chart view.

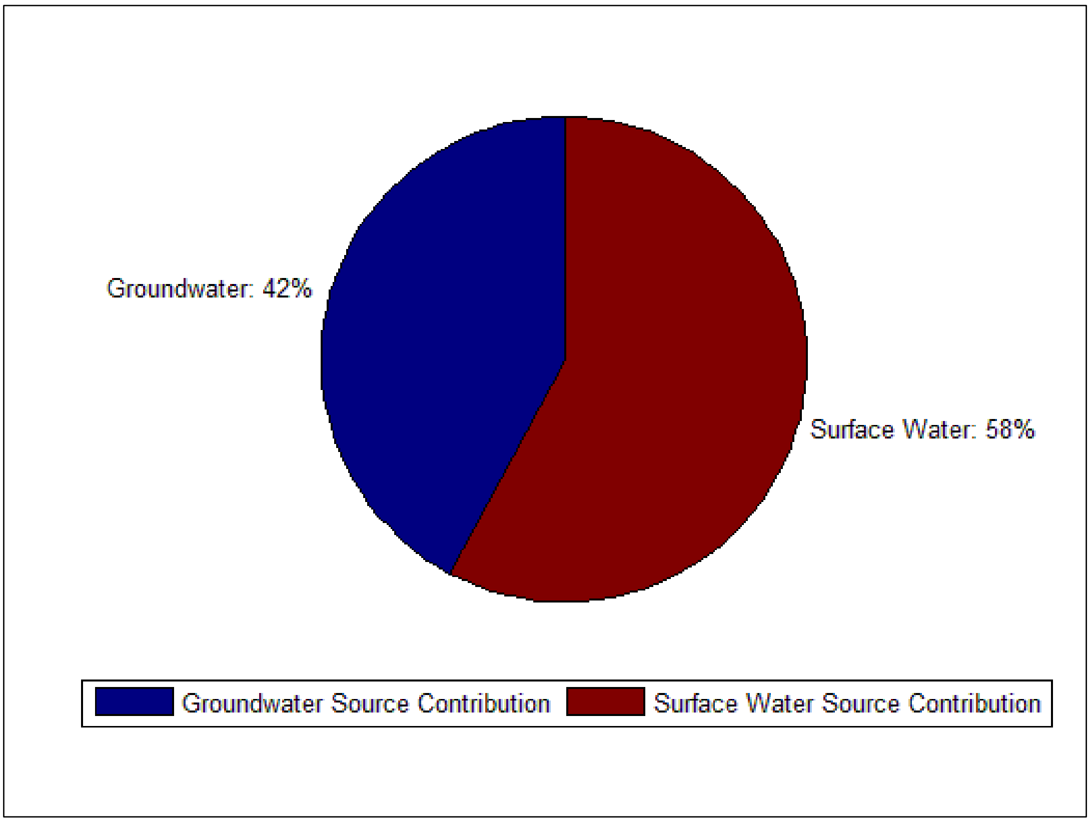


Figure 5.17: ROA Overall Percentages Contribution of Surface Water and Groundwater Sources to the Total Optimal Conjunctive Water Use

Figure 5.17 indicates that the overall percentages contribution of surface water and groundwater sources to the ROA optimal solution was about 58 percent and 42 percent, respectively. This implies that if ROA technique would be adopted for used in the study area, groundwater source can optimally (sustainably) be able to contribute up to 42 percent to the total water requirement. This is an increase of about 29 percent from the existing un-optimized contribution of about 13 percent. However, surface water source contribution would be reduced by 29 percent from the existing un-optimized value of about 87 percent. Moreover, comparing the percentage of water sources contribution of ROA approach with that of the deterministic approach, it can be seen that there is an increase of contribution of groundwater source to the total optimal water demand of about 27 percent (i.e., from 15 to 42 percent) and a decrease of surface water source contribution of about 27 percent (i.e., from 85 to 58 percent). Figure 5.18 compares the stochastic-ROA True optimization problem solution with deterministic solution.

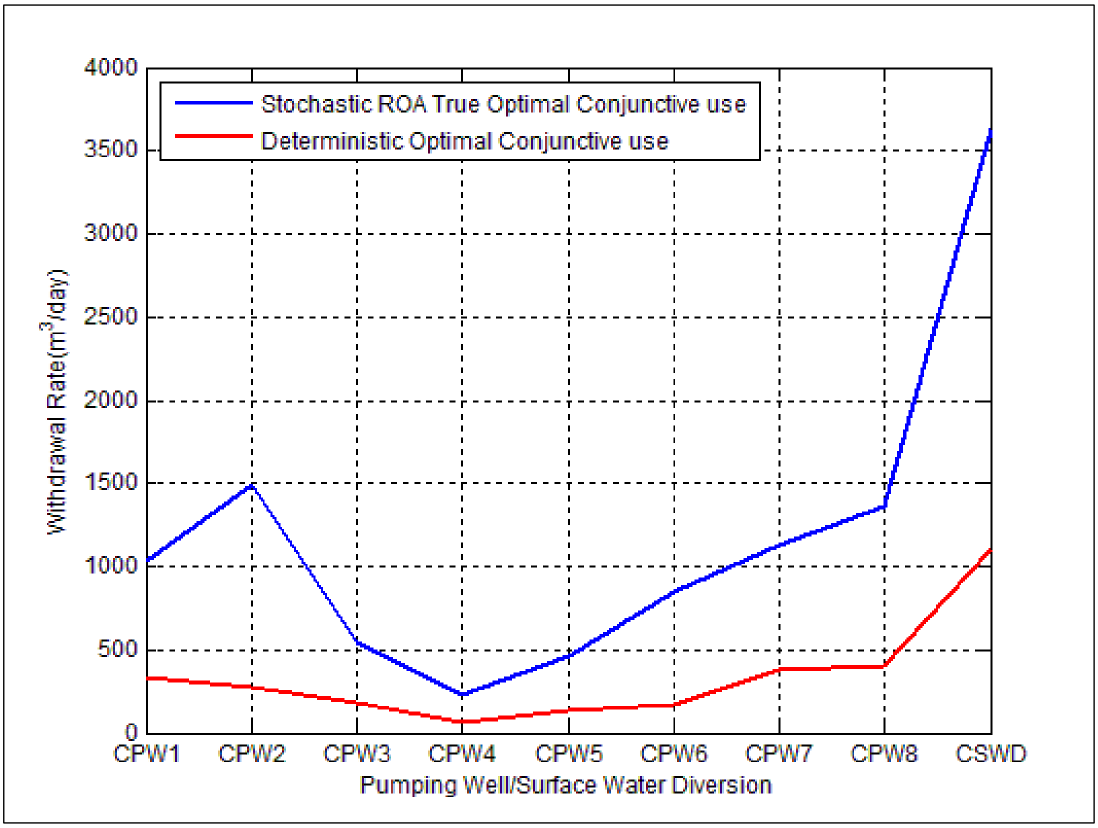


Figure 5.18: Comparison of Stochastic-ROA True Optimization Problem Solution with Deterministic Solution

It can be observed from Figure 5.18 that withdrawal rates of optimal solution designed based on stochastic optimization approach are higher than those of optimal solution designed based on deterministic approach. This is because optimal solution designed based on stochastic optimization approach depends on the outcomes of realisations of hydraulic conductivity values. Hence, it implies that deterministic approach solutions underestimated optimal solution withdrawal rates. Figure 5.19 compares existing un-optimized withdrawal rates with deterministic solution and ROA-True optimal solution.

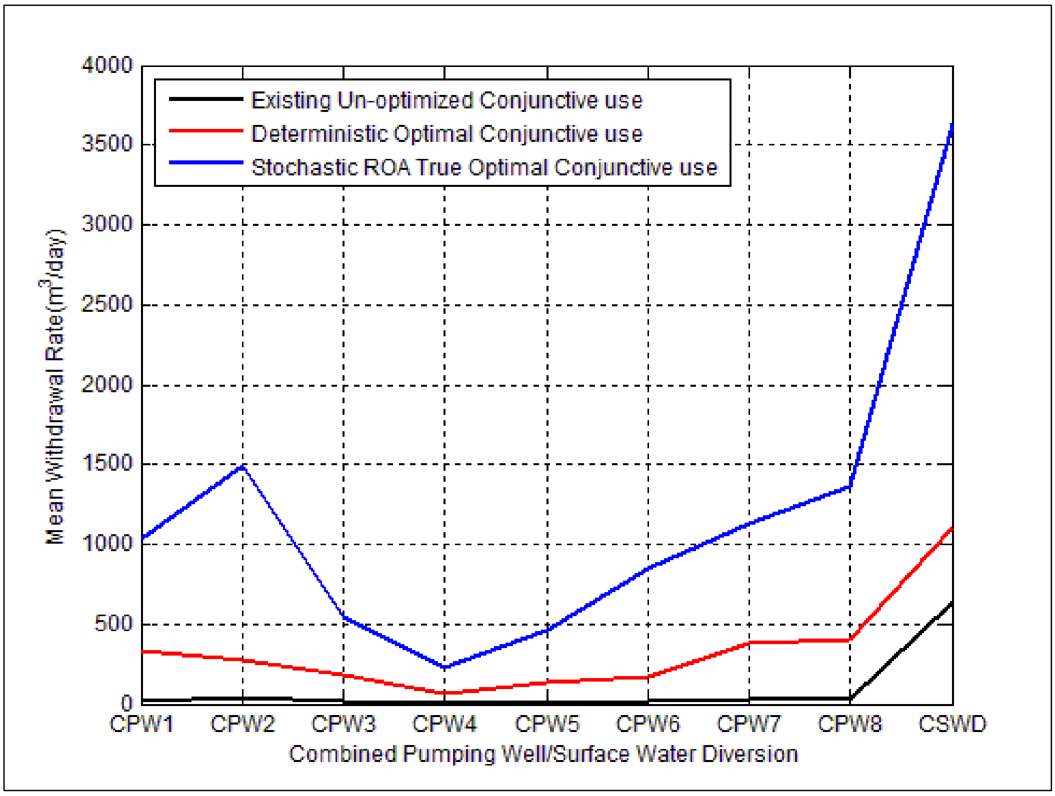


Figure 5.19: Comparison of Existing Un-Optimized Withdrawal Rates with Deterministic Solution and ROA-True Optimal Solution

From Figure 5.19, it can be noted that the optimal solution based on ROA approach results in higher withdrawal rates than those determined based on deterministic approach as well as the existing un-optimized withdrawal rates. This suggests that there is potential water resource undeveloped. It also means that optimal solution designed deterministically based on hydraulic conductivity mean values underestimates withdrawal rates. It can also be noted from Figure 19 that the pumping wells which had higher pumping rates in existing un-optimized pumping scheme also have higher values of pumping rates in deterministic and stochastic optimized schemes.

In Figures 5.18 and 5.19, it can be observed that high difference in groundwater withdrawal volume rates occurred in the quaternary catchments B81G, B81H, B81J, and B81D with combined pumping wells CPW1, CPW2, CPW7 and CPW8 respectively (see Table 5.1) while lower withdrawal volume rate occurred in quaternary catchment B81C with combined pumping well CPW4. This is because quaternary catchments with combined pumping wells CPW1, CPW2, CPW7 and CPW8 are characterized by hydraulic conductivity values of relatively high magnitude values while the quaternary catchment with combined pumping well CPW4 falls within a relatively low magnitude of hydraulic conductivity value.

Concluding Remarks

As it was previously found in hypothetical example (refer Chapter four), similarly in this real world case application, it was revealed that the optimal solutions designed based on ROA approach have higher withdrawal rates than optimal solutions designed based on deterministic optimization approach. This demonstrates further that deterministic approach underestimates withdrawal rates.

CHAPTER SIX: CONCLUSIONS AND RECOMMENDATIONS

The overall objective of this research was to develop a combined simulation optimization quantitative conjunctive water use management model which can sustainably manage water resources taking into account input parameter uncertainty. The broad aim was achieved through four specific objectives that sought: (1) to develop surface water conceptual model; (2) to develop groundwater conceptual model; (3) to integrate the surface water and groundwater conceptual models to develop a conjunctive water use conceptual model; (4) to test the conjunctive water use model by applying it to hypothetical example as well as a real world aquifer system. Principally, objectives one and two were designed to facilitate execution of objective three.

The conjunctive water use conceptual model developed based on objective three was designed to represent surface water and groundwater (aquifer) systems' hydrological and hydrogeological physical features in numerical simulation model. The simulation model was used to determine aquifer system responses (aquifer heads drawdowns) in deterministic and stochastic approaches. These responses were assembled as response matrices and incorporated into optimization (management) model through the response matrix technique. The fourth objective was achieved through application of the proposed methodologies (i.e., deterministic approach and stochastic approach – the Retrospective Optimization Approximation (ROA) method framework) to an hypothetical and real world case aquifer water systems and evaluation of their performance.

6.1 CONCLUSIONS

6.1.1 Development of Conceptual Models

In this research, surface water, groundwater and conjunctive water use conceptual models were developed to represent hypothetical and real world aquifer water systems. Model features of interest (including geometry, attributes and boundary conditions) were described using arcs, points, polylines, and polygons in a GIS database, and the process of discretizing these data sets to a grid was automated. GIS was used to generate river network shape files, groundwater geologic shape files and Digital Elevation Model (DEM) for regional aquifer water system. These shape files and DEM model were used to develop three conceptual models.

Based on objective one surface water conceptual modelling, hydrological components for estimating net recharge were used as an input into groundwater model. These hydrological components includes river network configuration of the system, water abstraction rates, location and rates of inflow and outflow.

Based on objective two groundwater conceptual modelling, sources and sinks of water, physical boundaries, distribution of hydraulic properties and groundwater use patterns were characterized. The overall objective was to develop groundwater conceptual model based on groundwater hydrogeological balance and the law of conservation of mass.

Based on objective three, the conjunctive water use conceptual model was developed by integrating the surface water and groundwater models. The basic idea behind the conceptual modelling was to develop simple but realistic conjunctive water use simulation model that could mimic as close as possible the physical systems natural behaviour when subjected to external stresses.

The conjunctive water use simulation model was built up using a modular three dimensional finite difference groundwater flow numerical model (MODFLOW-2000) together with RIVER Package (RIV) supported in Visual MODFLOW 2014.2 Classical Interface. The simulation-optimization problems were solved and analysed through “Active-Set” (Sequential Quadratic Programming (SQP) standard core optimizer (algorithm)) implemented under the MATLAB 2014a environment. The following sub-sections shall mainly dwell on conclusions based on results discussed in Chapters four and five.

6.1.2 Deterministic Conjunctive Water Use Management

In this approach, results from this study indicated that the optimized withdrawal rates solution were higher than un-optimized withdrawal rates. This suggests that there is potential volume of water resource in the river basin which is undeveloped. However, the drawback of tackling such optimization problems deterministically is the assumption that field measurements are adequate and known with high precision, which is practically uneconomical and infeasible.

6.1.3 Stochastic Conjunctive Water Use Management

ROA methodology was used to solve stochastic optimization problem. ROA framework solves and evaluates a sequence of optimization sub-problems in an increasing number of realizations. Results obtained based on stochastic approach (i.e., the ROA optimization approach) indicated that the optimal conjunctive water use solutions corresponding to different hydraulic conductivity realizations are different from one another as well as from the deterministic solution. The optimal strategies designed based on stochastic heterogeneous hydraulic conductivity fields were found to have relatively high withdrawal rates than optimal strategies designed based on deterministic homogeneous hydraulic conductivity field. This demonstrates that optimal solutions designed through deterministic approach underestimates the conjunctive water use withdrawal rates.

Moreover, results indicated that the overall percentages of contribution of surface water and groundwater sources to the total optimal water demand obtained through stochastic optimization approach – the ROA method was about 58% and 42%, respectively while the overall percentages contribution obtained through deterministic approach was about 85% and 15%, respectively. This is about $\pm$27% variation (i.e., differences between the approaches) of percentages of contribution of the two water sources to the total water demand. This therefore, demonstrates that through ROA management model the uncertainty realized was addressed.

Findings also indicated that if ROA approach would be adopted for use in the study area, the existing groundwater source percentage of contribution to the total water demand could increase up to 29% (i.e., from 13 to 42%) while the surface water source percentage of contribution could reduce by 29% (i.e., from 87 to 58%). This implies that currently the study area surface water system is water stressed and hence, reduction of 29% of the existing surface water source contribution could reduce stress on surface water system. This further demonstrates that ROA conjunctive water use management technique has potential to ensure sustainability of limited water resources of river basins.

Furthermore, it has been revealed that through ROA method the expected total optimal conjunctive water use objective function value converged to its maximum value within a relatively few iterations (6 to 8 iterations) in about 2.30Hrs computational time (Processor: Intel(R) Core (TM) i5-3320M CPU@2.60GHz, Memory: 4.00GB, Windows 7). In conclusion, results demonstrated that the ROA-stochastic optimization approach is a promising technique for use in managing conjunctive water use under uncertainty conditions.

6.2 RECOMMENDATIONS FOR FUTURE RESEARCH

Although our results demonstrated that the ROA approach is a promising technique for use in managing conjunctive water use under geological uncertainty, the following recommendations are thought useful.

i) Future research should be focused toward the establishment of guidelines for the determination of the sequence of sample sizes to be used in ROA framework;

ii) Explore the use of parallel computer processors so as to enhance the computational efficiency in terms of computational time for large problems;

iii) Explore quantitative methods for determination of the weights to be used in estimating values of objective functions; and

iv) Application of the approach to multi-objectives optimization problems.

REFERENCES

AGUADO, E. & REMSON, I. 1974. Groundwater hydraulics in aquifer management. *Journal of Hydraulics Division, ASCE,* 100(HY1): 103-118.

AHMED, S. & SHAPIRO, A. 2008. Solving chance-constrained stochastic programs via sampling and integer programming. *Tutorials in Operations Research*, 27(3): 261–269.

AJAMI, N. K., HORNBERGER, G. M. & SUNDING, D. L. 2008. Sustainable water resource management under hydrological uncertainty. *Water Resource Research*, 44(W11406).

ASTM INTERNATIONAL. 2002. Standard guide for defining boundary conditions in groundwater flow modelling. *Designation: D5609-94 Reapproved 2002*.

BAKR, M.I. 1995. A Multi-objective groundwater management approach: A case study of a Quasi-3Dimensional aquifer system, Vinaroz-Peniscola plain. MSc. thesis, International Institute for Aerospace Survey and Earth Sciences, Department of Water Resources, Enscheda, The Netherlands.

BANTA, E. R., POETER, E. P., DOHERTY, J. E. & HILL, M. C. 2006. JUPITER: Joint universal parameter identification and evaluation of reliability-an application programming interface (API) for model analysis, 6-E1.

BARDOSSY, M. G. & RAGHAVAN, S. 2017. An inexact sample average approximation approach for the stochastic connected facility location problem. *Networks*.

BARLOW, P. M. & GRANATO, G. E. 2007. Simulation-optimization modelling to assist conjunctive management of stream – aquifer systems of Rhode Island. *Issue Focus: Conjunctive water resources management, USA Groundwater News*, Nov. 2007, 4(2).

BARLOW, P. M., AHLFELD, D. P. & DICKERMAN, D. C. 2003. Conjunctive – management models for sustained yield of stream-aquifer systems. *Journal of Water Resources Planning Management,* 129(1): 35 – 48.

BASAGAOGLU, H. & MARIÑO, M. A. 1999. Joint management of surface and groundwater supplies. *Groundwater Journal*, 37(2): 214 – 222.

BASAGAOGLU, H., MARIÑO, M. A. & SHUMWAY, R. H. 1998. Stochastic conjunctive management of water resource in Yolo County. *University of California Water Resources Center*.

BEAR, J., BELJIN, M. S., & ROSS, R. R. 1992. Fundamentals of groundwater modelling. *Groundwater Issue*, Apr. 1992, EPA/540/S-92/005.

BETTER, M., GLOVER, F., KOCHENBERGER, G. & WANG, H. 2008. Simulation optimization: Applications of in risk management. *International Journal of Information Technology and Decision Making*, 7(4): 571-587.

BOISON, M. & CABAL, A. 1998. 3D Hydraulic objects for groundwater flow and contaminant migration modelling. In: *Proceedings of the Third International Conference on Hydroinformatics*, August 24 – 26, Copenhagen, Denmark, 1: 561-565.

BREDENKAMP, D. B., BOTHA, L. J., VAN TONDER, G. J. & VAN RENSBURG, H. J. 1995. Manual on quantitative estimation of groundwater recharge and aquifer storativity: based on practical hydrological methods. *Water Research Commission, South Africa*.

CAPE TOWN GREEN MAP. 2009. South Africa running out of surface water. Available from: http://www.capetownngreenmap.co.za/ blog/south-africa-running-out-surface-water [Accessed: 15/10/2012].

CHEN, H. & SCHMEISER, B.W. 2001. Stochastic root finding via retrospective approximation. *IIE Transportation,* 33(3): 259-275.

CHEN, Y., OLIVER, D.S. & ZHANG, D. 2009. Efficient ensemble-based closed-loop production optimization. *SPE Journal*, 14(4): 634-645.

CZARNECKI, J. B., CLARK, B. R. & STANTON, P. G. 2003. Conjunctive use optimization model of the Mississippi River Valley Alluvial Aquifer of Southeastern Arkansas. USGS Water Resources Investigation Report 03-4233, US Geological Survey, Reston, Va.

DATTA, B. & HARIKRISHNA, V. 2005. Optimization applications in water resources systems engineering. *Indian Institute of Technology, Kanpur*.

DRAPER, A. J. 2001. Implicit stochastic optimization with limited foresight for reservoir systems. PhD Dissertation, Davis, University of California, USA.

DUDLEY, T. & FULTON, A. 2006. Conjunctive water management: What is it? Why consider it? What are the challenges?. University of California, USA.

DWAF. 1990. DEPARTMENT of Water Affairs and Forestry **see** SOUTH AFRICA. Water resources planning of the Letaba River basin: Study of development potential and management of water resources. SRK Inc. Report No. P B800/00/0290.

DWAF. 1994. DEPARTMENT of Water Affairs and Forestry **see** SOUTH AFRICA. Letaba water resource development: Pre-feasibility Study. SRK Inc. Report No. P B800/00/0294.

DWAF. 1998. DEPARTMENT of Water Affairs and Forestry **see** SOUTH AFRICA. The Groot Letaba water resource development: Feasibility Study. Volume 2, Report No. PB B910/00/0398.

DWAF. 2003. DEPARTMENT of Water Affairs and Forestry **see** SOUTH AFRICA. Luvuvhu/Letaba Water Management Area: Water Resources Situation Assessment. Main Report October 2003. Report No. P02000/00/0301.

DWAF. 2004. DEPARTMENT of Water Affairs and Forestry **see** SOUTH AFRICA. Letaba catchment reserve determination study-Specialist Report: River Hydraulics, Final December 2004.

DWAF. 2006a. DEPARTMENT of Water Affairs and Forestry **see** SOUTH AFRICA. Letaba catchment reserve determination study: Resource Units Report. Final February 2006. Pulles Howard & de Lange Inc. Report No. RDM/B800/01/CON/COMP/0704.

DWAF. 2006b. DEPARTMENT of Water Affairs and Forestry **see** SOUTH AFRICA. Letaba catchment reserve determination study: Hydrology support and water resource evaluation. Final February 2006. P D Naidoo and Associates. Report No. RDM/B800/01/CON/COMP/1104.

DWAF. 2006c. DEPARTMENT of Water Affairs and Forestry **see** SOUTH AFRICA. Letaba catchment reserve determination study: Groundwater Report, Final February 2006. WSM Civil Engineers, Hydrogeologists & Project Managers (pty) Ltd. Report No. RDM/B800/02/CON/ COMP/0504.

DWAF. 2008. DEPARTMENT of Water Affairs and Forestry **see** SOUTH AFRICA. Water for growth and development in South Africa, Version 6.

FAYAD, H., PERALTA, R. C. & FORGHANI, A. 2012. Optimizing reservoir – stream – aquifer interactions for conjunctive use and hydropower production. *Advances in Civil Engineering,* 2012: 1 -10.

FOSTER, S., VAN STEENBERGEN, F., ZULETA, J. & GARDUÑO, H. 2010. Conjunctive use of groundwater and surface water – from spontaneous coping strategy to adaptive resource management. *Sustainable Groundwater Management Contributions to Policy Promotion*, Strategic overview Series No.2.

GELHAR, L. W. 1993. Stochastic subsurface hydrology. Prentice-Hall, Inc.

GORELICK, S. M. 1983. A review of distributed parameter groundwater management modelling methods. *Water Resources Research,* 19(2): 305 – 319.

GORELICK, S. M. 1990. Large scale nonlinear deterministic and stochastic optimization: Formulations involving simulation of subsurface contamination. *Mathematical Programming,* 48: 19-39.

GRIP – LIMPOPO. 2013. GRIP Database for borehole point data for the Limpopo province. Available from: http://griplimpopo.co.za [Accessed: 25/05/2016].

HAFI, A. 2006. Conjunctive Water Management: Economic Tools for Evaluating Alternative Policy and Management Options. *Australian Bureau of Agricultural and Resource Economics*.

HAIMES, Y.Y. 1973. Integrated System Identification and Optimization for Conjunctive Use of Ground and Surface Water: Phase I. *Ohio State University. Water Resources Center*.

HANTUSH, M. S. M. & MARIÑO, M. A. 1989. Chance-constrained model for management of stream-aquifer system. *Journal of Water Resources Planning Management,* 115(3): 259-277.

HANTUSH, M. M. & MARIÑO, M. A. 1995. Continuous-time stochastic analysis of groundwater flow in heterogeneous aquifers. *Water Resources Research,* 31(3): 565-575.

HARBAUGH, A. W., BANTA, E. R., HILL, M. C. & MCDONALD, M. G. 2000. MODFLOW-2000, the US Geological Survey modular groundwater model user guide to modularization concepts and the groundwater flow process, OFR: Reston, Virginia, USA.

HO, C. C., YANG, C. C., CHANG, L. C. & YEH, M. S. 2007. System dynamics modelling of the conjunctive use of surface and subsurface water. In: *World Environmental and Water Resources Congress 2007*: *Restoring Our Natural Habitat*, 1-10.

HOLLAND, M. 2011. Hydrological characterization of crystalline basement aquifers within the Limpopo province, South Africa. PhD thesis, University of Pretoria, South Africa.

HOMEM-DE-MELLO, T. & BAYRAKSAN, G. 2014. Monte Carlo sampling-based methods for stochastic optimization. *Surveys in Operations Research and Management Science*, *19*(1): 56-85.

HOMEM-DE-MELLO, T. 2003. Variable sample methods for stochastic optimization. *ACM Transportation Modelling Computation Simulation,* 13: 108 – 133.

HORNBY, D., VANDERHAEGHEN, Y., VERSFELD, D. & NGUBANE, M. 2016. A harvest of dysfunction: Rethinking the approach to drought, its causes and impacts in South Africa. Report October 2016, Oxfam South Africa.

HUGHES, D. A. 2005. Modelling semi-arid and arid hydrology and water resources: the Southern African experience.

INDRIYANI, BANJARNAHOR, T. P., NABABAN, N., YUSLENI, SAMOSIR, A. H. & MAWENGKANG, H. 2010. An optimization model for water resources management under Uncertainty. In: *Proceedings of the 6th IMT-GT Conference on Mathematics, Statistics and its Applications (ICMSA2010),* University Tunku Abdul Rahman, Kuala Lumpur, Malaysia.

IWMI. 2007. Improving the Management of Land and Water Resources for Food, Livelihoods and the Environment. International Water Management Institute, Olifants South Africa.

JOHNSON, E. 2010. Conceptual water model for the Horn River Basin, Northeast British Columbia. *Geoscience Reports,* 99 – 121.

JOLLY, I., RASSAM, D., PICKETT, T., GILFEDDER, M. & STENSON, M. 2010. Modelling groundwater-surface water interactions in the new generation of river systems models for Australia. *CD of Proceedings of Groundwater*.

JONES, N.L., BUDGE, T.J., LEMON, A.M. & ZUNDEL, A.K. 2002. Generating MODFLOW grids from boundary representation solid models. Groundwater, 40(2): 194-200.

JONES, N.L., LEMON, A.M. & TALBOT, C.A. 2000. Integrating GIS data with 3D finite element groundwater models. In: *Proceedings of the International Symposium 2000 on Groundwater*, May 8-10, Saitama, Japan, 327-332.

KATAMBARA, Z. 2011. Fuzzy and Conceptual – Fuzzy modelling of complex river systems with Scarce Data: Case of Letaba River. PhD thesis, Faculty of Engineering and the Built Environment, Johannesburg, University of the Witwatersrand, South Africa.

KIFANYI, G. E., NDAMBUKI, J. M. & ODAI, S. N. 2017. A quantitative groundwater resource management under uncertainty using a retrospective optimization framework. *Sustainability* 2017, 9(2).

KROM, T. D. & LANE, R. 2009. A novel approach to groundwater model development. In: *World Environmental and Water Resources Congress 2009: Great Rivers*, 1-9.

LABADIE, J. W. 1993. Combining simulation and optimization in river basin management. In: *Stochastic hydrology and its use in water resources systems simulation and optimization*, 345-371.

LE ROY, E. 2005. A study of the development of water resources in the Olifants River catchment, South Africa: Application of the WEAP Model. MSc. thesis and for the Diploma of Imperial College London.

LEON, E. & TY FERRÉ. 2003. A Conceptual model of groundwater flow in the Upper Agua Fria. *Arizona Department of water resources*.

LETTENMAIER, D. P. & BURGES, S. J. 1979. Reliability of cyclic surface and groundwater storage systems for water supply: A preliminary assessment. *National Technical Information Service, Springfield VA 22161 as PB 80-180870, Price codes: A 04 in paper copy, A 01 in microfiche Technical Report*, 64.

LÉVITE, H., SALLY, H. & COUR, J. 2003. Testing water demand management scenarios in a water – stressed basin in South Africa: Application of the WEAP model. *Physics and Chemistry of the Earth*, 28: 779-786.

MAHJOUB, H., MOHAMMADI, M. M. & PARSINEJAD, M. 2011. Conjunctive use modelling of groundwater and surface water. *Journal of Water Resource and Protection*, 3: 726-734.

MANDLE, R. J. 2002. Groundwater modelling guidance: Groundwater modelling program. Michigan Department of Environmental Quality, Draft 1.0, No.10/16/02.

MARIÑO, M. A. 2001. Conjunctive management of surface water and groundwater: Regional Management of Water Resources. In: *Proceedings of a Symposium, held during the Sixth IAHS, Scientific Assembly at Maastricht,* The Netherlands, July 2001, (268): 165 – 173.

MAYER, A. & MUÑOZ-HERNANDEZ, A. 2009. Integrated water resources optimization models: An assessment of a multidisciplinary tool for sustainable water resources management strategies. *Geography Compass*, 1176 – 1195.

MAZVIMAVI, D. 2003. Estimation of flow characteristics of ungauged catchments: Case study in Zimbabwe. PhD thesis, Wageningen University.

MCKINNEY, D. C., CAI, X., ROSEGRANT, M. W., RINGLER, C. & SCOTT, C. A. 1999. Modelling water resources management at the basin level: Review and future directions. *SWIM Paper 6, Colombo*.

MEIGH, J. R., MCKENZIE, A. A. & SENE, K. J. 1999. A grid-based approach to water scarcity estimates for eastern and southern Africa. *Water Resources Management*, *13*(2): 85-115.

MOK, C. M. 2007. Optimizing integrated water supply operations under uncertainty. *Groundwater News and Views USA*, 4(2).

NDAMBUKI J. M., OTIENO, F., STROET, C. B. M., TERLAKY, T. & VELING, E. J. M. 2005. Second-Order Cone optimization approach to groundwater quantity management. In: *Advanced Optimization Laboratory, McMaster University*.

NDAMBUKI, J. M., OTIENO, F., STROET, C. B. M., TERLAKY, T. & VELING, E. J. M. 2000. Groundwater management under uncertainty: A Multi-objective approach. *Water South Africa*, 26 (1).

NDAMBUKI, J. M. 2001. Multi-objective groundwater quantity management: A stochastic approach. PhD thesis, Delft University, The Netherlands.

NDAMBUKI, J.M., OTIENO, F., STROET, C.B.M., TERLAKY, T. & VELING, E.J.M. 2003. An efficient optimization method in groundwater resource management. *Water SA*, 29(4): 359 – 363.

ÖZDOGAN, U. & HORNE, R. N. 2006. Optimization of Well Placement under Time – Dependent Uncertainty. *SPE Research Evaluation Engineering,* 9: 135-145.

PALMER, J. B., BERGMANN, K. & SCHULTZ, C. L. 2007. Ground-water/stream flow model of the Monocacy River Basin, Maryland and Pennsylvania. Phase I: Steady-state model, *ICPRB Report,* 07-5.

PASUPATHY, R. 2010. On choosing parameters in retrospective – Approximation algorithms for stochastic root finding and simulation optimization. *Operation Research*, 58(4): 889 – 901.

PASUPATHY, R. & SCHMEISER, B. W. 2009. Retrospective approximation algorithms for multidimensional stochastic root finding problems. *ACM TOMACS,* 19(2): 1 – 36.

PECK, A., GORELICK, S., DE MARSILY, G., FOSTER, S. & KOVALEVSKY, V. 1988. Consequences of spatial variability in aquifer properties and data limitations for groundwater modelling practice. International Association of Hydrological Sciences, 175.

PERALTA, R. C. 2001. Optimal conjunctive water management in different legal and hydrologic regimes. Department of Biological and Irrigation Engineering, Logan UT, Utah State University, USA.

PERALTA, R. C., AZARMNIA, H. & TAKAHASHI, S. 1991. Embedding and Response matrix techniques for maximizing steady – state groundwater extraction: Computational comparison. *Groundwater Journal,* 29(3).

POOJARI, C. A. & VARGHESE, B. 2008. Genetic algorithm based technique for solving chance constrained problems. *European journal of operational research*, *185*(3): 1128-1154.

RANDELL, B. N. 1999. A review of conjunctive use and a proposed model. Water Systems Research Group, University of the Witwatersrand, South Africa.

ROBERTS, S. V. 2010. Conjunctive surface water and groundwater management: A new framework for strategic decision-making. PhD Dissertation, Texas State University, USA.

SAFAVI, H. R. & BAHREINI, G. R. 2009. Conjunctive simulation of surface water and groundwater resources under uncertainty. *Iranian Journal of Science and Technology*, 33(B1): 79-94.

SAFAVI, H. R., DARZI, F. & MARIÑO, M. A. 2009. Simulation – optimization modelling of conjunctive use of surface water and groundwater. *Water Resources Management*, 24: 1965-1988.

SAHINIDIS, N. V. 2004. Optimization under uncertainty: State – of – the – art and opportunities. *Computers and Chemical Engineering,* 28: 971-983.

SAHUQUILLO, A. & LLURIA, M. 2002. Conjunctive use as potential solution for stressed aquifers: Social constraints, Chapter 7.

SALIS, F., SECH, G. M. & ZUDDAS, P. 2006. Optimization model for the conjunctive use of conventional and marginal waters. CINSA-University of Cagliari Italy.

SAMI, K. & HUGHES, D. A. 1996. A comparison of recharge estimates to a fractured sedimentary aquifer in South Africa from a chloride mass balance and an integrated surface-subsurface model. *Journal of Hydrology,* 179: 111-136.

SHAPIRO, A. & WARDI, Y. 1996. Convergence analysis of stochastic algorithms. *Mathematics of Operation Research*, 21(3): 615-628.

SHAPIRO, A., HOMEM-DE-MELLO, T. & KIM, J. 2002. Conditioning of convex piecewise linear stochastic Programs. *Mathematical Programming,* 94(1): 1-19.

SHAPIRO, A. 2001. Monte Carlo simulation approach to stochastic programming. In: *Simulation Conference, 2001. Proceedings of the Winter IEEE*, 1: 428-431.

SIMONOVIC, S. P. 2009. Managing water resources: Methods and Tools for a systems approach, UK, UNESCO.

STILL, D. A., DICKENS, C., BREEN, C. M., MANDER, M. & BOOTH, A. 2010. Balancing resource protection and development in a highly regulated River: The role of conjunctive use. *Water SA,* 36(3).

THOMAS, A. 2015. Modelling of spatially distributed surface runoff and infiltration in the Olifants River catchment/water management area using GIS. *International Journal of Advanced Remote Sensing and GIS,* 4 (1): 828 – 862.

TODD, D. K. & MAYS, L. W. 2005. Groundwater Hydrology, Third Edition, John Wiley and Sons, Inc., USA.

TRAN, T. M. 2004. Multi-objective management of saltwater intrusion in groundwater: Optimization under uncertainty. MSc. thesis, Wageningen Agricultural University, Viet Nam.

TYAGI, N. K., SRINIVASLU, A., KUMAR, A. & TYAGI, K. C. 1995. Modelling conjunctive use of water resources: Hydraulic and Economic Optimization. In: *Bulletin No.6/95, CSSRI, Karnal*. Available From: http://krishikosh.egranth.ac.in/ bitstream /1/2046491/1/CSSRI201.pdf [Accessed: 28/06/2015].

UNDERWOOD, K. 2009. South Africa already using 98% of surface water. Available from:http://www.treehugger.com/clean-water/south-africa-already-using-98-of-surface-water.html [Accessed: 20/10/ 2012].

VAN LEEUWEN, M., TE STROET, C. B. M., BUTLER, A. P. & TOMPKINS, J. A. 1998. Stochastic determination of well capture zones. *Water Resources Research,* 34(9): 2215-2223.

VUCETIC, D. & SIMONOVIC, S. P. 2011. Water resources decision making under uncertainty. University of Western Ontario, Report No. 073.

WAGNER, J. M. & GORELICK, S. M. 1989. Reliable aquifer remediation in the Presence of spatially variable hydraulic conductivity: From data to design. *Water Resources Research*, 25: 2211–2225.

WAGNER, J. M., SHAMIR, U. & NEMATI, H. R. 1992. Groundwater quality management under uncertainty: Stochastic programming approaches and the value of information. *Water Resources Research,* 28: 1233–1246.

WAGNER, J. M., SHAMIR, U. & MARKS, D. H. 1994. Containing groundwater contamination: Planning models using stochastic programming with recourse. *European Journal of Operational Research*, 77: 1-26.

WANG, W. & AHMED, S. 2008. Sample average approximation of expected value constrained stochastic programs. *Operations Research Letters*, *36*(5): 515-519.

WANG, C., LI, G. & REYNOLDS, A.C. 2009. Production Optimization in Closed-Loop Reservoir Management. SPE Journal. 14(3): 506-523.

WANG, H., CIAURRI, D. E., DURLOFSKY, L. J. & COMINELLI, A. 2012. Optimal well placement under uncertainty using a retrospective optimization framework. *SPE Journal*, March 2012.

WANG, H.G. & SCHMEISER, B.W. 2008. Discrete Stochastic Optimization using Linear Interpolation. In: *Proceedings Winter Simulation Conference 2008*, Piscataway, New Jersey, IEEE Press, 502-508.

WARBURTON, M. 2011. Surface water - hydrologically relevant spatial data and projections. In: *SARVA eNewsletter,* School of Bioresources Engineering and Environmental Hydrology, University of KwaZulu-Natal, South Africa.

WATERWIKI.NET. 2006. Water: A shared responsibility.

WEIHUA LI. 2011. Uncertainty reduction in hydrologic modelling and regional water management utilizing inter-basin transfer. PhD Dissertation, Raleigh, North Carolina State University.

WILLIS, R. & YEH, W. G. 1987. Groundwater systems planning and management. Practice-Hall, Inc.

WOLTDT, W., BOGARDI, I., KELLY, E. W. & BARDOSSY, A. 1990. Simulation and analytical tools for modelling groundwater quality: Computational methods in subsurface hydrology. In: *Proceedings of the 8th International Conference on Computational Methods in Water Resources*, Venice Italy, 11-15.

WOODFORD, A., ROSEWARNE, P. & GIRMAN, J. 2007. How much groundwater does South Africa have?.

YETEN, B., DURLOFSKY, L. J. & AZIZ, K. 2003. Optimization of Nonconventional Well Type, Location, and Trajectory, *SPE Journal*, 8(3): 200-210.

YEVJEVICH, M. 1979. Conjunctive water use. *Water International*, 4(3): 17-30.

YOUNG, R. A. & BREDEHOEFT, J. D. 1972. Digital computer simulation for solving management problems of conjunctive groundwater and surface water systems. *Water Resources Research*, 8 (3): 533-556.